

so we have been having a series of lectures on systems of particles and rotational motion in the last lecture we asked the following question what is the rotational analog of mass and we found that this is the concept known as moment of inertia and we calculated moment of inertia for various objects like circular ring rod sphere cylinder and we also discussed two important theorems uh the perpendicular axis theorem and parallel axis theorem now we will proceed further uh it is almost that we have all the necessary concepts and uh techniques that are that are required to study the motion of a rotational motion of an object under essentially we are going to focus on rotational motion about a fixed axis this is a very this is a simpler form of the problem of rotating rotational dynamics we need to study both kinematics and dynamics kinematics means the study of motion without any specific reference to the forces which are acting on it and okay now ah we are as i told you again we are going to focus on rotational motion about a fixed axis the advantage is such a motion requires only one degree of freedom what is it suppose i have here an object and then i have here three axis x-axis y axis and z axis and so an object every particle it will go around in a circle so what happens is this is the axis going down and originally the point p is here let us say now it comes to p prime so it makes an angle this angle is theta so the location of a point can be specified by just the angle this is a fixed axis we need to keep this in mind just theta alone is sufficient to specify the position of the particle and first we will study kinematics then we will go on to dynamics we already have this is theta is the angular displacement we know this is the angular displacement it has the angular displacement we have this quantity is angular velocity or it has got another name rotational speed angular acceleration α is $d\omega$ by dt okay so in the case of linear motion we have in case of linear motion the case of linear motion the kinematical equations are v is equal to $u + at$ and yes namely the displacement is equal to whatever the initial displacement plus $ut + \frac{1}{2}at^2$ and then v^2 is equal to $u^2 + 2as$ the symbols are all very standard u is the initial velocity of the particle v is the velocity velocity at that particular instance time a is a constant acceleration uniform acceleration then s is the displacement these are all fairly standard things now the corresponding kinematical equations in the case of rotational motion because of rotational motion r we will write down ω is equal to $\omega_0 + \alpha t$ plus θ is equal to $\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ ω^2 is equal to $\omega_0^2 + 2\alpha\theta$ how are we getting these equations are is it just by analogy or is there any methodology and you can see that at the outset uh you see there's a remarkable similarity okay between linear motion kinematic equations of linear motion and kinematical equations of rotational motions how are they these equations arrived at first thing we need to remember is α is a constant okay so first we start with the definition of $d\omega$ by dt is equal to α which is a constant therefore integrate what do we get if i integrate i will get ω is equal to $\alpha t + c$ i am integrating between 0 and c 0 and t let us say that then at time t at time t is equal to t_0 at t is equal to $t_0 + t$ initial let us say ω is equal to ω_0 therefore this implies c is equal to ω_0 and hence i have the relation ω is equal to $\omega_0 + \alpha t$ okay so this is the first equation i will write here so from this i have i will make use of it later $\omega - \omega_0$ by

alpha α will integrate what is $d\theta$ by dt $d\theta$ by dt is equal to dt by dt is equal to ω naught plus αt okay then again i integrate integrate what will i get θ is equal to ωt plus $\frac{1}{2} \alpha t^2$ plus a constant of integration c a constant be a a constant it's a constant of variation let us say that at $t = 0$ θ is θ_0

so therefore i will have $\theta = \theta_0$ when i put that what will happen this implies c is equal to $\theta_0 - \omega t_0 - \frac{1}{2} \alpha t_0^2$ therefore θ is equal to $\theta_0 + \omega(t - t_0) + \frac{1}{2} \alpha(t - t_0)^2$ this is the second equation of motion this is second this is the third

so what did we do this for second one lead θ by dt this is where we started ω is $d\theta$ by dt from that definition and right then ah now what we need to do is this is the equation two if you eliminate from these two equations t okay

so that will leave it as a simple exercise we will not do it rather eliminate t between one and two you can do it as a simple exercise and you will get here ω^2 is equal to $\omega_0^2 + 2\alpha(\theta - \theta_0)$ okay

