so welcome all of you for the current lecture on gravitation
So this is a good time for us to recollect all that we have done
So far what we did was to first start with estimation of distances of astronomical bodies from the earth prior to that actually we also looked at how to estimate the radius of the earth assuming that it is perfectly spherical otherwise what you are going to get is the mean radius then of course we went on to discuss trigonometric methods for estimating the sizes of the moon the sun and the planets this required extensive and careful observations of the orbits of the moon the sun and the planets around the earth over many many centuries and by using simple plane trigonometry we saw that all these could be estimated and then what we did was to enumerate the planetary loss of motion planetary loss of motion which is due to kepler
So these loss of motion are very very important for us because there was a shift in the frame of reference whereas earlier everyone tried to describe find out or discover the systematics in the planetary orbits by looking at it from the earth frame of reference kepler shifted from the earth centered frame heliocentric frame kepler shifted to from the earth centered frame to the heliocentric frame and there he got an excellent fitting with elliptic orbits and he was able to enumerate the three laws
So if you remember the first law said that the orbits were all elliptical the second law said that the planets sweep equal areas in equal intervals of time and the third law related the period of the orbit orbiting planet to the distance from the sun t squared over r cube goes a constant and this came as a great surprise
So at this point we should remember that in india also there was the kerala school of astronomy who actually found that the algorithms for planetary orbits could be much simplified if one assumed the frame of reference which was fixed in the sun today it is a well established fact but anyway leaving that part of the history aside what we shall do is to continue with our revision
So these are the three laws that kepler obtained and to supplement it with dynamics because we want a theory of gravity we also discussed galilean law of falling bodies galilean law of falling bodies this law is very important for us because philosophically speaking although it may not matter much to us at this time it went against the aristotelian paradigm that later objects go upward heavier objects come downwards or to put it a little bit more quantitatively heavier objects fall faster than the lighter objects in the gravitational field of the earth but galerio did fairly careful experiments from the leaning tower of pisa
So he dropped two objects of two different masses he was intelligent enough to choose them such that the viscosity of the air the lift of the air is not very very important if you throw your piece of paper then of course it will not fall accord as given by the law of falling body its d will not be given by 10 meter per second squared or whatever he did that and we found that the acceleration is independent of the mass of the falling body
So we write m a equal to mg and we cancel and we get a equal to g this equation that i have written here m a equal to m j also brings out another very very fundamental fact or fundamental principle of physics namely that inertial mass is indistinguishable indistinguishable from gravitational mass
So we write m i equal to m g and that is how i was able to cancel and we had a fairly long discussion on this fact that m i equal to m g this is what is known as equivalence principle in physics and this is the bedrock of einstein's general theory of relativity we give up many many things but we do not give up mi equal to mg
So whatever i have listed
So far formed the basis for newton's formulation of law of gravitation
So newton looked at the galilean law of really falling bodies galilean law of freely falling bodies orbit of moon around the earth orbit of the moon around the earth orbits of planets around the sun
So this was the information that newton had and out of this he had to build a coherent theory all these were facts and the understanding was empirical there was no theoretical basis but newton provided a theoretical basis by using all these to formulate universal law of gravitation this was the first universal law the fundamental force to be discovered in physics i mean even today it is an extraordinarily fascinating interaction we do not completely understand that
So all of us know what the formulation of universal of gravitation is the most important thing is that it is an inverse square law and then there is a constant which characterizes the strength of the gravitation and that is the newton's gravitational

