so today we are going to discuss few problems on systems of particles and rotational dynamics

so before we proceed to discuss problems let me say a few things and first thing is problem solving is very important not necessarily from the point of view of competitive examinations just to understand the subject one needs to do problems if a person is not able to solve problems and he is able to he or she is able to reproduce only the theory then that knowledge is not very sound and feynman used to call such an order just for agile for example a glassware looks very beautiful glossy very costly also perhaps but if one drops it the whole thing has to go to the dustbin

so solving problems is very important i need not emphasize but however i stress it now while you solve problems what are the things one needs to take care it only comes by practice and experience we are discussing this particular topic on systems of particles rotational dynamics we will see gradually various things and okay we will go problem after problem now i am going to describe mostly the situations uh the physical situations then we will see how to solve this problems and okay

so there is a wire which is in the shape of a parabola wire bent in the shape of a parabola the axis are given here this equation is y is equal to k x square where k is a constant it has to be positive otherwise the parabola would look like this and there is a bead which can which is kept on this wire which is can slide along and it can be it can slide along without friction slide along slide along the wire bend in the shape of parabola slide along without friction that is important ok now what happens is that the wire is accelerated in this direction with a constant acceleration a the wire is accelerated parallel to the x axis the wire is accelerated accelerated parallel to the x-axis with acceleration

so find the new equilibrium position question to find the new equilibrium position position of the bead okay now if this is stationary if this parabolic wire is stationary then the particle will come and settle down at the origin that is the equilibrium position now what happens this is given an acceleration parallel to x axis therefore the bead will slide up what will happen is at any position there is this mass then therefore this mg acting downwards it can be resolved along horizontal and vertical components and right we will do that

so let us say this is a position and this is the tangent here mg emg is acting downwards and this normal reaction this normal reaction would be perpendicular to this tangent right

so this mg can be resolved along this direction and this direction okay i'm just

so what you have is that um this is uh this n is dissolved like this n is n cos theta this is n sine theta horizontally n sine theta

so the normal reaction is resolved along these two directions if this is theta then this is theta

so n cos theta and n sine theta this is m a okay

so it is a normal reaction at this point which is dissolved along x axis and y axis and right now the equilibrium the particle will be in this new equilibrium position provided when the forces balance and or equilibrium

so it is a mechanical equilibrium for equilibrium for equilibrium of the particle we need this n cos theta is the vertical component of normal reaction

so this n cos theta must be equal to the weight acting downwards weight on the beat then the weight of the beat the next is n sine theta this is a horizontal component of the reaction this must be equal to the force with

which the force which is acting on this bead ma now ah these are the two equations we have divide one by the other and you will get 2 by a by g so this is equation 2 divided by equation

so this is theta this is the this theta here therefore tan theta is nothing but derivative at this point derivative on this curve at this particular point therefore d y by d x is equal to y is equal to k x square is equal to k x square therefore d ah d y by d x is equal to 2 k x this is equal to tan theta therefore x is equal to tan theta is acceleration divided by acceleration a on the particle or acceleration of this wire divided by acceleration due to gravity that divided by 2k because for tan theta i have substituted

so therefore this is the this is the new equilibrium condition now we spent few minutes on analyzing this problem what are the various things that are expected from the student to know

so whenever a problem is given the student should ask this question why this particular problem has been given of course if you do it correctly and then you may get the you may get selected in that particular exam that's not the what i mean by this what i mean is that what are the concepts which are which the examiner is testing

so the examiner expects the students to know certain concepts to get qualified in this problem that is being tested is does the student know the conditions of equilibrium number one that means the forces should balance force should balance that is number one no external thoughts that is number two then after that this is also somewhat mathematical because one needs to know that ah at this particular point the slope can be obtained by differentiating this okay the next problem which we are going to ask is this on angular velocity okay let me explain this problem now sometimes when you read a problem it looks it strikes some kind of fear but you have to patiently look at it and divide it into various components here in this particular problem what happens is that there is a circle okay we have access this is uh this is the axis that's not very important to label them and then what we have is a particle is going round a particle is going round on this circle i got it here the a particle is going around this particular circle now what are the things given this is given the particle is going around the circle with the angular velocity is phi radians per second and then r the radius of the circle is 4 meters right now what is happening is i can draw the at this from this particular point this is the foot of the perpendicular p prime is the foot of the perpendicular on this x axis p prime is the foot of the perpendicular on the x axis now as the particle goes around the circle the foot of the perpendicular will go back and forth on this x axis now people would have recognized this is what you say the one to one correspondence between circular motion the particle which is going on a circle it has got circular motion whereas the foot of the perpendicular will have simple harmonic motion but even all these things are not necessary to lose this problem what are the things which are been asked one to calculate to calculate the speed of the foot of the perpendicular foot of the perpendicular that is we calculate the speed of p prime when when op sweeps 30 degrees that is theta is equal to 30 degrees 30 degrees with respect to sweeps theta is equal to 30 degrees