so this is the third equation only thing you'll find that when i wrote here third equation you only have two α θ of course it does not uh i can't say it does not matter what we have is a time t is equal to $t_0 + \theta - \theta_0$ θ is θ_0 in this particular equation 3 here at time t is equal to t_0 θ is θ_0 that's all the difference center however both these equations have the same spirit so what we realize from this is what we realize from this following from these two kinematical equations you look at it the role of v is taken over by the role of linear velocity is taken over by ω and the role of displacement is taken over by angular displacement the role of linear velocity is taken over by ω the the the role of acceleration is taken over by α

so this is the kind of correspondence we have here yes correspondence the role of s is taken over by angular displacement linear velocity is taken over by ω and then the linear acceleration rule is taken over by α okay now we have some comments here you take a look at this equation see each of this the first one is the definition of velocity of the particle at any instant of time this tells you how to calculate the distance traveled what does this equation say this we are going to come across the little later now uh i will take this particular space what i will do $v^2 - u^2 = 2as$ is equal to $m a s$ right displacement multiplied by m on both sides so $v^2 - u^2 = 2m a s$ yes what is that we have on left hand side it is the change in kinetic energy the initial velocity if the velocity is v and then it get reduced to u this is the drop in kinetic energy and that must be that must go as the work done on the particle that is by the force $m a s$ is force into distance

so this is the so called work energy theorem we are going to come across the rotational work of this little later now we have the we want to relationship between angular and linear quantities next topic is relationship between angular and linear quantities to some extent we have seen this in lecture three however we will have so what we have is we have a rigid we have an object which is going around this is the axis x-axis y-axis and i have here let us say this particle is i consider a circular motion of a particle this is r this is θ and this is the linear velocity v we have the standard thing $v = r\omega$ actually what we have is in vector form is we have seen it in lecture 3 $\omega \times r$ so this particle is moving on the circular path at this instant the angular

displacement is θ

so what is v v is by definition it is ds by dt rate of change of displacement
so we want to calculate $d s$ which is this arc length which is nothing but r
times $d\theta$ whatever is the change of angle by dt very simple $d s$ is
change in arc length is change in this angular displacement that divided by
infinitesimal and take the limit okay therefore v is r into $d\theta$ by dt now
tangential acceleration now tangential acceleration tangential acceleration
so let me draw this diagram in a better way this is some rigid body and i
have the axis x axis y axis

so i am considering the circular trajectory of a particle

so at this particular point at this particular point the tangential
acceleration will be like this a sub t i am using the same symbols as one
which i used in earlier one in lecture three this is the radial acceleration
and if i compound these two if i compound these two i will have this quantity
is actually the i will um i will call this as p and this has q

so p q sorry p q vector p q vector is the actual acceleration vector which
earlier i called it like this lecture 3 no problem

so the tangential acceleration is equal to tangential acceleration is equal to
ah this is the linear velocity in this direction therefore it is dv by dt
this is equal to d by dt of $r\omega$ this is r into $d\omega$ by dt which is our
 α this is what earlier we called it as α cross or which we call it as
capital a_r vector okay now radial acceleration there's an acceleration this
direction towards the center right this we must have the there should be a
centripetal acceleration otherwise we cannot keep the particle going around the
circular orbit

so a_r is equal to v squared by r this is the standard formula for centric
centripetal centripetal acceleration this is equal to $r\omega$ whole squared by
 r this is $r\omega$ squared what is $r\omega$ squared ω is $d\theta$ by dt
whole squared this is what earlier we called it as $r\dot{\theta}$ squared
remember what is $\dot{\theta}$ $d\theta$ by dt now earlier we had in lecture three
this formula a_r is minus $r\dot{\theta}$ square times e_r you may ask me what
is this minus sign sir now we are not having please remember it is minus e_r
that means if the this is the year direction unit vector minus e_r is towards
this

so my this is sorry if e_r is this direction then minus e_r is in this
direction

so what you have is the magnitude

so it is fine now now we have the we have the tangential acceleration term we
have the tangential uh tangential acceleration term here i should have
written t which is sorry and and we have the radial acceleration term
therefore we can calculate what is the actual a which we wrote it as earlier
so actual a is equal to which in our earlier notation is this

