

we'll start with the problem on how to use principle of conservation of angular momentum and

so the physical situation of the problem is like this i have a rod of length like this and then two it is rotating with angular velocity ω and then two small spheres of m each are attached gently to the two ends of the rod this one m gets attached to here two small spheres of mass m each are attached gently gently go to the ends of the rod what is the final angular frequency of the system to find to find the ω of the system right now there are no external torques

so first thing we notice that no no external torques

so principle of conservation of angular momentum applies

so what is happening is as the initially the rod is rotating about an axis with a particular angular velocity therefore it will have certain amount of angular momentum now without disturbing the system two masses m each are attached very gently to the rod at each end one at each end therefore since no external torques are there therefore the angular momentum of the final system should be same as the angular momentum of the initial system that's the idea angular momentum of the initial system is equal to $\omega_i I_i$ uh sorry moment of inertia initial times ω_i this is equal to $\frac{1}{12} M L^2 \omega_i$ now I_f is equal to $\frac{1}{12} M L^2 + 2 m l^2$ if ω_f this is equal to now if alone $I_i \omega_i = I_f \omega_f$ i will write here one is $\frac{1}{12} M L^2 \omega_i + 2 m l^2 \omega_f$ mass of little m is attached at each end therefore this will have two masses

so each will have a moment of inertia this whole thing will act with ω_f if we equate these two you will get ω_f is equal to $\frac{I_i \omega_i}{I_f}$ is equal to $\frac{\frac{1}{12} M L^2 \omega_i}{\frac{1}{12} M L^2 + 2 m l^2}$ now we notice that the angular velocity of the final system is smaller than the angular velocity of the initial system that is obvious because more mass has been added therefore the moment of inertia is higher okay i have uh the problem is like this this is length is then we have here this length is $4l$ this length is $4l$ this length is $2l$ and then this length is $2l$ this is three rods are joined three uh light rods first drive three joints one three light rods first i will take the rods to be right case 1 the connecting rod is light let m be the mass per unit length of a b as well as c d right now ah there is a force if it will act at a particular point this is at a distance x from here this is from the center

so this length would be $l - x$ now taking moments about p moment of a b will calculate taking moments about taking moments about p okay there is p this is this is the point p

so i will have this mass a b m into $4l$ that into x okay that must be equal to this is $4m$ times mass per unit length y m into l this is the mass per unit length is m therefore m into l then that is the mass that times x this distance similarly in the due to this rod's c d it is $4m$ into l for lm into the distance this distance would be $2l - x$

so this implies x is equal to $8l$ by 5 is equal to $1.6l$ the connecting rod has the same mass per unit length the connecting rod has the same mass per unit length okay then what will happen from the top four it will be the same and from the top portion means from the portion a b the moment due to a b will be the same moment due to c d will also be the same however there is going to be a moment due to this portion in the middle portion right

so therefore $2vlm$ into $l - x$ plus plus $4l$ m into $2l - x$

so x is equal to $10l$ by 7 principle of conservation of linear momentum and principle of conservation of orbital angular momentum i will explain the problem i have a rod a uniform rod is on a table okay and masses y and $2m$ strike there is a mass which comes and there is a mass $2m$ which strikes here