so when the particle is here what is the speed of this p prime that is the question asked the problem is very simple this i will call it as r therefore o p prime is equal to x into cos theta right o p prime is equal to x is equal to r cos theta now x is a function of time because that keeps varying d x by d t is equal to minus r sine theta d theta by dt d theta by dt is what is

omega minus r sine theta minus r omega sine theta
so this is the now foot of the perpendicular v is the velocity   people at p
prime the modulus of that time this is going to be equal to   modulus of r is
4 and omega is 5 that into sine 30. okay this is equal to 10 meters per second
 this is 10 meters per second now another thing here itself i will call this
point as q is q now what is the angular velocity of p   i mean angular
velocity of p you are required to calculate angular velocity of p of p about
point q and what we are doing in this we given the omega is 5 radians per
second is the angular velocity   with respect to o now you are asked to
calculate what is the angular velocity with respect to q right this is again a
very fairly simple one   even though it looks very daunting all that you need
to do is that join this card this is a card i will call it as m let us say
okay
so this card pm subtends an angle theta   therefore now we need to calculate
what is the angle this is the okay
so all that we need to calculate is though
so the calculate the angle required angle is required angle is equal to m q p
this we know very well this is a card in a circle   at the center it subtends
an angle theta therefore it will subtend off of that angle at any point on
the circumference this is equal to theta by 2 in this case it is going to be
equal to it is omega   is 5 radians per second 5 by 2 is equal to 2.5 radians
per second
so a simple property of circle   is made use of similarly you can calculate
what is the what is the acceleration of p prime   that you can calculate and
okay the various things once you go about doing a problem ask   what are the
various other things which you can calculate we'll do a problem on center of
mass
so this is on the center of mass problem this again i have taken this problem
from a complete examination
so problem is like this a uniform circular disc of radius r okay from this disc
what is happening is here another circular disc   of radius r by 2 is removed
okay this is uh so this circle is radius of the smaller circle is   r by two
obviously and this center i will call it as bow this has another point here
called sd
so from the bigger circle the smaller circle is cut out and it is a uniform
circular disc okay now um what is the task this you have to calculate   what
is that i mean to calculate to determine the center of mass of the   remaining
portion once this has been removed of the remaining portion okay it's very
easy what's the definition of center of mass   center you have two masses
which is which m1 is located at x1   and another mass m2 is located x2 then
the center of mass by definition is this quantity now i will i will take this
entire thing as m1 and then   the remaining portion is mass as m2 okay so let
sigma p the mass per unit area sigma mass per unit area of the material of the
disk of the disk okay
so m one is equal to mass of the smaller circle is pi e   r by two whole
squared that multiplied by sigma ok and where is x one x one is   r by two yes
suppose i call this as the origin with respect to this is at r by two then m
two m two is the remaining portion therefore i should from the area of the
entire circle i should   subtract the area of this smaller circle therefore pi
into bigger circle area is r squared minus   r by 2 the whole squared that
times sigma and where is its located it is at some point   it is at the point
x 2 is equal to i will call it as x this is vod od is x ok and what is this
area this area i can calculate   three pi r squared by four now x center of
mass is equal to x center of mass is equal to ah simply m1 is this quantity r

