so

so today is the sixth lecture to just to give you a summary of what we have done yesterday yesterday we discussed about
so called the condition for the translational equilibrium and the condition for the rotational equilibrium we have made use of this concepts to discuss a few problems and in this process we also came across the concept of the center of gravity and its relation to the center of mass problem today we will continue further
so far we have been seeing the rotational motion is somewhat similar to linear motion equations 5 for example what is known as velocity in the case of linear motion its role is taken over by angular velocity d theta by dt etc the the linear acceleration is dv by dt and the angular acceleration is d omega by dt etc today we will continue further
so far we have not asked we have not asked rather a very important question in the case of linear motion you have mass the concept of mass which comes in newton's equation everywhere and who takes the role of the linear mass in the rotational motion and so today's topic for discussion is moment of inertia basically this particular lecture moment of inertia and there are two rather important theorems and what i'll call this parallel and perpendicular access theorems this is what we are going to focus on and
so the question we are already asked is what is the what is the analog of mass a log of mass usually denoted by m in rotational motion this is the uh i won't call it as a motivation this is an intriguing question which everybody should ask it will come very naturally and we see uh what is the answer for it
so there's one more thing even yesterday we had seen rotation of a rigid body and then here after we are going to consider rotation about a fixed fixed axis when this is very important
so rotation about a fixed axis that's what we are going to consider the general rotation of a rigid body in all possible direction access is considered a topic for advanced study we are not going to consider it and
so what we have is a let us say rigid body it is one axis this is a fixed axis and you consider a particle here and then it will be making a circular motion its radius is let us say ri this particle has got a mass m i here right then the kinetic energy k the kinetic energy of the rotating body i will denote it by capital k this is equal to
so this whole body is uh can be viewed as different masses m1 m2 etc i am considering a typical mass which is located at distance r a from the fixed axis center it at the distance
so the kinetic energy of this particular object is like i can consider this as sum of all the kinetic energies i am not indicating irons over what over all particles this is equal to half of sum of sigma over i mi this is what is velocity this is v i r a omega this is whole square lambda right
so half mv squared v is omega cross r this is perpendicular it will have r a times omega and omega is the same for every particle within this rigid body and whereas this r i the distances will change
so this is equal to half of omega squared is common and you are simply left with summation over i my r a squared and this is the quantity which is known as the moment of inertia
so moment of inertia of a rigid body is simply uh sigma or summation over m i r a squared where r a is the distance from a fixed axis
so always talk about moment of inertia of your body about an axis that is important i can also consider the moment of inertia of same body about some other axis

so there is no point in simply mentioning what is the moment of inertia of a sphere or any object   you should say that the question you should ask is what is the moment of inertia of a body about an   axis center that is very important right okay this is something this equation is uh this is   usually denoted by i therefore i have k the total kinetic energy is equal to half of i omega square this equation reminds us of this is something similar to this is something similar to something   similar to in the case of linear motion we say that half m v squared

so therefore the when you see this equation immediately your bell should ring in your mind what is that   you would like to compare this in the case of linear motion the expression for kinetic energy is   half mv square this is something similar to that right and now we are going to consider there are certain important properties of moment of inertia

so that is the next topic for discussion

so properties of moment of inertia i i the symbol is it for moment of inertia i simply call it first one first thing is the kinetic energy of sorry the moment of inertia of your body   does not depend on omega namely the angular velocity does not depend on then what does it depend on it depends on mass it it depends on   actually mass distribution rather to say so mass distribution in terms of shape and size   okay this is the first property then second property   and it's a characteristic of the rigid body it is a characteristic it's very typical to each rigid body characteristic of a rigid body then and also not only that and also about an access and also about an access also an axis about which it rotates it means rigid body rotates now just like mass is considered to be the measure of inertia of a particle or a body similarly moment of inertia is a measure of the rotational inertia in the case of linear   motion you can call it as um it's a measure of translational inertia here it is a measure of inertia in rotational motion it's a measure of inertia in rotational motion and uh as already told it is better to keep in mind that it is also a measure it also depends   on mass shape size distribution of mass then one more property is there it says the mass does not depend on any axis or any anything