constant
So how do we write that if you have a body of mass m one if you have a body of mass m two
So let us say we ignore their sizes
So treat them as point masses and then if they are separated by a distance r
So what did we say we said that my force experienced by 2 because of 1
So 1 is acting on 2
So that is what we wrote was simply given by g m 1 m 2 by r square
So if i denote the unit vector r from m1 to m2 this will be directed towards m1 therefore i put r that is what i am going to do in a similar manner by making use of the third law the force experienced by mass m one because of m two
So how are we going to write two acts on one
So this is a good notation which causes no confusion this is nothing but minus of f 1 comma 2 this is what we have
So if we did that the only unknown quantity would be the universal constant this gravitational universal of the universal constant of gravitation and we also discussed how cavendish in his beautiful experiments was actually able to measure this g
So the value we go to cavendish and i gave a fairly long description of how cavendish was able to do his experiment in a barn trying to you know protect it from vibrations etcetera etcetera and he got a fairly good number please go back and listen to that and revise this part cavendish of course did not call it as the determination of the gravitational constant but he called it as weighing the mass of the weighing the earth finding the mass of this earth because this g is related to the mass of the earth through the acceleration due to gravity given by the galilean law of falling body
So that was a very great accomplishment because we cannot have ordinary balances in order to measure the masses but cavendish was able to do that
So essentially once you have gravitational law you can determine the masses of many many objects i discussed that also i told you how the sun can be made because of once we know g
So it is a great great accomplishment of the theory of gravitation
So much
So people thought that we have the key to almost all the secrets of nature perhaps all the secrets of nature
So there is a very famous poem due to alexander pope who wrote that nature and nature's laws lay hidden night god said let newton be and there was light
So it was newton who threw light on nature and her mysteries now all this has been a spectacular thing
So what we are going to do today as i discussed at the end of my last lecture is to show one very very important application of the law of gravitation and that is phenomena of tights
So all of us who have visited the seashore and spent some days and all of us who stay by the seaside we know that the height to which the water level rises or falls shows a regular pattern depending on the day depending on the time of the day and also depending on the phase of the moon it is of course intimately related to the face of the moon because the tides are the most spectacular on the full moon night and the new moon night there is a great difference between what happens during the day time and what happens during the night time and in almost all societies the moon has been associated with the mind and all kinds of powers
So for a long time people believed that the occurrence of trades was actually a supernatural phenomena a manifestation of the great power of the gods well it is indeed a manifestation of the great power but not of the gods but of nature if you replace the word of gods by nature and one of the important consequences or important applications of newton's law of gravitation is that it allows us to quantitatively understand tights in this lecture i am not going to tell you all about the quantitative nature because to do that would require lot more mathematical work and lot more information we have to know for example the compressibility of water
So on and
So forth
So it will be necessarily qualitative but i would also like to tell you that the other important thing that the phenomena of tides from the viewpoint of gravitation is that many times it is not the magnitude of the force that matters but it is the difference at two different points that matters

So tights are very peculiar in the sense they are not sensitive to the magnitude of the force but they are sensitive to the difference in the forces and this give rise to very interesting and surprising consequence and that is worth learning interestingly this kind of sensitivity to difference in gravitational force at two different points actually becomes very important in general theory of relativity

So i would like to go back to that and give you a an idea of what happens when you look at the tidal forces after describing the tides themselves

So let us start to get working on the tidal forces

So symbolically i am going to write earth as a very very big sphere

So the earth is treated as a sphere of radius r e the moon as we know is much closer to the earth than the sun

So let us say my moon which is a small object that is how we see on the sky

So is sitted here and let us say the sun which is far far away is here without any prejudice we are putting the sun at some other point that is what we have the moon has a mass which i will write mm

So i will write moon here the sun has a mass m s the distance between the moon and the earth i denote by dm and the distance between the sun and the earth i denote by ds when i say that i am looking at the distance between the sun and the earth what am i doing i am looking at the distance between the center of the earth with the sun we are not worried about the size of the sun because this one is far far away but if i move along the surface of the earth we see that the distance changes from d m minus r e to d m plus r e in a similar manner d s plus r e to d s minus r e

So there is a variation in the distance because of this variation in the distance there is going to be a variation in the force

So what are we saying for any point on the surface of the earth of the earth the true distance varies between d s m plus r minus r e that is what it is of course the distances dm and ds are much much larger than the radius of the earth

So it might appear that for all practical purposes this is of no consequence like for example when we look at the galilean law of falling bodies you may drop it from a height of 10 meters you may drop it from a height of 20 meters or even let us say 100 meters you would not care because the radius of the earth is about 6 or 6 400 kilometers