the rod and there is a mass m which strikes here as indicated in the bar below
 now this m the velocity of this mass is v the velocity of this mass is $2v$
 c is the center of the mass this distance is $3a$ and this distance is a this
 distance is a okay one determine to determine we see velocity of the center
 of mass that is first part okay right now we can make use of principle of
 conservation of linear momentum principle of conservation of linear momentum
 what does it say principle of conservation initially the rod is at rest
 therefore momentum of this rod is atm times 0 plus the $2m$ strikes with a
 velocity v with a velocity but it is in the opposite direction these two
 directions are opposite this direction is minus v plus this mass m it
 strikes with the velocity $2v$ this is what is going to be the the momentum of
 the initial momentum this is equal to 0 what is the final momentum final
 momentum is equal to final momentum there is one thing which i have not shared
 in the problem is the uh the bar is that the these masses $2m$ and m they get
 stick to the bar the $2m$ and m they stick to the bar after striking therefore
 now the total mass of the system is total mass system is atm atm plus m plus
 m the whole thing will have one velocity that is the center of velocity v_c
 so you equate one and two now this implies velocity of center of mass is equal
 to zero okay
 so system will have
 so since the there is no translational motion this implies no translational
 motion
 so it will have only rotational motion to calculate the angular velocity of
 the center of mass right
 so we have seen uh there is no translational motion there is only rotational
 motion or no external torques are acting therefore orbital angular momentum
 angular momentum is conserved therefore what is L_i L_f is equal to $2m$ as the
 mass times velocity times a plus little m times $2v$ times $2a$ is equal to is
 equal to what will be this value this value would be 2 plus 4 $6mva$ is the
 initial angular momentum
 so final angular momentum now after the masses get stuck to the rod the rod
 is going to rotate with ω therefore optical angular momentum is whatever
 is the moment of inertia times ω now for this moment of inertia uh the
 mass $2m$ will contribute mass m will contribute and also the rod will
 contribute because the entire thing will rotate
 so first $2m$ into a square this is the moment of inertia this is the moment of
 inertia of the mass $2m$ plus the moment of inertia of the mass little m
 m cross 2 here whole square okay $2m$ into a squared m times 2 i am calculating
 all this with respect to the center of mass plus by twelve six a square by 12
 this is the moment of inertia of the rod about the central axis whole thing
 times ω
 so this will give you $30m a^2 \omega$
 so equate this to $30m a^2 \omega$ is equal to $6mva$
 so m and m will cancel a will get cancelled i will have ω is equal to $\frac{v}{5a}$
 by v divided by $5a$ now what more can be calculated with respect to this
 problem now there is no translational motion it is not moving parallel to
 any axis for that matter but it is only rotating and it has got an ω
 angular velocity ω which are calculated therefore the whole system after
 the masses get stuck to the rod it will have a rotational kinetic energy
 that can be calculated so third thing is kinetic energy due to rotation what
 is expression even if you don't remember if you remember linear motion you
 can try to write down in linear motion the expression for kinetic is $\frac{1}{2}mv^2$
 squared here half the role of mass is taken over by moment of inertia and
 then angular velocity squared that is

so this is equal to half into we have calculated what is the moment of inertia in this case earlier that we have calculated $30 m a^2$ therefore $30 m a^2 \omega^2 v$ by ϕa the whole square that will give you 3 by $5 m p^2$ ok

so this is a good problem which requires the principle of linear momentum conservation it requires the conservation of linear momentum and also the conservation of angular momentum now we will do a problem which involves various concepts like slipping etc okay

so this is a problem involving slipping

so when an object moves on the whenever object moves on rolls on the other object it can simply slip that means there is no rotational motion for it and involving frictional force etc

so i have a rod ab there is a mass m ok it is a distance of l units so m is a bead which can slide along the rod okay this m is a bead which can slide along the rod without falling initially it is a distance l as indicated here it is distance this is the initial distance little capital l sorry the rod rotates about a with a constant angular acceleration

so it rotates okay with a constant angle acceleration α right now the rod rotates this is given about a constant angular acceleration the symbol for angular axial error is generally α which you must be knowing which is constant μ is the coefficient of friction μ the coefficient of friction who is the coefficient of friction between the rod and the bead okay

so we can neglect gravity then find the time after which now what is happening there is a rod on which this mass is there this is a bead the rod is rotating about a with a constant angular velocity so the

so as it rotates the bead can uh move along the rod it can slide along the rod there is a friction between the bead and the rod

so at some point of time the mass has to slip we have to find the condition for that condition for slipping now first thing we should note α is given to be a constant α you say α is constant therefore the angular velocity is not constant it uh it has to depend on it has to therefore the angular velocity has to be αt the reason is if i take $d\omega$ by dt which is i will get α

so first thing we should realize in this problem that angular velocity is not constant it depends it linearly varies with respect to time right so linear acceleration of the bead first linear acceleration of the bead this is here it is going to be how is linear acceleration defined linear acceleration is the length times α this is the definition then reaction force on the bead due to the rod the reaction force on the bead due to the rod is equal to this i will call the n this must be same as m into a this is m into l into α now and we have taken this angular velocity to be αt now there is a centripetal force on the bead centripetal force on the bead is equal to $m r \omega^2$ what is the expression for centripetal force or $r \dot{\theta}^2$ where $\dot{\theta}$ is if you had forgotten $d\theta$ by dt that is nothing but the ω right therefore this term is equal to centripetal force on the bead is equal to $m r \omega^2$ r is l $\dot{\theta}$ is αt whole square therefore it is $m l \alpha^2 t^2$