so here it depends on the   nature of rotation about an axis nature of rotation about an axis then it is good to do this exercise whenever you come across   any any physical quantity first time for the new is better to write down its units and dimensions   what are its dimensions the mass times l square

so therefore the units now in c g is kilogram meter square and remember it's a scalar quantity it's a scalar quantity that we need to keep in mind next we will proceed to calculate moment of inertia of a few   of certain objects which we come across very frequently in physics first a thin circular ring this is the first one thin circular ring

so i have a circular link like this it's a fairly simple calculation then i need to indicate an axis access is passing through the center

so its radius of the ring is r but this let us say that uh the total this is the axis of rotation and total mass is young now i take a typical point here and   let us say that mi is the mass now what's the definition of moment of inertia it's a small   element i'll take definition of moment of inertia is m i r a squared

so here this uh

so every point   on this circular ring is a distance r therefore let us say that mass element is m um mi i square this is equal to r square times summation m i   summation m i is the total mass of the particle therefore this is m r squared

so this is nothing but adding all the masses which are located on this ring and

so moment of inertia   of a circular ring about an axis passing through its center in a plane perpendicular to the plane   of the circular ring that is important okay i am considering an axis which is perpendicular to the   plane of the circle and it also passes through the center and

so this is the moment of inertia now as i told you we are going to consider few examples we will have one more   one simple example as an illustration i am looking for some space yes i have here suppose i have i have here three masses maybe i will divide this into two i can save the space

so i have here a triangle okay it's an equilateral triangle a yeah i have uh m1 is equal to m and m2 is equal to m here   at the vertices m3 is equal to m so what is the axis i am considering axis i am considering is   the altitude so therefore it is a by 2 this is a by 2 here

so moment of inertia of this   triangular lamina it's not a laminar sorry the moment of inertia of these three mass three masses   located at the vertices of this uh equatorial triangle

so i altitude what is the meaning of   i sub altitude means moment of inertia of these three masses about this particular altitude is m 1 into 0 squared plus m 2 into up from the axis it is a by 2 whole square plus again m3 into this is located distance of a by 2 whole square

so therefore this is equal to   2 times m into a by 2 the whole squared therefore it is m a squared by 2. it's a simple calculation to illustrate that how you how a moment of inertia of a distribution is   calculated and now we will consider the second example remember we are calculating the moment of inertia moments of energy of various objects which we regularly   come across and we are going to make use of this next is moment of inertia of a rod uniform rod lord of uniform cross section of mass is uniformly distributed

so i am going to consider an axis which is passing through the center center of mass this is the axis suppose i consider this as   this will come little later this will come little later what i will do is i will put masses here sorry it is a this is a massless rod sorry it's a massless rod light rod at the two ends of it we have two masses m1 and m2 and then   i want to calculate the moment of inertia of this this ah in principle it is almost same as this

so moment of inertia about this axis indicate m into ah l by 2 the whole squared   this is l by 2 this distance is l by 2 plus m into l by 2 the whole square therefore it will be m   l squared by two

so this is almost same as this only thing is uh the moment of   inertia of the this figure is uh turns out to be the same because we are considering the axis   passing through m one and ok now there is a concept called radius of gyration the radiation of gyration is like this

so one thing is clear whenever you calculate moment of   inertia of any object there is going to be a mass term times uh a quantity which has length squared   there may be some proportionality constants there may be there may be certain objects which are   going to be numbers

so i'll just call it as number now you treat this whole thing as m k squared so then   you have to calculate k for each of these cases this is then k is called as radius of gyration   why what is the idea what this means is that about a particular point the whole the mass of the   whole body is located at here at a distance of k because whenever you have moment of inertia to be   mk squared what it means there is a mass m which is located from the axis or from a fixed point   whoa at a particular distance k and

so the moment of inertia is either expressed as it   is or in terms of radius of gyration and we will see in problems and for example in this case when   i write it as m k squared then the radius of gyration k is equal to l by root 2 okay now i will consider the third example the moment of inertia of a rod

