so the topic for today's discussion is equilibrium of a rigid body
so here after mostly our focus is going to be on rigid bodies and well we have
seen that uh   there are the forces acting on rigid bodies can be divided into
two groups one external forces external forces this we generally call it as a
force ext or internal forces this we were we have been denoting it as force
internal we had seen that   these internal forces usually do not contribute
either they   do not contribute for they do not contribute either for rotation
either for translation or rotation this we had seen
so uh unless otherwise we specify most of the forces what we are going to
consider   are external forces
so we will simply denote it by f vector and now remember we have had
so far   we have dp by dt rate of change of momentum is equal to force and we
also have and we also have and the rate of change of angular momentum is what
we call it as torque which is responsible   for the rotational motion of the
object so whenever a body is subjected to force then   there is going to be
momentum or acceleration translational motion is possible   and due to torque
you will have the rotational motion   now we have the concept of under what
conditions the system is in mechanical equilibrium mechanical equilibrium
there are different kinds a body may be under different kinds of equilibria
there is something   else called thermodynamic equilibrium chemical
equilibrium right now we are concerned about   the concept of mechanical
equilibrium so what is this concept this is like this   if linear momentum is
a constant of motion this is that linear momentum is a constant of motion that
means linear momentum is conserved it is not changing then in that case what
happens on the other hand if the angular momentum is a constant of motion
will deter here it's a constant of motion then what can we have and
so what happens in this case this implies the sum of all the forces   acting
on the body there may be several forces force is a vector
so sigma over i   one is equal to n say this becomes zero
so for mechanical equilibrium   system of all forces acting on the body must
be equal to 0 this is what you called as   the translational equilibrium
so this is what you call it as this terminology translational equilibrium ok
then next is if tau is 0 maybe i write here if l is a constant of motion then
this   implies
so various stocks acting on the system depending on how many of them this is
equal to   zero this is what you call it as rotational equilibrium ok
so what is the meaning of the first equation the meaning of first equation is i
will write   here meaning of first equation is remember it is a vector
equation therefore ah the sum of all the   x component of all the forces is
equal to zero
so if f i is the ith force the x component and i   am summing over all the
forces this is equal to 0 similarly all the y components is equal to 0   and
all the z components is equal to zero now what this means is that torque is a
vector again   sum of all the x components of the torques over all for all the
torques acting is 0 and sum of   all the y components of torques is equal to 0
and sum of all the z components of torques and here i am writing in component
notation these two equations these two equations are   in component notation
therefore there is no need to write vectors we shouldn't write suppose we have
coplanar process essentially   a special situation a coplanar for case of
coplanar forces and let us say that the system is under equilibrium it is some
say essentially   like a two dimensional problem essentially a 2d problem two
dimensional means all the forces are   acting on let us say x y plane and then
what happens for the translational equilibrium f5 is equal to 0 this means 2
conditions what is it essentially what is it sum of the x components is equal

