

various examples of systems of particles and rigid motion and we realize that the central important concept to study such problems is the concept of center of mass then yesterday we further ah proceed we further went further to realize and we introduced the concept of the velocity of centre of mass similarly the acceleration of the center of mass these two concepts were introduced and then we also discussed a case in the case in the simplest of ah multi-particle system a two particle system where the motion was separated or split into the motion of center of mass and other one is what is called as relative motion or ah the concept of effective mass and so we had calculated the total kinetic energy of the system of this two particle system then we realized that this kinetic energy of this two particle system can be split up into that corresponding to the center of mass and that corresponding to the reduced mass and ok and it appears as if the reduced mass rotates with the relative velocity between v_1 and v_2 then yesterday we proceeded further to study the systems of particles and we realized that we required some additional concepts like how do we generalize the concept of momentum acceleration in the case of systems of particles so we had introduced the notion of the velocity of center of mass the acceleration of center of mass etcetera we considered a very interesting example the case of a two particle system how does it go and it turned out to be that the motion of this two particle system suppose i look into the kinetic energy this total kinetic energy can be split into two parts one corresponding to the kinetic energy of the centre of mass and the other one corresponding to the kinetic energy of reduced mass what is the velocity of the reduced mass it is the relative velocity between v_1 and v_2 these are the velocities corresponding to two particles and today we proceed further to study ah rotational motion of systems of particles remember in the case of systems of particles it can be either pure translation or it can be pure rotation or both

so ah we need what i can say that how to deal with rotational motion and we need to equip ourselves in today's topic we are going to consider ah vector product which is the when we have two vectors a and b what is the cross product between these two vectors we need this vector products and angular velocity if a body rotates about an axis then it will have every point on it will have an angular velocity correspondingly it will also have angular acceleration every point on this body

so we realize you see that we are gradually equipping ourselves with various concepts and methodologies has to deal with systems of particles and rigid motion and ok

so this angular velocity is general denoted by omega vector and angular acceleration is usually denoted by alpha these are fairly standard notations and now we have to do some little bit of you may think that it is mathematics but it is not as i keep repeating in my lectures do not be scared of mathematics at least at this level treat whatever mathematics you are coming across as a tool to study physical problems and

so first we will have vector products now ah before that just me let me suppose i have two vectors a and b earlier you would have come across what is called as the dot product between these two vectors the dot product the so called the dot product dot product of two vectors it is defined as $a \cdot b$ is equal to modulus of a that is the length of the vector a times the length of the vector b times the angle between them these two vectors now one simple example for this is ah suppose a force is acting on a particle let us say the force vector is acting on a particle this is the force vector let me say and then it moves by a small distance displacement ds let us say then the

work done by the force on this particle is $\int_a^b \mathbf{f} \cdot d\mathbf{s}$ in moving the small infinitesimal displacement is $\mathbf{f} \cdot d\mathbf{s}$ right then we are moving this particle from let us say a particular point a to particular point b then the work done by the force on the particle is $\int_a^b \mathbf{f} \cdot d\mathbf{s}$ integrate from a to b

so these things you would have come across now we are going to so the dot product between two vectors is a scalar quantity its a scalar it is not a vector it will be a number now we are going to consider what is called as vector product between two vectors it is defined like this suppose i have a vector \mathbf{a} i have a vector like this sorry this is vector little vector a this is little vector b see there this vector a and vector b they are not perpendicular to each other they make some angle they can be perpendicular in general we need not take it that way

so the angle between the vector a and vector b is θ then the cross product between these two vectors is denoted by another vector \mathbf{c} which is perpendicular to both vector a and as well as vector b therefore this vector is perpendicular to the plane formed by this vector little a and vector little b

so this is denoted like this and we have to give a direction to it i will explain what it is in this now we need what is the concept of a right handed screw i will explain here what is a right handed screw suppose i have a screw like this this is the tip of a screw

so these are the what you call as ah screw edges and then this is the axis the concept of right handed screw is like this suppose this denotes the direction of a and then you have this is direction of b i am denoting the same thing actually actually i could have drawn a screw here itself but i did not want to complicate the diagram now when you when you rotate from a to b the screw has to advance in the forward direction the screw has to advance upwards