so i will put it inward commas is equal to a our unit vector e_r the radial
component of the acceleration tangential component acceleration θ right
so therefore the magnitude of the vector a acceleration is equal to a_t
squared plus a_r squared $r\alpha$ squared plus $r\omega$ squared whole squared
this gives you r times α squared plus ω to the power of 4 this is the
magnitude of the acceleration of a particle which is moving in the circular
orbital now the advantage of studying rigid body about a fixed axis is that
in a plane perpendicular fixed axis every particle goes around in a circular
motion next is ah we have to ask an important question there is a concept
called torque which we had introduced and studied and there is something
called angular acceleration what is the relationship between these two objects
and

so this next one this relationship between torque and angular acceleration it's a it's a very important topic

so we will show you'll see that the angular acceleration is α it is a vector quantity generally

so we will see that first we discuss the case of particle rotating about a fixed point under the influence of an external force then we extend the results to the case of rigid body rotating about a fixed axis first we consider a particle which is going on a circular orbit there is a tangential force this is radius is r this is radius is r yeah and then this is a mass m here it is the tangential force of the tangential force

so there must be a centripetal force otherwise it will not move on the circular orbit as we stated earlier it is necessary i am not indicating the existence of f of r it must be there otherwise you cannot keep it in you cannot keep the particle moving in a circular path and

so the tangential force gives raise to tangential acceleration

so the tangential force of t the magnitude is equal to mass times tangential acceleration

so the torque about the origin due to the force f_t the torque this force act on the particle therefore i can talk about torque on this particle about this particular center namely the origin torque about the origin talk about the origin due to force f_t now is equal to i am writing only the magnitude because this directions are perpendicular f_t times r this is equal to m into a t times r okay now what is eighty in the earlier section also we calculated it is r times angular acceleration with this we know the last section also we calculated therefore τ is equal to therefore τ is equal to $m r^2 \alpha$ okay

so this is same as what is $m r^2$ moment of inertia times α

so we have this important relation this is for a particle which is moving on which is moving due to a tangential force and then there is a centripetal force to keep it on a circular path τ is equal to $I \alpha$ in other words we say that the torque acting on a particle is proportional to α is proportional to the angular acceleration α therefore I is the proportionality constant

so I is proportionality constant yes this is the rotational analog of newton's second law f is equal to $m a$ now we extend the discussion to a rigid body of any shape but rotating about a fixed axis

so now we extend this discussion to rigid body of any shape but rotating about a fixed axis

so i have some arbitrary rigid body and i can set up the axis O is the origin and x i have here a small mass dm this dm will sweep a circular orbit and this is the tangential force the tangential force is $d f_t$ ok typical mass element is dm this is r ok

so i have this $d f_t$ i am writing only the magnitudes $d f_t$ is equal to dm times mass times tangential acceleration now now i can calculate torque detail i am writing i am writing parallelly here by the side therefore you can compare the torque you the torque detail due to the force the force $d f_t$ about the origin about the origin is equal to $d \tau$ is equal to r times $d f_t$ so the direction of force and r are perpendicular therefore it it simply goes to this r times what is $d f_t$ $d f_t$ is dm times a_t a sub t tangential acceleration and we know what is tangential acceleration is $r \alpha$ therefore it is therefore it is $\int r^2 dm$ into $r \alpha$ this is equal to α times integral $r^2 dm$ right this is for τ

so this implies this implies τ is equal to $I \alpha$ strictly speaking I should write in a more generic way τ is a vector it is proportional to it