to   0 and sum of y components of the forces equal to zero
so this is essentially two conditions and and you need to look for a direction
which is perpendicular to this 2d plane there is no   rotational motion about
an axis therefore tau is going to be only tau about an axis tau about an axis
perpendicular to perpendicular to f1 f1 and f2 vanishes
so essentially it is only three conditions essentially okay right now now let
us say that we consider an important case   suppose some of all the talks are
acting on this body they vanish now you may say that   i'm going to calculate
this torques with respect to some other origin   is it possible that the body
will have rotational motion the answer is no the reason is   we'll see that
how i'll consider i have some of tau i is equal to 0 this is for i will call
it as rotational equilibria r g right is it going to remain valid is it
remain does it remain valid does it remain valid does it remain valid if i see
if the origin is changed if the origin is changed you'll see that because it
is very easy simple calculation i have the origin here   and one of the forces
this is the power a   b the two points here remember it is a it's a couple
acting minus f and on this it will be f remember these two have to be
parallel   i mean these two lines but in opposite directions this is b i will
join this this is position vector r one   this va vector is called position
vector r1 similarly ob vector the position vector is r2
so these two forces f and minus f constitute a couple   on a rigid body
so now let us calculate the moment of the couple moment of the couple this
couple is what is the moment of the couple this is   r1 crossed with the force
 crossed with minus f1 then plus this r 2 crossed with f actually it is here
the force is   minus f therefore i should not write this one here simply write
minus f and f right this is nothing but this is this is what will happen here
this is equal to what is um this quantity this is going to be r2 minus r1
crossed with f what is r 2 minus r 1 from this triangle   o a b it will be a b
this is equal to a b cross f
so this simple calculation shows that when you   calculate the moment of a
couple that is going to be independent of the origin   whatever origin you
choose again it is the moment of the couple is a b cross f   okay
so we can say that this translational equilibrium is independent of the
location   of the origin under
so translational sorry rotational equilibrium audi remember what it is is is
independent of of the location of the origin
so if a particular if a body is under rotational equilibrium with respect to
a particular coordinate system and then what you do you change the origin and
look at it still it   will remain the same that is the message and okay now we
will consider various cases as possible   the first case is like this i will
you can treat it as a kind of illustration or   i will treat it as an
illustration that means explaining the basic concepts i consider a rod a b
this is a b here and you have the center here at c   this is little a this is
literally a the distance this is a uniform rod of uniform   cross section now
we will say that there is a f acting and then there is a f acting here
so this particular rod is subjected to two forces f here of here   is there
any now it will induce a torque like this this force will induce a torque in
this direction   this is going to be anti-clockwise this is the clockwise both
of them they cancel   therefore tau is equal to zero but the tau is equal to 0
means it is under   rotational equilibrium sorry so it is under rotational
equilibrium   and what about sigma forces total forces is equal to f total
sorry is equal to f total f total is not equal to 0 in fact that is equal to
2f
so obviously what will happen this is a case where is it under rotational

equilibrium yes   what about translational equilibrium no it is not on a transmission now we will   consider the other case this other case is like this we'll consider the same rod and there is a at this end i have a force acting like this there is another force acting like this   it's again it's a couple strictly speaking that's the definition have the center here   now only thing i changed reverse the direction of one of the forces in this case what will happen   uh sigma f5 is equal to f total is equal to 0 they are in opposite direction therefore it vanishes however the sum of torques is is equal to the torque total what is the total torque acting on it

so this will with respect to this this will   have a torque in this direction it is in the anti-clockwise direction it is anti-clockwise   direction it is 2 times the torque due to each force

so this is not equal to 0   therefore what about translational equilibrium of the system it is under translational equilibrium   whereas what about the rotational equilibrium of the system no this will rotate and

so such   a situation is what is known as such a situation where the body does not have   translational motion ah where i just under translational equilibrium   however it rotates about a particular point and axis this is what is called as a pure rotation okay we will uh one special case we will see this is what is called as a liver problem   common liver which we use this is what is called principle of moments actually this is start even   at a very element even at the early stage in school however we shall discuss this from the point of view of translational equilibrium and rotationally

so i have a simple   lever like this what you have is you have a fulcrum this is what you call it as a pivot or fulcrum as it is known and this is subjected to here there is a force this distance is t1 and then this distance is d2 call this point as a call this point as b okay right now this divide point you can call it as wow about this particular point now what happens   now this body this whole rod will have a   ideally speaking a liver should not have any mass

so it is of negligible mass

so an ideal liver has negligible mass

so there is going to be the two forces are one force is acting here another force f2 it will have a moment like this it will reduce a moment this will induce a moment like this finally it will be there so   generally what you do is this is the low dom this section is known as nodum this portion is known as   load load um this a to y is known as lodam then what to be is known as this effortum see what you do is you there is a weight here which requires to be lifted or moved   you have here effort on some force you are going to apply okay now there is   there is a forces are acting here therefore there will be a reaction at this particular point   reaction at the fulcrum reaction of the support at the fulcrum this is a vector quantity

so the reaction at the fulcrum is the support   reaction of the support at the foreground okay

so for translational equilibrium for for translational equilibrium what we need this r must be equal to f1 plus f2 now taking moments now there are three forces acting essentially f1 f2 and then the   reaction it's also of a force variety now taking moments about this particular point for moments about whoa what do you get here this moment is moment duty f1 is f1 into d f1 into d1 and this is uh because it's in the translation equilibrium now you will see that this is must   be equal to f2 into d2