so there is a frictional force between the bead and the rod and we know what is the reaction force on the bead due to the rod that the reaction force on the bead due to the rod n is there therefore the frictional force in the limiting case limiting frictional force therefore the limiting frictional force is equal to μ times n that is equal to μ times n is $m \alpha l$ ok

so first condition for slipping is for sleeping is equal to this frictional force must be equal to the centripetal force this frictional force must be equal to the centripetal force

so from this we get this at this particular time t is

so m and m will get cancelled l and l will get cancelled when α will get cancelled i will have μ by α i need to take a square root of that

so what are the concepts which are tested in this particular problem first thing is you should realize that since α is constant ω is not constant many students know by what i can say default they will put an arrow around this rod rotating and put an ω take ω to be constant which is wrong right then second thing is as the bead moves along the rod okay there is a reaction force on the bead due to the rod that is there then there is also a centripetal force on this bead right

so for slipping what will happen the friction limiting frictional force must be equal to the ah must be equal to the centripetal force only then till then it will survive

so this happens at this uh survive in the sense that this mass will remain on this rod after that it will slip off okay now we will do another problem which involves there is one more concept which is called toppling slipping and toppling we will explain what it is

so what i have is i have a cubical block i have a cubical block there is a force acting on this cubical block here if

so length of this cubical block is l edge rather

so this is a horizontal surface but there is a its a rough surface its a cubical block resting a rough horizontal surface the coefficient of friction is such that the coefficient of friction is

so high such that the block does not slide before toppling the coefficient of friction it is given the coefficient of friction is

so high is such that the block does not slide before toppling

so as the horizontal force is supplied there is a tendency for this block to move along translate on the other hand the coefficient of friction is

so high then what will happen the block would only topple

so we need to calculate the minimum force calculate to calculate get the f minimum for the block to top block to topple rather okay it's a fairly simple problem but we need to realize now let us mark the various forces acting on it we will again redraw the diagram block force right now this is the center of mass mg

so the initially the normal reaction when when you are not applying any force the f is not there then the normal reaction would be at the center of mass of the cube such that it opposes mg however since the horizontal force is there gradually the normal reaction would move and it will it will exactly topple when the normal reaction is coincides with this side of the cube now

since it is it is has a tendency topple like this the frictional force has to act down in this direction opposing the motion right ok now Σf what are we going to do the sum of all the forces along y direction is equal to there is normal reaction acting upwards and then mg acting downwards

so is equal

so we write all the this this gives these two balance or n minus mg θ i

should write i am writing like this similarly when i consider the forces all the forces acting along x direction now the horizontal force is capital f this must be balanced by frictional force that's it now we will take torques

we will write the torque equation about the c we will write the torque equation the torque equation about about the center of mass center of the cube right

so f into l by 2 this this distance is l by 2 . then plus f into l by two
 the f into l by 2 is equal to the normal reaction n into l by 2
 so this distance is also l by 2
 so this gives capital f plus frictional force is equal to n and we have got it
 earlier capital f is equal to two f therefore two f is equal to n and what
 is n normal reaction is f plus s is n therefore this implies f is equal to n
 by 2 and we are showed that n is mg we already have it when we balance the
 forces along the y direction what is interesting about this problem is we need
 not know what is the value of μ because the question is what is the minimal
 force that is required to top lift right and what are the things we have
 made use of uh it is essentially force balance equation along x direction and
 force balance equation along y direction and taking and there are
 essentially speaking there are three forces one is capital f the horizontal
 force and little f which is the frictional force and then which is the normal
 reaction
 so take the torques and equate them and the problem is solved yes minimum in
 fact it is amazing that minimum force that is required is half of the weight
 of the body now we will move on to a problem sometimes no people would ask
 questions involving rotational motion and clubbing it with something else it
 all depends on the ingenuity of the examiner there is one problem involving
 atomic physics diatomic molecule in a diatomic molecule rotational frequency
 diatomic molecule rotational frequency and quantum theory sometimes such
 problems which club uh concepts from different branches of physics they
 would strike terror however one needs to look at them rather carefully with
 some patients right now what does a diatomic molecule do a diatomic molecule
 you have two atoms
 so they can rotate about an axis with the frequency of ω this
 so this distance is x
 so what is this this diff uh this is the uh this is a separation of atoms x is
 the atomic separation separation between the atoms okay now atoms are we are
 going to treat atoms as pine particles but they have masses and now we will
 take the case of oxygen atom oxygen atom for this for the for the oxygen
 molecule you know two oxygen molec atoms can combine to give you oxygen
 molecule the the separation between the atoms is 1.20×10^{-10} meters
 all this data will be provided to you and mass of the
 atom mass of the oxygen atom rather mass of mass of oxygen atom is equal to
 two points this data is also given two point six six into 10^{-26} kilograms
 now what is that you are asked to calculate you are asked
 to calculate what is the frequency of the what is the calculate calculate
 the rotational frequency calculate the rotational frequency now how are you
 going to do what first thing you realize that these two atoms sorry this
 molecule has a moment of inertia about the center
 so moment of inertia is equal to the standard $m_i r^2$ a squared this is equal
 to m into x by 2 whole squared plus m into x by 2 whole square right
 so just leave it as it is mx^2 by 2 now according to quantum theory
 the the fundamental unit of angular momentum right the fundamental unit of
 fundamental unit of angular momentum quantum according to quantum theory is h
 cross what is the value of h cross this data will also be given to you 1.054
 into 10^{-34} kilogram meter squared per second okay
 therefore i from the given data i can calculate what is the moment of inertia
 of the object therefore moment of inertia into the ω namely the
 rotational frequency this must be of the order of h cross therefore ω is
 equal to h cross divided by i is equal to 1.054×10^{-34} kilogram meter squared per second this divided