So we are speaking of 6.4 into 10 to the power of 5 meters

So you say 10 meter 20 meter 30 meter is of no consequence and that is the reason why we use small g the acceleration due to gravity without committing much error in a similar manner if you have to look at the radius of the earth and the distance between the moon and the sun it might appear to be a small correction but as i told you when we are looking at the difference in the forces this is of quite significance large significance and that is what we want to understand

So what i will do is to start with the moon earth force and then i will look at the sun earth force normally when we look at the earth moon system we always speak the language of the force exerted by the earth on the moon because the moon is in an orbit and earth is very very heavy

So at some point you will learn that both the earth and the moon are moving around their common center of mass but then the earth is

So heavy the center of mass is practically in the rest frame of the earth that is exactly what we do in the case of earth sun system it is practically in the sun in your hydrogen atom for all practical purposes the electron is moving around the proton that is what we do but here we are not interested in that we are actually interested in the force exerted by moon on the earth

So there is a shift in our emphasis

So we are interested in the force exerted by the moon on the earth now when i am making this statement this makes sense if i also remember that a large surface of the earth i think two thirds if i remember correctly large part of the surface of the earth is covered by water otherwise for us the earth is a rigid object therefore the difference between the force on different points on the surface of the earth would not matter because it is a rigid body the distance between different or the various points are fixed but the water that we are looking at is not rigid it is going to respond to the forces it is a fluid therefore we are interested in the gravitational force of the moon acting on the water part of the earth

So we are interested in the gravitational force acting on water now you see how it naturally connects to the concept of a tide that is something that you have to understand

So now let us look at what is going to happen i am going to draw the figure again quite exaggerated

So there is this small point which represents the moon

So as i told you this is my radius of the earth and this is dm distance between the moon and the earth i am going to plug in all the values numerical values of the radius and the distances and the masses at the end of the calculation but right now what we shall do is to find the difference between the force acting at this point and the force acting at this point

So i will call it as f1 and this i will call it as f1 prime that is what i will call this

So the moon is exerting a force at this point the moon is exerting a force at diametrically the opposite point its attractive force at both the points and the force of attraction at this point the closest point is larger than the force of attraction at this point because the fact that this is a farther away from the moon

So we have to write down the distances

So what are we going to write my f_1 i am only going to write down the magnitude we will not worry about the signs we know that it is attractive is simply given by g me the mass of the moon divided by d m minus r e whole square that is what we have in a similar manner if i were to write the force f one prime prime is at the farthest point what would be the force now that would again be g m e m m over d m plus r e whole square what would happen if it were the sun and not the moon well the mass of the moon would be replaced by the mass of the sun the distance of the sun from the earth ds would replace my dm

So that is what i would have and we are going to use it in the next step

So i am going to repeat whatever you people are thoroughly familiar with the magnitude of the force keeps on increasing when the mass increases mass of what i am interested in the force of the earth on the earth

So if i look at the sun the sun is much much heavier than the moon therefore it tends to increase the force on the earth but on the other hand if i look at the distances the sun is much farther away therefore the inverse square law tells me it tends to suppress the force

So what we are interested here is in the interplay between the competition between the masses and the distances larger mass but larger distance smaller mass but shorter distance

So we are interested in that and we are interested to see how it manifests when i look at the distance between the two forces i am interested in these forces

So what i am interested actually is in delta f1 which is f1 minus f1 prime this is what i am interested in at two different points two diametrically opposite points on the surface of the earth which are collinear with the location of the moon that is what i am interested in when i am doing this calculation i would like you people to remember that d is much much greater than r e d is much much greater than r a distance between the earth and the moon is of the order of a 10 to the power of 5 kilometers okay and here we are speaking of 6 400 kilometers

So and the sun of course is much much faster farther away

So what are we going to do in doing these calculations we make a binomial expansion

So this is always the trick whenever you have a small correction to a large number

So let me rewrite that again my f_1 is given by some constant k divided by d m minus r e whole square where k is gravitational constant mass of the earth and mass of the moon we are interested in the moon at this point

So what do i do i first obtain the mean force and then obtain the correction the mean force is what is acting at the center of the earth that is what i have therefore i will write k over d m squared into one minus r e by d m whole square that is what i have

So we are saying that r e over d m much much less than one this approximation is important for our understanding and eventually we are going to substantiate this claim by putting in the values

So let me open it up

So this i will write f naught k over d m square everything is fixed in this and i will write this as 1 over 1 minus 2 re by dm minus re square over dm square that is what i am going to write r a by d m is a small quantity r a squared by d m squared is an even smaller quantity

So 2 r a by d m minus r e squared by dm squared is a small correction to 1.