so this is for ah for rotational equilibrium you have this condition for translation equilibrium you have this condition and

so from this you can

so this object   is not rotating at all it is in equilibrium it is neither rotating like this nor it is rotating   like this both these moments are cancelling with each other

so from this i have we have f1 by f2   is equal to d2 by d1 under

so this is what is known as mechanical advantage see what we want is ideally if if f1 is much much larger than f2 let us say then   in order to maintain this equilibrium we need to have this distance d2 much much larger   that is the idea

so d1 is much smaller and this is a common sense experience center   okay for battery line input western now we will go into concept of center of gravity all the time the concept of centre of gravity it is a common experience that would have seen   everybody can do it if you have a notebook or a cardboard it can be held on a at a particular point at a particular point where one can vertically hold it

so that this but this book or cardboard it is balanced okay

so the how does this happen

so that   there is a going to be reaction at the tip this reaction at the tip what we call it as arm   this reaction at the tip is going to balance the the total weight the total weight of the book mg   of the material of the book or material of the book let us say

so the notebook is under translational equilibrium it is under translational equilibrium   not only that it is also under rotational equilibrium why otherwise if there is uh   the different forces acting here they can tilt like this or tilt like this it's not happening

so so not due to uh it is not due to unbalanced torque if there is going to be one it will tilt so   what happens is the center of gravity this is now we define what is known as the center of gravity cg the center of gravity is located such that the total torque on the body center of gravity   located such that the total torque on the body due to the forces let us say that there is a   young one g there is something else m2g etc so the total torque due to different forces they   balance they cancel out and

so torque is equal to summation over i r i

so this is the r one vector okay that crossed with whatever is the mass at this particular point and acceleration due to gravity that is equal   to zero

so happens is the center of gravity of a body is that point where the total gravitational   torque on the body is zero

so this is this definition of center of gravity the total   gravitational torque acting on the body must be equal to 0 and

so ah r i and g are perpendicular   to each other therefore essentially you are left with m i or a summation is equal to zero okay at this stage one would think that this is same as the center of mass but is not remember the center   of mass definition is this quantity divided by the total mass however it it will turn out to be the   same if the the origin is the center of mass of the body right if origin is going to be the center   of mass of the body then it will turn out to be the same

so what happens is that the center of   mass and center of gravity both of them will turn out to be the same if the body is subjected in a   uniform gravitational field

so center of mass is similar to center of gravity in a uniform gravitational field now if g on the other hand if g varies from point to point   if the g is going to it is point to point then center of mass and center of gravity do not go inside now how is center of gravity of a body determined it is a again a very standard   one suppose i have a cardboard or something and i want to find the center of gravity so   this is a very standard procedure what you