which is uniform rod about an axis which is at the one end
so it's a uniform rod rod of uniform cross section
so there is a concept called mass per unit length the whole length of the
rod is l let us say
so what i will do i will consider an element here this is a distance x dx okay
now uh rho is the mass per unit length it is one dimensional therefore it is
the mass per unit length we consider this is the axis of rotation ok
therefore if rho is the mass per unit length what is the total mass total mass
is length times mass per unit length now moment of inertia of this element
this particular element dx is is equal to let us say this whatever is the mass
here that times x square the small element is located distance of x now what
is dm it is dx times rho times x square now i want to calculate the total
moment of inertia therefore i have to integrate it rho x square dx and at
this end it is x is equal to 0 at this end it is x is equal to l therefore i
have to integrate with 0 to l
so rho into x cube by 3 therefore l cube by 3 this i can write your rho l into
l squared by 3 rho l is m therefore m l squared by 3 so m l square by 3 is
the moment of inertia of a uniform rod of length l about an axis which is
located at one ear indent right then next one will move little faster now we
will calculate the moment of inertia of a rod uniform rod about an axis
passing through the center this is l by 2 this is l by 2. therefore this i can
view it as ah moment of inertia of two rods each rod about an end of the rod
therefore it is um it is is equal to m by 2 into l by 2 whole squared by 3
into 2 therefore it is ml squared by 12 phi we are going to calculate moment
of inertia a few more objects therefore i will divide this space
so now moment of inertia of a circular disk it's a circular disc moment of
inertia of a circular disk
so i have a circular disc
so this is the center center okay now what i will consider this radius is
capital r its radius is r i will consider a typical disk here it has an is
an annular space if i take this as r then the this annular portion has a width
dr
so i consider the moment of inertia to be now what is the circumference of this
it is 2 pi r 2 pi r then area is d r then what is the mass of this this is
area into mass per unit area rho is the mass per unit area mass per unit area
of the material and that times this mass is located distance of r square
therefore i is equal to 2 pi rho i can take out integral rho cubed dr r
going from 0 to capital r
so this is equal to pi rho r to the power of 4 by 2 this is equal to m r
squared by 2 why the total m is equal to what is the mass of the disc area
into mass per unit area therefore from here i can split it out pi r squared pi
r squared rho when pi is missing here i can write pi r squared rho will
become m the remaining terms right
so like this we have now that of a solid cylinder similarly you can calculate
moment of inertia of a solid cylinder which i am not going to do it i of a
solid cylinder about an axis passing through the about an axis about an axis
passing through the center about an axis passing through the center passing
through the axis of the cylinder this we can work it out this is again is m r
squared by 2. now i will do this calculation for hollow cylinder okay
so i have this hollow cylinder like this finite all the cylinder and then this
is the axis ok now i consider a small element it's a hollow cylinder it's a
what i call a circular strip band rather which is lying on this cylinder now
what is the length of this length of this is 2 pi r because the radius is r
and then this length this is of width dl i will take then ah this hollow