is proportional to α vector then the constant of proportionality is it can be this is the moment of inertia times α when you go for higher studies you will realize that in general τ is proportional to α and then I is not simply a constant it will be a three by three matrix right now we are not concerned about it now we will come to ah perhaps the most important equation we have is this is something similar to this is something similar to linear motion f is equal to m times a so force vector is proportional to the acceleration vector the rule of mass is to play that of the constant of proportionality here the moment of inertia plays the role of the constant of proportionality between τ and α and now we have one more thing to do namely what is the relationship between what is the role of work and energy in rotational motion work and energy in rotational motion ok what is the definition of torque torque is defined as r cross f what about the dimensions of torque does that of work or energy uh however it's a vector quantity so the torque can rotate an object by θ so if a torque and when a torque is acting on a body and it rotates the object and it rotates the object rotates the object about an axis let us say by $d\theta$ then the infinite symbol the work done for this infinite symbol rotation is then then then the work done for the infinitesimal rotation for this infinite symbol rotation is what is that dW is equal to sorry work done so dW is equal to $\tau d\theta$ so this is something similar to in linear motion $f dx$ this is the analogy in linear motion f times the force acting on it it moves it by a small amount dx so here it is to torque so it rotates the body by $d\theta$ therefore the amount of work done is infinitesimal rotation is τ times $d\theta$ now i can calculate dW by dt sorry dW beta rate at which the work is done is equal to τ times $d\theta$ by dt $d\theta$ by dt is ω therefore it is τ times ω so what is this quantity rate at which the work is done is the what you call it as instantaneous power what is the rate at which the work is done that is what you call it as the power so i have this we have this power is equal to τ times ω it's a scalar quantity now we can ask what is the corresponding equation in linear motion what is the corresponding equation to this in linear motion i should not put in linear motion i will put it power is equal to yeah if i differentiate this will be d by dt of this will be the it will be it will be f into v ok so you can see that there is hardly any distinction between linear motion and rotational motion with respect to either kinematical equations or dynamic equations there is a one to one correspondence now we will have there is only one thing which is left out in the last lecture okay see in the last lecture we had seen the so called work energy theorem in the case of when you look at the kinematical equations i hope i have it here if i have it here i can show i am not sure if it is not that yes ah this particular equal the last kinematical equation v^2 is equal to u^2 plus $2as$ this is what we called as the work energy theorem in the linear motion now we would like to interpret give a similar interpretation the case of rotational motion and what is it right work energy theorem work energy theorem let me write little better work energy in rotational motion okay so where shall we start we start with we have τ is proportional to α and constant of proportionality I this is the basic equation

so this is same as $I \alpha = \frac{d\omega}{dt}$ rate of change of angular velocity this I can write it as little bit of calculus chain rule $\frac{d\omega}{dt}$ we want to bring in the basic variable angular displacement is $\frac{d\theta}{dt}$ this is what you call it as chain rule $I \frac{d\theta}{dt} = \frac{d\omega}{dt}$ and I will have $\frac{d\omega}{d\theta}$

so we have now I bring the $\frac{d\theta}{dt}$ to this side

so so I have $\tau \frac{d\theta}{dt} = I \frac{d\omega}{dt}$ is equal to $\tau \frac{d\theta}{dt} = I \frac{d\omega}{dt}$ is equal to $\int \tau \frac{d\theta}{dt} dt = \int I \frac{d\omega}{dt} dt$ but what is $\tau \frac{d\theta}{dt}$ remember this is the torque τ acting on the body induces an angular displacement $\frac{d\theta}{dt}$ therefore it is the the amount of work done in by the torque in rotating the body by $\frac{d\theta}{dt}$ now we can integrate on both sides on both sides we get the integral $\tau \frac{d\theta}{dt}$ this is from θ_0 to whatever point θ this is equal to $\int_{\theta_0}^{\theta} \tau \frac{d\theta}{dt} dt = \int_{\omega_0}^{\omega} I \frac{d\omega}{dt} dt$ this is $I \omega^2$ by 2 so what I will have is uh $\frac{1}{2} I \omega^2$ by ω_0^2 so what is that when when the particle is θ_0 when sorry θ_0 what is $\theta - \theta_0$ that is the change in angular displacement there is a change in angular displacement okay when the particle is at θ_0 the angular velocity is ω_0 sorry when the particle is at θ the angular velocity is ω therefore the corresponding angular displacement angular display corresponding change in angular velocity is given by $\omega - \omega_0$ the corresponding corresponding change in angular velocity angular speed rather right okay

so uh what I have is this is the work energy theorem

so it is something like the change in kinetic energy what we have is uh this is the this is the you can compare this with particle kinematics a particle moving having a linear motion the corresponding equation is what corresponding equation is the work done is $\frac{1}{2} m v^2$ sorry I forgot to write this $\frac{1}{2} m v^2 - \frac{1}{2} m u^2$ this is in linear motion linear motion okay so this is the so-called work energy theorem in rotational motion now we have to do we have introduced almost all important concepts now we can compare the similarities between rotational motion and linear motion