do is you suspended it from a particular point a suspended from a particular point a this point is a
so the whole   the weight is going to act along this though
so it will be along this direction now you take some other point b and suspend it again um then its weight will act along this when you put it this way when you   when you fix this whole body about uh fix it about the point b
so similarly you will find that   various lines will intersect suppose i have another point c here and do this
so this is the center of gravity
so this o is the cg of the body okay we will work out an illustration you'll work out a simple problem and it will illustration you can treat it as an example various concepts which are involved with
so i have a rod this end i will call it as a this end i will call it as b
so there is a pivot here k1 there's a pivot here k2 it's obvious that there will be a reaction here this is r1 this is r2 r2
so this will indi um i have the cg of the body
so this is uh g the weight will act down right let us say that this weight is weight remember mass times weight is what mass times acceleration due to gravity this is 4   then i have another weight at this particular point p let us say that this is the point p there is a weight w1 this is w1 is q1 to be 6 times g okay now k1 and k2 are pivots or knife edges whichever way you want to take   there are few dimensions given in the problem i will specify that a b is 70 centimeters namely the length of the rod then a g a g is because this is the center of gravity 35 centimeters right and ap is given to be 30 centimeters
so obviously pg would be five centimeters ok then we need certain distances ah what is this ak1 kk1 aka 1 is equal to and ak1 is equal to bk2 is the location of knife edges is 10 centimeters
so we know what is k1 g and k2 g k1 g is equal to k 2 g is equal to
so 35 minus 10 is 25 centimeters right now we have it is under translational equilibrium therefore the forces acting upwards   r1 and r2 must be equal to forces acting downwards
so translational equilibrium implies r one plus r two two reactions must be equal to w 1 plus w
so we can have this
so r 1 plus r 2   is equal to w 1 is 6 w is 4 therefore 10 g okay this is one equation then i'll label it as one now taking moments about taking moments about g for rotational equilibrium delta now it is going to be in the clockwise whereas this will be in the anti-clockwise   and w1 with respect we are taking moments with respect to the point g
so this will rotate in this direction so minus r1 into what is the distance k1g plus r2 into r2 into k2g now this plus there is one w1 acting here w1 into pg sum of the all the moments is equal to 0
so we can substitute the numbers this is 0.25 in meters k 2 g is   0.25 meters p 1 g is just 5 centimeters here p 1 g is 5 centimeters therefore is 0.05 meters from this you will get an equation r1 minus r2 is equal to 1.2   g strictly speaking i should put if i want to write the units i should write newtons here and some little bit of arithmetic is involved do it now from these two equations   you can calculate r one and r two r one is equal to if i had these two equations   two r one you will get that from there to two r one is equal to eleven point two then r1 would be 54.88 newtons and r2 is equal to 43.12 newton center right
so it is very easy to do these kind of problems all that you need to do is not going to better ah

so all that you need to do this kind of problems is to write the balancing equations for forces and write the balancing equations for dots namely translational equilibrium condition and rotational equilibrium condition when you take when you write down the rotational equilibrium condition you need to take talks about a suitable point where your algebra would be simpler and now we will consider an illustration another problem but there are different kinds of problems that are involved which are repeatedly asked in examination various things can be asked one is a ladder problem this i will call it as an illustration or a problem can treat it as a problem or example this is what ladder problem led up right now

so the situation is like this i have a wall i have a ladder here this is a ladder ladder is a b the wall is smooth the wall is smooth whereas the floor is rough i can have this also rough this also smooth will do this

so this point i will call it as d the this is c call this point as c

so the weight will act downwards m into g and now what are the various forces acting that's what first we should sketch very clearly indicate very clearly then indicate the directions of torques also very clearly now because the wall is smooth there is going to be a reaction here this i will call it as n2 i need to have a little longer one for reasons you will realize why this is cn2 then because it is rough here what happens is you have here a frictional force f and this is what is going to be the normal reaction of the floor on the foot of the ladder now these two forces can be combined into one i mean i need to i'm not indicating it uh very clearly

so this is going to be the f this particular one is what is going to be the f sorry i need to do it this is f now when uh for equilibrium when i produce this when this force is produced they must meet at a particular point not a bad diagram and this point i will call it as go why should they meet there in that case what happens if they don't meet what will happen because there if they meet what will happen is all the torques about this particular point will be zero therefore it will be under really rotational equilibrium and it's also under we will see this okay a b length of a b is l right we will call this angle as theta which angle the the angle made by the ladder with the floor as theta right

so what is the f actually f is nothing but the reaction of reaction of the floor reaction of the floor on the foot of the rad ok now let us workout it is under it is under translational equilibrium and also under rotational equilibrium

so translational equilibrium implies sigma of all the forces is equal to zero what are the forces acting third the we will have it as we can have there are two kinds of forces one along horizontal direction one along vertical direction therefore we will have it as two equations and sum of all f y is equal to zero f of x is equal to zero implies is equal to uh there are only two forces uh the forces along horizontal direction n2 is one force it is acting at a there is another force namely the frictional force f which is acting at b therefore f is equal to n2 what about the forces acting along y direction only two one is the weight of the uh ladder which is acting at the center of gravity and the reaction at this particular point this implies n1 is equal to mg two important equations we have got all that we have done is to make use of translational equilibrium nothing more now rotational equilibrium

