

so so

so today we are going to continue our discussions on the use of cross product the topic for today's discussion is torque and also we will spend some time on angular momentum and conservation loss

so essentially today will be first we will be focusing on torque and its role in the study of rotational motion of particles yesterday we introduced the cross product between two vectors  $\mathbf{a}$  and  $\mathbf{b}$  and we showed that this vector is perpendicular to both the vectors  $\mathbf{a}$  and  $\mathbf{b}$  in the sense that this vector is perpendicular to the plane constituted by vector  $\mathbf{a}$  and vector  $\mathbf{b}$  then we introduce the notions of angular velocity or the magnitude of it is rotational speed then angular acceleration we also introduced other concepts like  $a_r$  radial acceleration and transverse acceleration and  $a_t$  before we proceed on to today's topic let me spend a few minutes on an important problem which we discussed yesterday let me again restate it  $a_t$  we have here a rigid body this is the  $x$  axis here  $y$  axis here and  $z$  axis here and then these are the standard equations which we derived yesterday the expression for the linear velocity in terms of it is written as a cross product between angular velocity and the position vector etcetera and transverse acceleration is  $\boldsymbol{\alpha}$  vector crossed with  $\mathbf{r}$  what is  $\boldsymbol{\alpha}$  vector it is the angular acceleration vector it is  $d\boldsymbol{\omega}/dt$  and then radial acceleration is  $\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}$  these expressions were we were not we didn't give you very rigorous definitions sorry rigorous derivations of this but we wrote by some kind of analogy and now these are the equations for rigid dynamics however you would have studied earlier  $a_t$  the motion of a particle in one dimensions and two dimensions suppose i consider the motion of a particle in two dimensions the particle can go anywhere along therefore the this distance  $r$  can change what you called as radial distance and this is what you call it as  $\theta$  vector  $\theta$  sorry it is already there given this is the direction of  $\theta$  once you rotate from here to here

so there are three directions essentially two directions in two dimensional plane the radial direction and  $\theta$  direction or the transverse direction you also have an axis which is coming out of the board that is the  $z$  direction so just like  $x, y, z$  from a triad just like in the cartesian plane  $x, y, z$  from a triad you have  $r, \theta, z$  forming a triad here ok now we derived those equations here  $\mathbf{v}$  the velocity vector is  $\mathbf{r} \cdot \dot{\theta}$  times you start from this position vector and differentiate it with respect to time and differentiating with respect to time you get this quantity now this you realize the coefficient of unit vector  $\mathbf{e}_r$  is the radial velocity whereas the coefficient of  $\mathbf{e}_\theta$  is the  $a_t$  angular velocity and anyway it is only the magnitude this whole thing is angular velocity this is the rotational speed that is the terminology now since the particle can go anywhere in this two dimensional plane if i fix  $r$  if i say that the particle has to only rotate along a circle that is what happens in the case of a rigid dynamics even though the body is rotating about an axis i consider a part position here then this is the radial direction this particular point is going to sweep a circle that circle is what going to correspond to the circular motion of the particle here once i fix  $r$  once i fix  $r$   $\mathbf{r} \cdot \dot{\mathbf{r}}$  is zero  $d$  by  $dt$  of the radial coefficient is zero then automatically you will get here the radial velocity is zero for a rigid motion in a you consider a particle which is fixed radial velocity is  $0$  but whereas the angular velocity is not  $0$  fine now what is  $\boldsymbol{\omega}$   $\boldsymbol{\omega}$  is this is the this is the  $\boldsymbol{\omega}$  vector how do you define this  $\boldsymbol{\omega}$  vector  $\boldsymbol{\omega}$  vector is defined as its magnitude times the unit vector this is your sense of rotation is described like this now i can calculate what is  $\boldsymbol{\omega} \times \mathbf{r}$   $\boldsymbol{\omega} \times \mathbf{r}$  is  $\boldsymbol{\omega}$  is  $\boldsymbol{\omega}$  times  $\mathbf{e}_z$  times the

vector  $\mathbf{r}$  is  $r$  times unit vector along radial direction now this is  $\omega$  times  $r$  times what is  $\mathbf{e}_z \times \mathbf{e}_r = \mathbf{e}_\theta$   $\mathbf{e}_z \times \mathbf{e}_\theta = -\mathbf{e}_r$   $\mathbf{e}_z \times (-\mathbf{e}_r) = \mathbf{e}_\theta$  would be  $v_\theta$  something like  $\mathbf{i} \times \mathbf{j}$  unit vectors  $\mathbf{i} \times \mathbf{j} = \mathbf{k}$   $\mathbf{j} \times \mathbf{k} = \mathbf{i}$  in that sense  $\mathbf{i}$  will get here  $\mathbf{e}_\theta$  now you see here ah the whatever equations we have here whatever results you get from here they they are obtainable from the free motion of a particle in two dimensions subject to the condition you are fixing the radial coordinate ok