by 2 whole square sigma into rm sorry all squared by e r squared by 4 rho um
i will call mass per unit sigma i call it as sigma sigma that into r by 2 plus
pi 3 pi r squared by 4 into sigma into x okay that divided by pi r squared
rho the center of my system is at the origin therefore it is 0
so it has all for this r by 8 plus three x by four is equal to zero
therefore x is equal to minus r by six so this is the ode is the distance r by
six therefore the center of mass of the remaining portion lies along x axis
at a distance of r by 2 on the left hand side ok
so so the coordinates of d are minus r by six and zero these are the
coordinates of t coordinates of t okay
so this problem illustrates ah a simple concept like center of mass which will
be repeatedly coming across
so next problem i have chosen is that is involving some concepts involving
moment of inertia omega linear velocity rotational kinetic energy orbital
angular momentum etc this is the problem now okay say given a symmetric you
have been given this you have been given some information asymmetric body is
rotating about it is rotating about an axis all that you have been given is
its rotational kinetic energy its rotational kinetic energy what you will
call it as simply ke it has been given it has been given to you then what
more is given its orbiting angular moment has been given to you now since the
rotational kinetic energy is given rotational kinetic energy is equal to
half i omega square remember this is an analogy with what happens in linear
motion half mb squared orbital angular momentum i omega right these are the
things which are given then now calculate what is kinetic energy by l that is
equal to omega by 2
so this implies omega is equal to 2 k e by k
so given rotational kinetic given the given that you are given that a symmetric
body is rotating about an axis and you have been given this data the value
of rotational kinetic energy and the value of orbital angular momentum from
that if they simply ask you if they ask you to calculate omega this is this
is simple like this okay and you can also calculate what is i i is equal to
l by omega is equal to l squared by 2 k e now in the case of the first one
when they asked you what is the value of omega let us say that it's a
multiple choice question omega is a 2 k e by omega is 2 k e by l then b k e
by l and c k e by 2 l and d k e by l now uh in all the cases uh we have ke
by l sometimes what students do is know they they are taught that uh he did
not work out all the problems just by eliminating all the wrong wrong
answers you might you can get the correct answer and that is true provided the
problem is framed that way if out of the four choices three choices are
obviously dimensionally wrong or then by looking at it you can come to the
conclusion the remaining answer is correct but in this problem it is not
possible because the dimensions of omega ah because omega has dimensions of of
course energy divided by l but there is a proportionality factor that is
involved here correct proportionality factor is 2 k e by l therefore a is the
correct answer
so only way you can do this is by working out and
so this problem looks very grave at the outlet oh we are given kinetic
energy and then we are given orbital angular momentum how are we to calculate
um the angular velocity that is very simple
so but one needs to know these are the things now which i keep repeatedly
telling what are the things you should know the rotational energy is half i
omega squared this is something similar to what happens in a linear motion
case we have seen this analogy we have seen this analogy repeatedly
similarly orbital angular momentum is i times omega this is something like

what happens in the case of linear motion that f is equal to sorry the momentum of the particle is equal to mass times velocity and okay so one should ask whenever cfm what are the concepts tested that is what are the concepts which the examiner expects the students to know next we will move on to a problem on moment of inertia

so calculating moment of inertia of various body is important and you should practice now okay the question is like this i have a three rods are there the three rods each of length l therefore when they are joined like this you have an equilateral triangle first thing p q and r

so mass of each rod is m mass of each rod length is l okay length of right okay then find the moment of inertia of the system about an axis passing through its center of mass and perpendicular to the plane of the diagram okay now let us say that the center of mass is somewhere here see

so this axis comes out of the it is not in the plane i can only indicate like this this is z axis let us say that so you can have your x axis like this y axis like this oh my god did you already complain um

so we have three rods um they are pq that's our problem then the mass of each rod is m length of each rod is l

so this is a problem on moment of inertia as already stated then now you are required to calculate the let us say that you are required to calculate the moment of inertia of this triangular of this triangle with respect to an axis which is coming out of the plane of the paper ok that is important now this is x axis y axis okay ah you have to carefully erase this y-axis right now suppose i produce this it will meet at the point d this is little d let us say that od is little d

so again i repeat you are required to calculate the moment of inertia of this triangular figure figure with respect to an axis passing through the center and perpendicular to the plane this is another axis z prime right now uh the moment of inertia of i of this very simple problem what we are going to do is that we calculate the moment of inertia of one rod that will be equal to the moments of inertia of other two rods for that but obviously the moment of inertia is going to be calculated about the point d okay in an axis which is uh passing through axis d which is lying in this plane therefore moment of inertia of

so this is z is equal to moment of inertia moment of inertia about bo is equal to moment of inertia about z prime plus we are going to make use of uh parallel axis theorem m d square this is the this distances d okay

so this we have to calculate angle d q o is equal to 30 degrees therefore tan 30 is equal to tan of that degrees is equal to d divided by l by two that is equal to one over root three this implies little d is equal to l by two root three okay i z is equal to i z is equal to ml squared by 12 m l squared by 12 plus this m into l by 2 root 3 whole square this is equal to m l square by 6 okay

so i of the system is equal to 3 times ml square by 6 is equal to ml square by 2 right now we will move on to other problem this again a problem on moment of inertias two spheres two solid spheres have the two solid spheres are given this is a problem on moment of inertia two solid spheres have the same mass they are made of materials of different they are made up made of made of materials of different densities which one will have larger moment of inertia question which will have larger mi about an axis about an axis passing through the origin okay i one is equal to you know moment of inertia of a sphere of mass m of radius r one is two by five m r square this is the moment of inertia about the center