so this situation is denoted like this what is a right handed screw ok now is a let us say that this middle finger it can point out in any direction this denotes little a ok and then this vector i mean this is very difficult as if you have to view this this entire thing is a point here and it is pointing in some direction this vector is little b

so the angle between these two is some θ depending on how i fold it now the thumb denotes the direction of motion of advancement of the screw when i rotate from a to b the screw has to advance upwards let me again do it and when you rotate from a to b the screw advances further this is what is called right handed screw under you can also have a left handed screw we are not worried about it and we will follow this standard convention so the cross product between these two vectors little a and little b is modulus of vector a into modulus of vector b into $\sin \theta$ and remember this rotation is indicative of the direction its a vector quantity i need to denote i will put a unit vector here

so this is the unit vector the unit vector is such that it follows the convention of a right handed screw and ok this is now how do you take this angle θ how do you take this θ now ah depending on the angle between a and b θ can be less than 180 degrees or θ can be greater than 180 degrees the convention is θ is taken θ is taken through the through the smaller angle which is less than one eighty degrees ok

so when two lines intersect you will have two angles one is θ and another is opposite one

so it depends which one you will take its always taken as a smaller angle which is less than 180 degrees this is the concept now ah this vector product

between two vectors they have various conventions first one is sorry various properties various properties first one $a \times b$ is not same as $b \times a$ suppose was taking $a \times b$ it is not same as $b \times a$ when you take $b \times a$ rotating from b to a through the other way right so this is same as $-\mathbf{b} \times \mathbf{a}$ these are the things which you can convince yourself now under reflection what do i mean by reflection a goes to $-\mathbf{a}$ and vector b goes to $-\mathbf{b}$ then $-\mathbf{a} \times -\mathbf{b}$ same as $\mathbf{a} \times \mathbf{b}$ with $-\mathbf{b}$ so under reflection the cross product remains the same it remains invariant now from $\mathbf{a} \times \mathbf{a}$ one easy property is third property what is $\mathbf{a} \times \mathbf{a}$ because the angle is going to be zero before it is zero so cross product of any vector with itself is zero now we come to $\mathbf{i}, \mathbf{j}, \mathbf{k}$ unit vectors of a coordinate system if i have here let us say this is $\mathbf{i}, \mathbf{j}, \mathbf{k}$ here the angle between each of this any two of these vectors is 90° these are the unit vectors so this is what you call it as $\mathbf{i}, \mathbf{j}, \mathbf{k}$ system remember sometimes this also used as \mathbf{e}_x unit vector along x direction unit vector along y direction and unit vector along the z direction such a convention is also there so you should get confused when people use different notations so you can see that what is $\mathbf{i} \cdot \mathbf{j}$ you take $\mathbf{i} \cdot \mathbf{j}$ automatically it will be \mathbf{k} so it is cyclic similarly $\mathbf{j} \cdot \mathbf{k}$ is equal to \mathbf{i} what is $\mathbf{i} \times \mathbf{i}$ \mathbf{i} crossed with \mathbf{i} what is it it is zero the cross product of any vector is itself therefore there are three vectors there are nine products so you can realize that $\mathbf{a} \times \mathbf{a}$ only two of them suppose you take $\mathbf{j} \times \mathbf{i}$ if you take $\mathbf{j} \times \mathbf{i}$ it will be definitely the unit vector the vector represented by $\mathbf{j} \times \mathbf{i}$ it will be in the direction perpendicular to both but you are taking the cross part in the opposite direction therefore by this property it is $-\mathbf{k}$ ok right so these are the various properties of dot products which will be using extensively now there is a general $\mathbf{a} \times \mathbf{b}$ mnemonic formula when i have let us say \mathbf{a} is equal to $a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k}$ and vector \mathbf{b} is these are all in cartesian notation so x component times \mathbf{i} plus y component times \mathbf{j} plus z component times \mathbf{k} then $\mathbf{a} \times \mathbf{b}$ it is calculated as \mathbf{a} there is a formula this is a kind of mnemonic its a determinant $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$ how you calculate i am going to tell you first you know determinant can be expanded through any row or any column but it is only a mnemonic you cannot do it here you have to always do it through first row its a kind of way to remember a methodology to remember \mathbf{i} so leave this column and this row you will be left with this determinant so what will be this $a_y b_z - a_z b_y$ it will be $a_y b_z - a_z b_y$ then minus \mathbf{j} actually what you do now i will erase \mathbf{a} what i have done here this thing so i can remove \mathbf{i} i am going to write the second component so i am \mathbf{j} i take this as lead element therefore this column i should leave and then this row i should leave when i do that i will have i put a minus sign this will be $a_x b_z - a_z b_x$ plus loss component so $a_x b_z - a_z b_x$ plus loss component so for the last component what should i do i should leave this column and this should grow it will be $a_x b_y - a_y b_x$ it is $a_x b_y - a_y b_x$ ok this is how you calculate and so you should try to do various such problems from books now i will take a simple example and illustrate an important property of cross product which already we have said but just as an illustration suppose i take two \mathbf{a} and \mathbf{i}