so therefore it starts off its ah what we are doing here its a sort of vindicates our derivations here this equations then you take i want to calculate the acceleration vector acceleration vector is  $d\mathbf{v}$  by  $dt$   $d\mathbf{v}$  by  $dt$  is  $d$  by  $dt$  of  $\mathbf{i}$  will write down the full expression here i am not going to fix the position of the particle right now i will derive the full thing and see what i am going to get

so  $r \dot{\mathbf{e}}_r + r\dot{\theta} \mathbf{e}_\theta$  differentiate this and i get here once i fix the position of a particle then i will get here therefore it is it is a surprise that the acceleration will have the radial component as well as tangential component whereas velocity have only tangential component and no radial component for a rigid motion now i should be able to get it from here yes i have this expression for  $a_t$  which we yesterday derived it and  $\alpha \times \mathbf{r}$  is what is  $\alpha$   $\alpha$  vector is the angular acceleration therefore  $\dot{\theta} \mathbf{e}_z$  double dot times  $\mathbf{e}_z$  times  $r$  times  $\mathbf{e}_r$  or this gives you  $r\dot{\theta} \dot{\mathbf{e}}_r$  times  $\mathbf{e}_z$  this is same as this expression is same as here now the radial acceleration looks somewhat complicated actually not  $\omega \times \omega \times \mathbf{r}$   $\omega$  vector is this quantity and  $\omega \times \mathbf{r}$  vector we already calculated here

so that i can put it here then i will get here  $r\dot{\theta}^2$  times  $c_r$  which is same as i think  $\mathbf{e}_z \cdot \mathbf{e}_\theta = 0$   $\mathbf{e}_z \cdot \mathbf{e}_r = 0$   $\mathbf{e}_z \cdot \mathbf{e}_z = 1$  is the  $d\dot{\theta}$  i am doing it in the other way the therefore i should have one minus sign here therefore this expression is same as what i have over here this sort of tells you that even though rigid dynamics is a special subject you can always study this in a sense if one axis is fixed from the motion of a particle in two dimensions and

so with this i will proceed further for today's topic today's topic for discussion is torque and angular momentum ok now what is the motivation for it now how do when to displace a particle i have a particle here i want it to move only way i can do it by a force should act on it

so it is the force which is responsible for the translational motion of a body if the body is here if i apply a force it goes to some other position from here to here this body has been translated

so force is the one which is responsible for the translation translational motion now the question is it also sorry it also produces an acceleration on the body now we consider the following situation suppose i have a door i have a door like this these are all hinges there is one in the one here there is one here and ok now when i want to rotate this door this door when i want to rotate this door if i apply the force here in whatever angle the door is not going to rotate if i apply the force here normal to it the door will rotate but our practical wisdom tells us from our day to day life if i apply the force here normal direction very close to this ah very close to this edge then the rotation is very elegant and easy to produce ok now this is what is called as a moment of a force this concept there is a force acting on a body and then there is a rotational effect happening this concept this effect is described by what is called the moment of a force moment of a force or it is also known as torque it has got several equivalent names moment of a force or also a couple we will see what it is center

so just like if i have a force it can produce acceleration on a body it is the force that is responsible for causing acceleration of a body i have here a torque can cause angular acceleration

so when the force acts at this particular one normal let us say that this door is on the plane of this board then a normal force acts on it then the door will rotate and when the door rotates it is going to sweep an angle with respect to the frame of this board

so what is the rate at which that angle will change what is the rate of change of rate of change of that angle that introduces the angular acceleration and now we will derive an expression for moment of a force will properly define it ok moment of a force on a single particle ok

so let me fix the axis like this this is x axis this is y axis

so i have the particle here this i will call it as  $r$   $r$  vector and ah this is  $r$  vector is the particle here then there is a force acts like there is a force acting like this this is  $f$  vector i can produce this i will also ah now the torque it is denoted by  $\tau$  torque  $\tau$  it is described as it is defined as the cross product of the position vector and the force acting on it and so ah one thing straight away clear is this this torque the direction of a torque is both it is perpendicular to the the position vector as well as the force vector and right and and it is given by right hand this given by right hand screw this you must keep in mind for the direction right hand screw which we described in a status class now therefore by definition this is nothing but  $r$  times  $f$  times  $\sin$  theta or  $r$  times what is uh what is  $f \sin$  theta this is this whole magnitude is  $f$