by moment of inertia is i've calculated this in the $m x^2$ by 2 m is 2.66
 into 10 to the power of minus 26 kilogram that by 2 into x^2 square square is
 the this atomic separation square of the atomic separation that is 1.20 into
 10 to the power of minus 10 meters second square tada you can do this
 simplification and then you will get to be it should be of the form its value
 is about 5.2 into 10 to the power of 11 radian per second in fact it is
 amazing that this value more or less agrees with the experimental one it
 really proves that this molecule actually has a rotational frequency most of
 these molecules they have rotational frequencies okay see something which i
 can do a simple problem a 58 clipper will do a problem involving angular
 momentum torque etcetera let me state the problem a rigid rod of mass
 capital m and length l rotates in a vertical plane about a frictionless
 pivot this is the origin frictionless pivot through the center rod of mass m
 and length l rotates in a vertical plane in a vertical plane about about a
 frictionless pivot through the center okay now this is uh
 so this frequency is given let us say so once the ω is known linear
 velocities of m_1 and m_2 can be calculated what are the various quantities
 which can be calculated first there is the moment of inertia of the system
 first I of the system in my system what am i of the system so my system is
 equal to the moment of inertia of the rod about the center is $m l^2$ squared by 12
 plus detail m_1 is the one of the masses that into both the center l by 2 whole
 square plus m_2 into l by 2 the whole square this is equal to this value
 would be l^2 squared by 4 into m squared by 3 into m_1 by three plus little m
 one plus little m two
 so number sixty minutes require editing time to waste now the system can rotate
 with a constant angular velocity of ω that is one data
 so so once ω is known ω can be related to the angular momentum
 so the angular momentum of the system is once ω is known I can be
 calculated therefore what we have I is equal to $I \omega$ right
 so this is equal to we already calculated I l^2 squared by four into m by three
 plus little m one plus little m two that times ω now there is a torque on
 the system right because there is one $m_1 g$ there is another $m_2 g$ forces
 therefore torque on the system is equal to three the torque on the system
 first τ_1 is equal to $m_1 g$ into this angle suppose i call it as θ
 therefore this is l by 2 $\cos \theta$ okay $m_1 g$ into l by 2 $\cos \theta$
 so if i l by 2 \cos this distance similarly this distance
 so what about this this is out of the paper the torque no it has a it has got
 direction
 so this is out of the paper out of the plane what it means is out of the plane
 τ_2 is equal to similarly $m_2 g$ into l by 2 $\cos \theta$ but this is into the
 plane therefore total torque is equal to is equal to half of m_1 minus m_2
 into $l \cos \theta$
 so this will act out of the plane if m_1 is greater than m_2
 so this is if m_1 greater than m_2 into the plane if m_2 less than m_1 since i
 α is equal to l we can calculate α namely the angular acceleration
 so α is equal to τ total by I
 so this you will get 2 times m_1 minus m_2 into $g \cos \theta$ divided by m by
 3 plus m_1 plus m_2 . you