So let us write this as f naught over 1 over 1 minus x that is what we are going to write

So when x is very very small compared to 1 we know how to make a taylor expansion or a

binomial expansion r a by d m is certainly greater than r e squared by dm square x is a positive quantity that is what i have

So i will write 1 over 1 minus x is 1 plus x plus x square plus higher order terms you may wonder why i kept the quadratic term and did not just stop s x because after all i have been asserting that x is a very small number the answer to that is that i am looking at the difference between the forces and that will manifest only at the level of x square this is the lowest order term that will contribute to the difference of the forces whereas here there is going to be a cancellation when i subtract

So what is my x my x is given by 2 re by dm minus re square by dm square is what i have

So perhaps i should introduce a notation r e by d m if i denote its my small r

So this quantity is nothing but it is a ratio 2 r minus r squared whole square this is what i have

So what is it going to be sorry there is no square here 2 r minus r square x squared will obtain the term now i want to evaluate x square

So 1 plus x plus x squared

So 1 plus 2 r minus r squared is what i am going to get and x squared is going to be 2r minus r squared whole squared plus higher order terms why do i keep this term because if i want to keep this r square term then there is a contribution to r square term coming from this also otherwise i would have to keep only the linear term r

So x is a small quantity but that itself is a linear combination of atom which is linear in r and quadratic in r

So if i want to keep a term which is quadratic in r in the linear term in x i have to necessarily keep the quadratic term in x square because i have to collect coefficients of all powers consistently that is what i have to do

So what am i going to get i am going to get 1 plus 2 r minus r squared now you see the first term is going to be 4 r square and all the other terms are of higher order therefore i am only going to keep 4 r squared and i will write terms of the order r cubed etcetera because the cross term will be of the order r cubed and the direct term will be of the order of r to the power of 4 which i am going to ignore therefore what i am going to get is essentially 1 plus 2 r plus 3 r squared this is what i have this is always the principle of expansion we should consistently keep terms of a given order by looking at contributions coming from every other term notionally x squared to be appears to be of a higher order compared to x but actually it is not because x itself is a combination of a linear term in r and a quadratic term in r that is what i have

So now we are doing fairly well i have to write my force here my f one is therefore given by f naught dimensionally there is no problem about it and then i have one plus two r plus three r square plus higher order terms this is what we are going to get i hope that i have done all the numerical parts correctly it is very important please verify that now what happens to f1 prime my f 1 prime is k whatever that g m m e is divided by what am i going to write i am going to write d m plus r e whole square

So let me do it let us not be impatient this is what my force is this quantity is k over d m square into 1 over 1 plus r e by d m whole square this is what i have

So my identification of x is different

So ideally speaking in terms of my notation i should write k over d m square into 1 over 1 plus r whole square that is the correct thing to do because i denoted r e by r m this is capital r this is my capital r r a by r m to be small r now i can compare it to this expression this was the earlier expression came with a minus r this comes with a plus r therefore i get my f1 prime by simply replacing r by minus r everywhere in this expression that is what i should do

So let me collect that my f 1 is given by f naught into 1 plus 2 r plus 3 r square is what i am going to get and my f 1 prime is going to be f naught into 1 minus 2 r plus 3 r squared that is what i am going to get

So i am afraid that i made an incorrect statement i was under the impression that i would get a correction from the quadratic term it is the opposite it is a linear term that is going to contribute

So anyway because when you do this subtraction please notice the quadratic term does not contribute it was some kind of elapse but never mind about that

So my delta f1 is simply given by four f naught r and my r is simply given by r e by d m r is a dimensionless quantity

So we unnecessarily kept a term which is of higher order i need not have done that there was a momentary elapse but it does not matter