so if i drop a perpendicular from here this is if i drop a perpendicular from here this is the theta ah i will take this as

so this is theta on this

so this is going to be the  $f \sin$  theta is what you called as  $f$  perpendicular  $f \sin$  theta this is  $\cos$  this is  $\sin$

so this is written as  $r$  times  $f$  perpendicular this is what  $f$  perpendicular you know this i can also write it in a different way how i drop a perpendicular from here this is theta

so when i drop a perpendicular from here this i can write it as  $f$  into  $r \sin$  theta  $r \sin$  this triangle right angle triangle i can take this is  $r$  therefore this is  $r \sin$  theta i can view it in both ways

so this is written as  $f$  into  $r$  perpendicular this is this quantity is known as or perpendicular right now what about the dimensions of torque force we know the dimensions and  $r$  is the distance therefore force is to distance this has the dimensions of energy or work work done right now whereas the work done is a scalar quantity if a particle moves by a distance ah if  $d x$  let us say if a particle moves by distance  $d x$  the force acting on it is  $f$  then the work done in moving this is  $f \cdot d x$  infinite similar right however it has the dimensions of energy but torque is torque also has dimensions of energy but it is a vector quantity

so you need to keep in mind we have seen similar pair one scalar and another vector which have both the same dimensions we had seen in couple of classes earlier for example the angular velocity  $\omega$  angular velocity of  $\omega$  it has got dimensions of ah it is what  $d \theta$  by  $d t$   $d \theta$  is in radians therefore it is time inverse what about frequency angular frequency i mean angular frequency is what how much angle it sweeps in whatever the time duration

so this also has  $t$  inverse whereas one is a vector quantity this is a scalar quantity

so such pairs do occur in physics ok now i will give you a simple

illustration and i will give you a simple illustration and suppose i have a screw like this i want to rotate it what do i do i want to rotate it so we have a spanner like this i will take a different color chalk just to indicate this spanner

so this is how a spanner will look like right now if i want to rotate it i will apply a force here if i want to rotate this i do like this this is our day-to-day experience on the other hand this is the force now if i apply the same force here it is not that easy to make the screw rotate advance forward so longer the distance it is better

so if i want to do it in a very elegant way easy way what i need to do is i need to slide a pipe of considerable length then apply it if i do it even less force is sufficient because it is  $r \times f$  since i am increasing  $r$   $\tau$  is going to be equal to  $r \times f$  if i have a long arm even less force will be sufficient enough right

so if you ah when is  $\tau$  zero when is the rotational effect zero in order that to happen if the force is zero if there is no force then certainly you cannot rotate there is no torque at all on the other hand if  $r$  is equal to 0 that is the line of action of force passes through the origin the line of action of the force passes through the origin this is the here this is the line of action if the line of action is also along this then there is no way it is or right other one is the angle between these two that is what is zero or one eighty degrees is it right

so  $r \times f$  we need to keep in mind that this is a vector product therefore we need to keep in mind the various properties of  $r \times f$  they do hold we have seen it what are the various things right if  $r$  reverses the sign  $f$  reverses the sign then the  $\tau$  will remain the same i will do a simple illustration you need to keep in mind this  $r \times f$  when you take the product what is the direction of  $\tau$  the direction of  $\tau$  is as specified by the right hand rule which we had seen yesterday  $r \times f$

so one is  $r$  this if this is going to denote  $r$  middle finger is coming out of the board  $f$  is actually from here to this side perpendicular to this then the thumb will tell when  $r$  to  $f$  when you rotate the thumb will tell it will advance in the forward direction we will do a simple illustration this an example

so i have a point a here and then this is a particle p here here is the axis coming out i will set up the axis as this is x axis y axis

so it is ah it is simply the particle is falling  $m \cdot j$  its a particle which is falling the mass is  $m$  gravitational fall and right

so i will take a point here this is  $m \cdot j$  i want to calculate what is the torque on this particular part with respect to a due to the gravitational force now this is a quadrant

so here i will have it this is point a  $x$  this is y axis below therefore minus y i will say a typical point