so I will have here here on the left hand side I will have rotational motion here we will have linear motion one what are the various quantities angular velocity what is the definition of angular velocity $\frac{d\theta}{dt}$ this also denoted by $\dot{\theta}$ now one the linear motion the role of θ is taken over by x therefore linear velocity is linear velocity v is equal to $\frac{dx}{dt}$ then angular acceleration angular acceleration α this definition is $\frac{d\omega}{dt}$ this also $\ddot{\theta}$ here linear acceleration here here means in the corresponding situation the linear motion curve is linear

acceleration is a is equal to $\frac{dv}{dt}$ of course I am doing in one dimensional no problem we can extend it to general case also now torque τ is equal to $I \alpha$ since we are considering mostly the rotation about the fixed axis that's why I am writing it otherwise I need to put proper vectors so in this case linear motion force is equal to F is equal to $m a$ remember we had seen moment of inertia takes the role of mass the role of mass in linear motion is taken over by the moment of inertia in the rotational motion that we have seen actually we are summarizing whatever we have done

so far in a sense kinematic equations $\omega_0 + \alpha t$ there are several what v is equal to $u + a t$ then next is θ is equal to $\theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$ here it is s is equal to some initial displacement which is already there in the system for the particle $v t$ this is equal to s is equal to $u t + \frac{1}{2} a t^2$ then finally ω^2 is equal to $\omega_0^2 + 2 \alpha (\theta - \theta_0)$ here it is v^2 is equal

to $u^2 + 2a \int dx$ then in a sense this equation is what you call energy conservation okay this is also rotational energy whatever loss and rotation energy this must go as a work done then 5. what is the work done expression for work done is τ is the torque acting on the body infinite displacement is $\int d\theta$ therefore this is the amount of work done in shifting by $d\theta$ therefore total work done is from a particular value to particular value now here in linear motion case work done w is equal to this is a one dimensional case we are writing here $\int f dx$ into $\int f dx$ into $\int f dx$ in general case force is a vector that should be dotted with the displacement vector then sixth one kinetic energy expression for kinetic energy $\frac{1}{2} I \omega^2$ now in the case of a linear motion kinetic energy is equal to $\frac{1}{2} m v^2$ there is one more equation which i will write but its derivation we will see in the perhaps the next lecture before that will have the power the power p is equal to $\tau \omega$ 7 power p is equal to next is

so this is the equation which i am going to write just to complete the analogy but we are going to see it little later it is the angular momentum angular momentum L is equal to $I \omega$ here it is linear momentum the linear momentum p is equal to $m v$ since i am dealing mostly one dimensional here i am not writing vectors otherwise one need to write okay then this we will this i will cross it here to show that we will be doing it later and nine then τ is equal to this we had seen in the much earlier how is τ defined dL/dt rate of change of angular momentum is what known as τ and similarly the rate of change of momentum is what known as force in essence this is the newton's second law this is the newton's second law in rotational dynamics now with this the analogy is complete let me quickly summarize what are the things we have done in this first lecture sorry in this particular lecture we started with the rotational motion about the fixed axis we had seen the basic first we did the kinematics then we moved over to the dynamics then we took a stock taking in the case of linear motion we have v is equal to $u + at$ at the kinematical equations in the case of rotational motion we have got this equations in fact all these kinematic equations we have shown the derivation of course we always start with a simple definition of we start with the simple definition of ω the angular speed is $d\theta/dt$ then relationship between angular and linear quantities it was earlier done in a different in a in lecture three we are taking a stock taking and right and this is the expression for the acceleration after that we went we discussed the relationship between τ and α which is a very important relationship it says that τ is equal to $I \alpha$ moment of inertia times α and this we did in at the kinematical level it is at the at the level of dynamic center okay this is the

so called newton's equation in the rotational motion work and energy in a rotational motion it is fairly simple if τ is the torque acting on it and it induces a displacement of $d\theta$ then $\tau d\theta$ and we can integrate it

so what we have is the $p = \tau \omega$ right this is the work energy theorem in the case of rotational motion we have seen that and then finally we we made a table comparing the basic equations occurring in rotational motion with those of equations occurring in linear motion and it is very remarkable that there is a similarity there's a fair there is an exact similarity between these two and we'll stop at this stage you