so i two is 2 by 5 since both of them have the same mass but different radial

so i 1 by i 2 is equal to r 1 squared by r two square
so what is m one mass of the first one   m is equal to four by three pi r one
cube rho one this implies rho 1 cube is equal to 3 m by 4 pi rho 1 then m 2
that is the mass of the second sphere is again 4 by 3   pi r 2 cube times rho
2 this implies r 2 cube is equal to r 2 cube is equal to 3 m 3 3m by 4 pi rho
2 right
so from this what is r1 square r1 squared is equal to three m by four pi rho
one   whole to the power of two by three
so i will have i one by i two   is proportional to rho 2 by rho 1 whole to the
power of two thirds ok
so this implies   the moment of inertia is proportional to 1 over rho to the
power of 2 by 3   from this you can argue whether the sphere with greater
density will have   higher moment of inertia among these two spheres which
will have   higher moment of inertia that i will leave due to little bit of
thing having done this much okay now next problem this again a simple problem
in moment of inertia but it is it is good that you know it okay i will do
another problem involving talks let us do a problem involving torques the
physical situation is like this i have a rod it's a uniform rod of length a b
regents label like this now a b is 10 meters right now
so this d the midpoint here there is a 30 newtons acting now here this
distance there is a force of 10 newtons there is a sorry not here the 30
newtons there is another one at c here it is 20 newtons okay
so this distance is 2 meters this distance is 3 meters and then this point ev
will come later now this is x
so you are to find to find the point of application of the force point of
application is a concept this concept is like this in this particular problem
on this rod   two forces are acting one is at uh c another is at d at c 20
newton's is acting upwards   at d 30 newton's is acting downwards now these
two forces do not balance   there is a difference about 10 newtons therefore
the point of application   is that particular point is on that rod where if
you apply this difference of 10 newtons   then what is happening is the
whatever resultant torque that is produced that is   balanced okay
so i will calculate taking moments about c taking moments about c we can take
any point that does not matter
so moment   about c is equal to 0 the moment of 20 newtons due to then tau d
tau d is equal to 30 into 3 is equal to 90 this is clockwise this is 19 this
is clockwise
so i need to find a point x   such that at which when 10 newtons acts this is
going to correspond to this 90 newtons 90 the torque of 90 units
so this gives us x is equal to 9 meters in fact one can put one can put
wherever you want x newton's let us say that x newton's at b then i have
determined x then overall is 90 torque is 90 units that should be equal to x
into 10 this implies x is equal to 9 newtons
so i can put at b 9 newtons downwards
so that   this entire the earlier system of forces is equivalent to one single
force right
so if on a rigid body various forces are acting and they can create a certain
amount of torque   this same amount of torque can be generated by application
of one single force   at a suitable point that is the concept that is tested
in this problem now we will do a problem   of course even though we have been
using uh the conditions for translational equilibrium and   rotational
equilibrium we will do an explicitly a simple problem this problem is like this
 okay there is a tangential force if   the mass of the shell is m and radius

is capital r now it is rotating in this direction now i have a thin thin spherical shell we need to find the acceleration of the shell now whatever whatever the the shell rolls without slipping therefore question to find the acceleration of the shell linear acceleration of the shell now there is a frictional force acting frictional force will act here   in this direction because here the motion is like this now there is a thin spherical shell which is uh rolling without slipping   that is the physics part of it i need to find what is the linear acceleration first for translational motion translational motion what are the forces acting along x direction f plus if it will have friction   f plus f is equal to match times acceleration this is one equation and then for rotational motion now this uh tangential force f will it will   there will be a torque on this shell due to the tangential force f that torque is f into r this will be in one direction what about frictional force f this will also induce a torque on this body it is in the opposite direction
so this will be a minus f r is equal to
so this is the total torque torque value of torque is   i times alpha this is m into a kind of thing this is into because there is the shell is rolling without slipping the condition is
so from these two equations from this a second equation is f minus little f is equal to   i into a by r squared add equations one and three then straight away the frictional force will cancel then i will have   2f is equal to 2f is equal to m plus i divided by r squared times k
so so this implies a is equal to 6 f by phi m okay
so i can have this as m plus i value is two by three 2 by 3 m r   squared that is the moment of inertia of a spherical shell that divided by this times a
so i'll have r square and r square cancel therefore this is how i will have
so what are the things that are expected in this particular problem   you need to realize what are the physical conditions for rolling without sleeping for rolling without sleeping the condition is velocity of the center of mass is same as   r times omega
so this we can extend it to acceleration also we'll stop at this change
so
 do you