so this is  $r$  vector this is  $\theta$  that is now what is  $\tau$   $\tau$  is equal to ah whatever it is radius of ah the magnitude of this times the force times sine of  $\theta$  this is same as this remember we always take the small angle whenever we take the cross product this is the  $\theta$  now you may ask me sir why don't you make why don't we make use of this definition of the cross product right we will do it and when we do that what will happen  $\tau$  is equal to  $r \times f$  remember yesterday we wrote down this definition i j and k this is this coordinate is  $x$  this coordinate is minus y this is 0 and  $mg$  has only the y component therefore 0 is a positive force when it moves towards it towards the ground towards the center of the earth but this when you calculate you will have ah i into this is zero j into this is also zero and

only surviving thing is  $k$  into  $x$  into  $m$   $j$   
 so this is nothing but  $mg$   $x$  into  $k$  what is  $\sin \theta$  sir this is  $\theta$  here  
 $\sin \theta$  is  $ah$  what is this is opposite this is adjacent therefore this  $x$  by  
 $x$  by  $r$  right fine the now um this is this is  $x$  sorry this  $x$   
 so this is opposite  $x$  by this radius this one will give you  $\sin \theta$   
 therefore this is same as  $ah$  this is  $\sin \theta$  is  $x$  by  $r$  into  $mg$  into  $\sin$   
 $\theta$   
 so the  $ah$   $x$  by  $r$  what is happening i am i am making some mistake ok i will  
 correct it  $r$  into  $m$   $g$  into  $\sin \theta$  is  $x$  by  $r$   $r$  and all will cancel  
 so i will have  $x$   $m$   $g$  here i am writing only the magnitude if i do like this  
 then direction i have to fix again right hand thumb rule if i write this  
 vectors properly then automatically the direction will emerge out that is  
 the moral ok you can do it either ways the next concept is angular momentum  
 of a particle now we are talking about the moment of a force similarly we  
 can talk about the moment due to the angular momentum  
 so let us consider a particle of mass  $m$  and its position it is with respect  
 to coordinate system it is at a particular position vector  $r$  and its  
 momentum is  $p$  then its orbital angular momentum the torque or moment of a  
 force force is now let us compare this with linear dynamics torque or  
 moment of a force is the rotational analog of force it is generally said  
 rotational analog of force what do we mean by this just like forces what that  
 is responsible for a body to move have translational motion it is torque  
 that is responsible for an object to rotate about an axis or a door to swing  
 etcetera etcetera now what about angular momentum the angular momentum is the  
 rotational analog of  $s$  ah  
 so angular momentum is the rotational analog of yes linear momentum yes now  
 you might notice that we are gradually equipping ourselves with various  
 concepts and their mathematical definitions  
 so that we can handle problems involving systems of particles and rotational  
 dynamics and now you may ask me sir today we have got two quantities  $\tau$  is  
 defined like this  $l$  which is defined like this which of course even in  
 linear motion we have it to be like this we have defined it earlier is there  
 any connection between these two that is what we are going to next topic for  
 discussion it is fairly simple connection between relation between is fairly  
 simple  
 so we start with  $l$  is equal to  $r$  cross  $p$  this is the definition of orbital  
 angular momentum  
 so let us differentiate let us differentiate with respect to time how is it  
 justified sir in general neither the position of particle is going to be  
 constant it will vary or the momentum of the particle also is subjected to  
 change therefore there is meaning in talking about  $d$  by  $d$   $t$  of  $l$  this is  $d$  by  $d$   
 $t$  of  $r$  cross  $p$  this is equal to  $d$   $r$  by  $d$   $t$  its a distributive derivative is  
 what you do is keep this constant and differentiate this plus keep this  
 constant differentiate this that's what we are going to do plus  $r$  times  $d$   $p$  by  
 $d$   $t$   $d$   $r$  by  $d$   $t$  is going to give you velocity vector therefore  $d$   $r$  by  $d$   $t$   
 will give a velocity vector velocity crossed with momentum vector is what  
 momentum is  $m$  times  $v$  therefore that will vanish  
 so you are simply left with what is  $dp$  by  $dt$  rate of change of momentum known  
 as by newton's law force good right therefore the what is  $r$  cross  $f$   $r$  cross  $f$   
 is the  $\tau$  torque acting on the body due to this force therefore we have a  
 very elegant relation  $dl$  by  $dt$  is equal to  $\tau$  you say that the time rate of  
 change of angular momentum the time rate of change of angular momentum is  
 equal to torque under ok this is something similar to when you compare this  
 equation with ah what you call the kinematics in one and two dimension general