So your delta f one is simply given by four f naught r e by d m now i can find out in a similar manner what my delta f 2 is

So what would delta f 2 be this would come from the force on the earth due to the sun at diametrically opposite points opposite points colony collinear with the sun

So what is the geometry that we have the geometry that we have is this is the sun radius r e moon sorry the earth radius r e this is the sun and i have my distance d s

So by the same token my delta f 2 will be given by 4 f naught prime because the distance from the moon to the earth will be replaced by the distance from the sun to the earth and i am going to get re over ds this is the correction that i am going to get

So let me write everything in full detail now delta f 1 is given by 4 g mass of the earth mass of the moon divided by d m squared into r e by d dm this is what we are going to get and delta f2 is given by 4g me m s d s squared r e by d s it is a very simple exercise that you people have to perform to check that if you look at this point and if you put the moon somewhere here the force exerted by the sun at this point is much much larger than the force exerted by the moon obviously we always worry about the motion of the earth around the sun and not the earth around the moon now if you come to the opposite point also diametrically opposite point the force exerted by the sun at this point far far exceeds the force exerted by the moon the question that we are asking is how does the force of the sun vary when i move from this point to this point how does the force of the moon vary when i move from this point to this point in other words we are asking how homogeneous is the force or the gravitational field produced by the sun how homogeneous is the gravitational field produced by the moon that is what we are asking because we are looking at the difference now the force by the sun can be very very large if it is homogeneous the difference would be equal to zero the force produced by the moon may be small but if it is inhomogeneous the difference can be large therefore while the absolute values of the forces can be larger there is per se no reason to assume that delta f1 is smaller than delta f2 f1 is smaller than f2 does not mean that delta f1 is smaller than delta f2 and that is exactly what we are interested in

So what do we do in order to appreciate that we calculate the ratio because we want to get rid of all the unwanted factors i am going to look at the ratio delta f1 over delta f2 please remember the numerator is due to the moon the denominator is due to the sun that is what i am interested in

So if i did that whole lot of things will get cancelled and what i am going to get is this will be mass of the moon divided by mass of the sun that is what i am going to get r a will cancel then i am going to get d s by d m whole cubed that is what i am going to get

So because the force due to the moon comes as dm cube the force to the sun comes as d s cube

So this is what i am going to get all the other fellows will cancel this is what exactly what we have

So as i was telling you there is a competition between the ratio of the masses of the moon and the sun and the ratio of the distances between the sun and the moon we have to worry and there is a cubic factor which is there which can actually upset the situation we have to be aware of that now this is the right time for us to plug in the numbers clearly it is quite independent of the radius of the earth it is quite independent of also the mass of the earth or gravitational constant now i will start plugging in numbers which i have noted down here

So let me start writing this mass of the moon is 7.3 into 10 to the power of 22 kgs mass of the sun is 2 into 10 to the power of 30 kgs

So you see sun is really really heavy compared to the moon a million times actually almost a million times is what we have

So this ratio favors that delta f1 should be smaller than delta f2 but now let us look at the distances distance of the sun is 150 into 10 to the power of 6 kilometers i hope i have written it correctly and distance of the moon from the earth is 0.3 into 10 to the power of 6 kilometers

So what do we do you take the ratio this 10 to the power of 6 cancels you are dividing 150 by 0.3

So 1500 into 3 by 10 is what you are going to do divided by 10 whatever you are going to get that number then you are going to look at the ratio 7.3 7.2 with this factor of 10 to the power of this one ultimately if you were to calculate the ratio you will find i am not going to work that out the ratio turns out to be this quantity turns out to be

something like 3.5 i hope that this calculation is correct therefore the difference in the gravitational force produced by the moon at the two ends is much much larger than the difference in the gravitational force produced by the sun it is of the order of three point five four let us say

So what we can do at the first starting point is to ignore this one and then what ask what happens i have the earth here i have my moon here and let us say it is covered by water

So there is a inhomogeneity in the force gravitational force produced by the moon and that delta f1 is there which we have not calculated but you can calculate that