cylinder this is of the area   therefore the mass per unit area i will take it
as rho mass per unit area and let me say that this height of the cylinder is
h i will take the length of the cylinder rather ok this row and this is
located at a distance of r square
so this is what is d i
so if i want to have i i have to integrate it
so when   i integrate it whatever i get 2 i can take out pi i can take out r i
can take out row this is r cube integral d d l is simply l therefore 2 pi r
cube rho l ok now now what is the mass of the cylinder mass of the cylinder
is 2 pi r the circumference that into l into rho
so this is 2 pi r l rho
so 2 pi r yellow if i factorize it is 2 pi r l rho into r square therefore it
is   m r square this is what i call it as m the same way you can calculate
moment of inertia of   various objects but i am going to do the moment of
inertia of one object then we will proceed further
so this is the moment of inertia of a solid sphere is an important quantity we
will be using it again and again
so i have a solid sphere about an axis passing through the center
so i have what i consider so this i don't need perhaps oh no right now i will
consider um a small sphere i'll consider a sphere i'll consider a sphere of
radius r and small increment d r and consider this portion
so what i will have this surface area of the smaller sphere is 4 pi r square
therefore the volume of this region is 4 pi r square d r   and mass of this
region is mass of this region is 4 pi r square d r times rho this whole thing
 is located at the distance of r
so i want the moment of inertia therefore i want to integrate it this is r
squared sorry i forgot right
so this will be 4 pi 4 pi   0 to r integration r to the power of 4 therefore
it will be r to the power of 5 by 5   and rho what is the mass of the sphere
mass of the sphere volume 4 by 3 pi r cube times rho therefore   i can write
this i in terms of i can factorize this in terms of i will have three m three
by five m r square ok this is the moment of energy of a sphere about an axis
passing through the center now we are going to have we will consider two
important theorems one is called as there are two important theorems which are
repeatedly used in the calculation of   moment of inertia problems one is
called as perpendicular axis theorem this is valid for planar objects is valid
for planar objects   we state the theorem and proof is not necessary at this
stage however not complicated one can   learn from some advanced books now what
it says is suppose i have a planar object it has three axis x axis y axis and
z axis i want the moment of inertia of this planar object one wants the moment
of   inertia of the planar objects about the z axis this is equal to what it
says is   you need to consider to access perpendicular passing through the
planar object then   this is one is this ix another is this moment of inertia
about this y in other words if i want the moment of inertia of a planar object
passing through an   axis perpendicular to the plane of the object then one
needs to consider two   perpendicular directions which are concurrent with
this axis   located on the body then if i know the moment of inertia of this
this is let us say this is i   x if i know then i y here i know then i know
the moment of inertia about z that's the idea
so the the moment of inertia the mi of a planar body about an axis
perpendicular to its plane is equal to the the sum of moment of inertias
about the two perpendicular axis about the two perpendicular axis concurrent
with with the perpendicular axis and lying in the plane now we will as i
told you proof is not required but you will make use of it i   will give you

two illustrations suppose i consider example one i consider circular disk
right we have
so i have a circular disc and x y z right
so i want the moment of inertia about z axis that's what i want i said it this
is same   as moment of inertia but i x moment of inertia about y these two i
have to add right
so i z i   know what is the moment of inertia about z axis of a circular disc
where we have calculated earlier of a circular disc that we have calculated
as   m r square by two moment finish of a circular disc therefore i said i
know but i don't know what is i x i x and i y you remember this diameter is
symmetric it divides the circle into two   therefore moment of inertia i x
should be same as moment of inertia i y therefore two times   moment of
inertia of ix is same as two times moment of inertia of i y this is equal to
is equal to m r squared by two this implies ix is equal to iy is equal to mr
squared by 4 right now i will do one more simple problem
so it makes our life easy if i want to calculate the moment of inertia of the
circular disc   about this axis x axis or y axis now circular ring one more
example circular ring again no the moment of inertia of a circular ring we
have calculated circular ring x axis y axis z axis i need not write
everything   circular ring i think we already done moment of inertia of a
circular ring where is it uh probably that was this circular ring first we did
mr square   that was the first example we considered moment of energy of a
thin circular ring this is a mass cut
so therefore i used z equal to m r squared
so what do we have moment of inertia of x plus moment of inertia of   y
perpendicular axis theorem this must be same as moment of inertia angle this by
symmetry 2   times i x is equal to m r squared therefore i x is equal to i y
is equal to m r squared by two next we are we have to discuss what is known as
parallel axis theorem parallel axis theorem what the parallel axis theorem
says is   ah it it is a sorry before it is applicable to a body of arbitrary
shape applicable to applicable to body of arbitration unlike the perpendicular
axis theorem which is valid only for planar objects and
so what we are going to do is that the idea is this is a solid um what we want
is that now we want the moment of given the moment of inertia suppose this
is the center this this is the center let's say center of mass this is cm of
this object given   the moment of inertia about the center of mass about an
axis passing through the center of mass   i want the moment of inertia of the
object about some line l let us say this is known known we want to calculate
moment of inertia about l this is equal to what answer is provided by
so called this parallel axis theorem what it says is that i l is equal to i
suppose i call this as z axis this this i will call it as let us say z prime i
z prime i z prime is equal to i z i sub z plus mass of the object then the
perpendicular distance between them right let me again repeat i know the
moment of inertia   of this object about an axis passing through the center of
mass then i want to calculate   if one wants to know the moment i finish of
the same object about some other axis   let us say this is the z prime then the
moment of inertia about z prime is same as the moment   of inertia of the
object passing through about an axis passing to the center plus this
particular   product of mass into the distance between square of the distance
between them and okay now we will we need to fix the axis if you required
just x axis here and y axis here just in order to avoid confusion right now we
will do two examples we will do two examples first an example one is already
known this is z axis this is l by 2 this is l by 2 i want the moment of
inertia about the line   which is this z prime from one end in other words