newton's equation what is that you get this is something compare this with  
 with  $\frac{dp}{dt}$  is equal to  $f$  data now this is not enough for us that  $\tau$   
 acting on one single particle is not sufficient because we are studying  
 system of particles and in general a big rigid body rigid body therefore and  
 what about the orbital angular momentum etcetera other quantities you  
 introduce for a system of particles its again fairly simple because  $L$   
 is a vector quantity  
 so vectors are ah you can add it if  $L_1$   $L_2$  etcetera or the angular moment  
 of a system of particles  $n$  particles then its total angular momentum is  
 given by this  
 so what is  $L_i$   $L_i$  is the position vector of  $i$ th particle crossed with the  
 momentum vector of that particular particle  
 so just for clarity  $i$  will write you can think  $m_i$  times  $v_i$   
 so the total angular momentum is  $L$  can write this way  $i$  already written it is  
 $L = \sum_{i=1}^n r_i \times p_i$   
 running from one to  $n$  this you can also write it like this  $r_i \times p_i$   
 running from one to  $n$  this is this is the ah this is the definition of angular  
 momentum of a system of particles how is it justified  $L$  is a vector quantity  
 and vectors you can add just like on a particular particle force  $f_1 + f_2 + f_3 + \dots$   
 you can add all of them and now we will talk about what is the  $\frac{dL}{dt}$   
 of this quantity  $\frac{dL}{dt}$  is  $\frac{d}{dt}$  of summation over  $i=1$  to  $n$   $L_i$   
 this is equal to  $\sum_{i=1}^n \frac{dL_i}{dt}$  ah differentiate each of the terms and then add what  
 is  $\frac{dL_i}{dt}$   $\tau_i$  is a vector sum over all vectors therefore ah the the some  
 of the ah sorry the rate of change of total angular momentum is what is  
 equal to sum of all the torques this you can call it as a capital  $T$  capital  
 tower rather it is the total torque that is acting on the system of  
 particles it is the total torque that is acting on the system of particles and  
 other now we will  $i$  take  $i$  consider  $\tau_i$   $i$  consider  $\tau_i$  what is  $\tau_i$  is a  
 ah the ah this is the position vector of height particle times  $f_i$  right  
 so this is equal to  $r_i \times f_i$  times we had already seen the forces are there are  
 two kinds of forces one external forces like gravitation if you put a charge  
 particle in electric field or a magnetic field etcetera and they can be  
 internal forces like tensions compressions inter body attractions etcetera  
 so  $i$  will say that  $f_i$  external plus the internal forces corresponding to the  
 height particle see the generally you look at it the internal forces are of  
 something like action react and reaction type you take in a gas two  
 molecules whatever force one of the when two ball two of these molecules to  
 collide one of them is going to accept a particular force on the other  
 molecule it will you will get a reaction on this first one due to the other  
 molecule therefore the internal forces are of action reaction type and then  
 and these forces do not contribute they are in opposite directions and then  
 they do not contribute  
 so we can say that  $\tau_{internal}$  the the  $\tau_{internal}$  is extended due to how  
 external one therefore  $i$  can write now  $\frac{dL}{dt}$  is equal to  
 so this is this  $\tau$  has reduced to just external because the internal forces  
 are not going to contribute they are action reaction type you can write as  
 $\tau_{external}$  ok  
 so this is something ah a bell should ring when you see this this is something  
 similar to we have earlier seen what is that ah the rate of change of total  
 momentum is equal to  $f_{external}$  right despite the translational motion  
 rotational motion or of different nature you can see the remarkable  
 similarities with respect to the basic governing equations in both the cases  
 right  
 so you say that now we can say that the time this is rather important the time  
 rate of the time rate of total orbital angular momentum ok you have a kind

of a mnemonic ohm not very common the time rate of total orbital angular momentum of a system of particles about a point or about a line or about a line or an axis about a line is equal to equal to the sum of the some of the you should not say simply talks external talks that is important because internal forces are not going to contribute anything external talks that is that is the talks due to there is a curse due to just complete the sentence external forces alone under ok

so what about now we will talk about conservation of angular momentum when conservation of angular momentum conservation of angular momentum ok if  $\tau$  external is equal to zero then implies zero then  $dL$  by  $dt$  is equal to zero this implies what  $L$  is a  $L$  does not change or  $L$  is a constant of motion  $L$  is a constant of motion  $L$  is a constant of motion you say that in technical language which you should learn to use  $L$  is conserved now what is the next what is the natural question you would ask you you would like to compare this situation with linear motion situation right