So what happens because there is a greater force of attraction and water is a fluid water would like to move in this particular direction of course there are forces of reaction and there is an increase in the height and there is a corresponding decrease here that is what is going to happen the sun is also going to do that therefore an interesting question is what happens when i look at the period of tides at different points of the that if on different days of a given fortnight that is what we are interested in

So what we shall do is to therefore look at a few scenarios and the most interesting scenario is the new moon phase

So in the new moon phase the moon and the sun are on the same side of the earth that is what we have both of them cooperate tights are strong in the full moon phase the moon is here and if you look at somewhere in the middle of the fortnight on the eighth day let us say a quarter the moon will be somewhere here now the forces partially cancel

So essentially what happens is that depending on the day and the night obviously because the sun is moving is that okay the forces are going to change and corresponding there is going to be an increase in the height there is going to be a decrease in the tide of course during the day time the sun and the moon or in the night time the sun and the moon are together because i never able to i am never able to see the moon because it is completely blocked what we see is a real high tide is that ok and this qualitatively explains what happens when the sun and the moon are at different phases and this was the great explanation given by newton in fact newton did not bother to work it out it was probably one of his students who did this and this takes away the need to have the so-called super natural explanations for the phenomena of types

So this is one of the important things what we have to now do is to move on to a different topic which is at the basis of the analysis of a whole lot of applications and that is the concept of gravitational energy potential energy

So let us recall what is going to happen

So imagine that you have a floor there is a spring there is a mass and this spring is compressed this spring is compressed and held by a stop here this is a stop now what happens as soon as the stop is removed the block moves the block moves as soon as the block moves that means it acquires energy

So a good question to ask is where did it get the energy from our experience tells us that this energy came from the fact that i did some work in order to compress the spring my muscles got exerted let us say there was a spring and i pushed it and then i put a heavy stop i did the work

So i can account for the energy my kinetic energy whatever i pushed it i did some work to the kinetic energy of the my muscular energy i should not use the word kinetic energy to the kinetic energy of the block but then where did the energy get stored in the intermediate process that is the question that we are asking all of you know the answer to that from hooke's law whenever you disturb the spring from its equilibrium position

So you move in this direction then there is a force f is equal to minus kx there is a force that is a restoring force and this restoring force wants to move the block away in this direction and you are stopping it therefore this corresponds to a stored energy of half k x squared this is the stored energy

So if this block is oscillating about its equilibrium position let me call it as x small x is the displacement at capital x there is no stored energy the force is zero all its energy is completely kinetic then when it is oscillating let us say it comes here and it comes here these are the two end points of oscillation at this point there is no kinetic energy at this point p it is completely stored energy it is all potential energy and similarly in the completely compressed position it is all potential energy

So there is a continuous exchange between what is stored as a potential and what is manifest as the kinetic half mu squared and kx square and the interplay between them is such that the total energy is always a conserved quantity and that is the energy that i

supplied if i assume you know there was zero energy at this particular point when it was at rest that is what we do therefore we write my a total is nothing but half mv squared plus half kx square equal to constant now one way of appreciating that which all of you know is to actually use this to obtain the law of motion what do you do if it is indeed a constant then d e by d t must be equal to zero it is a constant of motion and this tells me m v d v by d t plus kx into v equal to 0 i have differentiated x squared 2x dx by dt cancel v on both the sides and lo and behold you get the hook's law m d b by d t equal to minus k x of course if you integrated this expression you would get this if you differentiated this expression you would get this now this is not that is something peculiar to springs you should be peculiar to all forces because newton says in his law of gravitation that all forces behave in the same fashion now i can imagine that i did exactly the same thing in the case of gravitation also i picked up a pawl i did a lot of work lifted my hand and i placed it there on a shelf or some such thing and when i dropped it the ball fell down that is what i would like to say

So i can again ask the same question where was the energy stored because as soon as the ball hit the earth by galilean law it has acquired a lot of velocity

So this gives to the question of re gives rise to the concept of gravitational potential energy i guess we shall discuss that in the next lecture and i will use that to discuss escape velocities and launching of satellites

So on and

So forth

So but that we shall post one for the next lecture

So please revise these topics before you come for the next class thank you have a good day you