so sum of all the torques this must be equal to 0 about which point i would like to calculate and i would like to calculate that talks about b and so i will say that uh net torques about the point b you can choose any point you want earlier we already seen it then what will happen then this will

rotate in this uh the weight of the body will rotate in this direction in the clockwise direction whereas this n2 will rotate it in the counterclockwise anti-clockwise direction

so mg into mg into the foot of the perpendicular from this particular point that will be equal to   what this length will be ah l by 2 this whole length is l by 2 this is l by 2 into cos theta minus n2 into n2 into ah what did i do i should drop the perpendicular to this that would be   whole thing into sine theta this length into sine theta into sine theta l sine theta   is equal to zero

so i will have n2 is equal i will cancel this n2 is equal to   i know it is already equal to f here that will be equal to mg into cot theta by 2 now i know what is n1 i know what is n2 therefore i can calculate   what is the total force f is same as n2 therefore total force what is the total   force actually it is the reaction of the floor on the foot of the ladder this will be equal to   square root of ah as from this particular figure this particular figure i have here n1 squared   plus f square this is same as square root of n1 squared plus   n2 squared this is equal to n1 squared is mg squared 1 plus cos square theta by 4

so this is nothing but   mg into square root of 4 plus cos square theta divided by 2 this is the magnitude of the force you can also do a little bit of calculation to what is the direction of the force   the direction of the force is given by what suppose this is going to meet   at this particular point i need to give some name

so the direction of the force the direction of the reaction right direction of the reaction of the flow reaction r is given by this angle ah o b e that can also be calculated from little bit of geometry and now we will uh we will consider one more example now this again one of the typical problems is ladder problem another typical problem   is what we call it as you keep a heavy block on an inclined plane this kind of problems are called as   inclined plane we will call inclined plane and i will sorry i would call it as block or blocks placed on some inclined plane this another typical problem let me i am not going to write down the problem but   i will describe the physics of it physics office like this i have an inclined plane   this is theta and i have a block which is placed on it h is the height of the block and

so called length of the block or a side of a block is b okay the the weight will act from the through the center of gravity this will be mg this is uh maybe i'll reduce it a little bit and this can be resolved along two directions this is mg cos theta and this will be mg sine theta this is m j into sine theta okay one now when the let us say that suppose this particular inclined plane you can rotate about   this void let me say then the this plane this particular top portion will coincide like this   then the block is placed on this now i can rotate it let us say that i can rotate the inclined plane that means the angle theta can be increased when the block is on the floor mg will act downwards   the reaction will also act the normal reaction will be here now if i keep rotating what will happen at a this there is a situation when this block can   topple when the block will topple this normal reaction will no longer act through the center   of the cg

so normal reaction will be in some other point this is going to be the direction of normal

so as you keep rotating   when you reach a particular point let us say as you keep rotating this n will move

so this   distance we will call it as this distance is what x i will produce it

so that you will know clearly   this distance i will call it as x

so the ah the point of application of n will shift from   this line towards
this and the body will topple when n exactly coincides with this particular
side okay now this kind of problems can be done this i will call it as a this
point i will call it as a this point i will call it as   b that's all i would
need now i will write down the the block has two tendencies one   the block
can slide down it is the translational motion the block can slide down
so one can write down the for translational equilibrium for for translational
equilibrium sigma of f must be equal to 0 then rotational equilibrium we don't
want this to topple   therefore sigma of all torques i can take it about a
particular point i am going to take it about c this must be equal to c i am
going to take talks about this   now what about the translational equilibrium
what does it mean it means it means first horizontal forces what are the
various horizontal forces mean horizontal in   the sense horizontal to the side
of this block here there is a frictional force f   therefore this f must be
equal to this this is the only force that is acting along this direction
along this direction see mg sin theta therefore f is equal to mg sine theta
then n is equal to this n is equal to mg cos theta now what about the thoughts
about the c it will be equal to this will have a torque about this point c
that is this implies n into x this must be equal to r   and f it is also going
to create a torque f into f into perpendicular distance from here h by two
so from these two eq from these three equations we can discuss when the when
the top link will take place whether the toppling can take place before
sliding or sliding can take place without toppling etc
so various such situations will discuss the problem session thank you you