so what is that you have  $dp$  by  $dt$  when you have  $dp$  by  $dt$  is what you call it as  $F$  the force acting on it when  $F$  is zero if  $F$  is equal to zero then yes the linear momentum is is a constant of motion or equivalently you say that  $p$  is conserved ah it is true even in the case of a system of particles because even though  $i$  written small  $p$  denote the momentum of a single particle this relationship is true even in the case of system of the reason is momentum is a vector quantity you can add the total momentum we have done it the total momentum of the particle just few minutes earlier is vector  $p_1$  plus vector  $p_2$  etcetera all of them and right this is so when is orbital angular momentum a constant of motion when the external torques vanish now we will do a a simple problem we will do an illustration and then now let us consider a particle which is moving with a constant velocity this is an example particle moving with a constant velocity this one of the this is a simple system and let us consider the particle is moving in this direction let us say that ok at some instant of time the particle has to be somewhere here right now with respect to ah with respect to ah at some instant of them the particle is here

so this is the position vector of the particle this is the ah origin of the system coordinate system ok then  $i$  will ah  $i$  will project this  $i$  will produce this sorry this is the particle this is theta strictly speaking not so you will see that  $i$  will drop a perpendicular from here so this is  $r$  therefore this is opposite corresponding  $\sin \theta$  all right so the description is like this a particle is moving with a constant velocity constant velocity is  $v$  therefore its  $v$  is a a constant velocity therefore its momentum is  $m$  into  $v$  that is fairly simple now  $i$  want to calculate  $L$   $L$  is equal to  $L$  is equal to  $r$  crossed with  $m$  into  $v$   $r$  cross  $p$  this is this is by definition what do we have ah  $r$  which is the magnitude of vector  $r$  then ah  $m$  into  $v$  into  $\sin \theta$  and you know what is little  $r$  little  $r$  is this quantity and little  $v$  is magnitude of this vector is very standard what is its directions are this is the position vector this is this ah this is the direction of position vector and momentum is along this therefore ah when  $i$  take this right hand thumb rule  $i$  will look into the smaller angle  $\lambda$  so this will ah the direction is what is it coming out or into the page yes into the page the direction of  $p$   $i$  will write here the direction of  $p$  into the page integer page at this particular point and right now what  $i$  will do is as the particle moves along the straight line what is  $ym$   $i$  want to calculate so what is  $i$  want to call this is  $i$  will call this as  $m$  let us say  $\sin \theta$  is equal to  $om$  divided by  $r$  this implies  $om$  divided by  $r$  so  $i$  will calculate sorry  $i$  have not indicate the direction this is a this

vector this vector if i simply write this this is not correct this is the direction ok fine

so now i will have  $l$  into this is the magnitude alone i will write so that i need not worry about this  $r$  into  $m v$  into  $ah$  into  $om$  by  $ym$  is  $r \sin \theta$   $om$  is  $r \sin \theta$   $o m$  is  $r \sin \theta$

so therefore i have  $l$  is equal  $ah r \sin \theta$  one minute plus  $ym$  is  $r \sin \theta$  theta yes this  $r \sin$  i will write it as  $r \sin \theta$  into  $m v$  this is  $om r \sin \theta$  times  $m v$  now we are done the reason is the as the particle moves along the straight line ah we have calculate what is the magnitude of angular momentum its magnitude of angular momentum is this particular length that is what  $ym$  times  $ah mv$  right

so this is  $ah \sin \theta$  is equal to  $om$  by  $r$  that is what we have done so from this you come to the conclusion what is the conclusion whether the particle is here or the particle is here or the particle is here it does not matter it is always this  $vm$  is always this distance  $ym$  when you calculate the magnitude of the orbital angular momentum theta will vary as it moves along however this is always the same is  $ym$  is the same therefore this implies ah as the if a particle with is moving with a constant velocity then its orbital angular momentum with respect to an origin remains a constant of motion

so  $l$  is a constant of motion therefore what will happen to  $d l$  by  $d t$  e so there is no torque ok here you can see that ah

so with this i think i can let me summarize today we started with the definition of a torque and angular momentum then we showed that what is the connection between torque and angular momentum then under what conditions  $l$  is conserved under what conditions the torque is then we had considered a few simple examples of the situation involving cross products calculating torques and this particular problem we realize that  $l$  is a constant of motion

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so you