given the moment of inertia of a rod about an axis passing through the center i want to calculate the moment of inertia of the rod about an axis which is passing uh through the one end of the rod but both these axis are parallel that is the situation right then i said prime is equal to if we had we have i said we had calculated sorry i_z is ml squared by 2 ml squared by 12 which we had calculated where we had calculated this this was the example ah this is ml squared by 2 12 now i want the moment of inertia about z prime

so moment of inertia i about z prime is equal to i_z plus total mass is m distance between these two lines is l by two whole square i_z is ml squared by 12 plus ml squared by 4

so this is this is uh this is ml squared by this is 4 12 1 3 4 ml square by 3 this we had done moment of inertia of a rod about an axis for about an axis passing through one end this recalculation we have done so it is a very simple verification of perpendicular axis theorem so one more illustration we will do one more illustration we will do that now i will consider under circular ring moment of inertia of a circular ring example circular ring about about a tangent

so i have a circular ring okay this is the diameter i have a tangent okay so and for this now i need to know the moment of inertia about the diameter i diameter i dial right under now i need two perpendicular axis

so one axis here other axis have to let us say that i have to i will just indicate it here like this ah i don't need this oh my god

so we have to be careful with the diagrams

so this is the iron okay i diameter

so uh by perpendicular axis theorem two times i diameter is equal to the mass square this we have already done therefore i diameter what is m r square m r square is the moment of inertia of the circular ring about an axis which is perpendicular to the plane of the circular ring that is m r square from this i am calculating the moment of inertia of the about the diameter this is equal to m r squared by two

so here i have made use of perpendicular axis theorem now i need to know the moment of inertia of this circular ring about this tangent

so i tangent now i will make use of parallel axis theorem because i know the moment of inertia about the center right is equal to what is that i want um is equivalent about an axis which will be m r squared gamma square by two this is i will write a step therefore it will be i about a diameter plus mass into distance between these two lines is r

so r square

so this is equal to m r squared by 2 plus m r squared this is equal to m into r square by 3 this is an interesting problem in the sense that we are making use of both the this is parallel axis theorem here we are making use of both the theorems okay

so in this we are going to see some more examples at a later stage let me repeat what we have done in this problem i want to calculate the moment of inertia of this circular ring about the tangent i have to make use of parallel axis theorem i can for that i need to know the moment of inertia of the circular ring about the diameter that we have not calculated

so what is that we know we know the moment of inertia of the circular ring about an axis which is perpendicular passing through the center ok that is what is ah we have it therefore i have that is m r square therefore the i about diameter moment of inertia about diameter is m r square by two now i can calculate the moment i finish about the tangent it is moment differentiable diameter plus mass times the square of the distance between these two parallel lines m r squared for m r square by 3 and to summarize what we have

done is we found that the moment of inertia   is the the concept of moment of
inertia is the rotational analog of mass in linear motion and   then we have
calculated the moment of inertia of various objects and we have also seen two
rather   very important theorems which are repeatedly one makes use of
so called the perpendicular   axis theorem and parallel axis theorem the
perpendicular axis theorem is valid for   planar objects parallel axis theorem
is valid for object of any arbitrary shape and size only thing   we need to
know the moment of inertia of about an axis passing through the center of mass
of that   systems life is very simple if the systems are symmetric and we will
see in next class
so you