take two vectors one vector is \hat{i} one vector \mathbf{a} is $\hat{i} + 2\hat{j} + 3\hat{k}$ and then \mathbf{b} vector \hat{i} can take arbitrary any two vectors is $2\hat{i} + 3\hat{j} + 4\hat{k}$ take any two vectors so I want to calculate what is $\mathbf{a} \times \mathbf{b}$ it is very easy $\hat{i} \hat{j} \hat{k}$ and then component is one here two this is three the components of \mathbf{b} I should write here two three and four I guess you realize that these two vectors are not perpendicular to each other right now this will be \hat{i} times \hat{i} times four into two eight eight minus nine plus minus \hat{j} into four minus six four minus six plus \hat{k} into unit vector \hat{k} into three minus four so this is minus \hat{i} then this is plus two \hat{j} minus \hat{k} this is the $\mathbf{a} \times \mathbf{b}$ vector now what I will do I say as I tell you when I have two vectors one is \mathbf{a} another is \mathbf{b} this is $\mathbf{a} \times \mathbf{b}$ this is going to be the \mathbf{c} vector right and this is the \mathbf{c} vector since the \mathbf{c} vector is perpendicular to both \mathbf{a} and \mathbf{b} that is what our definition says let us check up so I shall calculate $\mathbf{c} \cdot \mathbf{a}$ when two vectors are perpendicular the dot product should vanish is it right let us check up minus \hat{i} plus two \hat{j} minus \hat{k} dotted with \hat{i} take \hat{i} so $\hat{i} + 2\hat{j} + 3\hat{k}$ so this will be minus one plus four minus three it is zero so it is clear that when I have two vectors \mathbf{a} and \mathbf{b} whatever angle they make when I calculate the cross product $\mathbf{a} \times \mathbf{b}$ here then it will be perpendicular to both so this vector dotted with either \mathbf{a} vector is zero this vector dotted with \mathbf{b} vector is also zero ok so you can ah you can you need to master this kind of calculations by doing various problems from standard textbooks and ok now why why did we introduce this vector product sir now it makes our life rather easy to study angular velocity and angular angular acceleration and this vector product is a very convenient tool to study these things let us see how we do that so remember we are going to study the motion of a rigid ah rigid object so we will see so I am going to consider the motion of a rigid body about an axis I need to draw a reasonably good diagram so I have an axis let us say this is the axis about which it is rotating then I consider ah I will draw this a different color chalk I will explain what it is in a minute I am going to consider a point here I am going to consider a point this I am going to consider a point I will let me have the axis this is x axis sorry this is y axis this is x axis this is z axis so about this axis this is the bow origin about this axis where the rigid body is rotating I consider a point P that is what I am doing here ok now it let us say that as it as the rigid body rotates this particle P will move on the tip of a circle what I have denoted here is tip of a circle usually in text books they denote it by slotted lines ok this is a plane this colored circle the rim this is actually in the plane here like this I will call this point as C now when it when the when it rotates a little it goes by an amount $\Delta\theta$ angular displacement is $\Delta\theta$ so this point is P' so ah by the when the particle moves from P to P' the angular displacement is $\Delta\theta$ so $\Delta\theta$ angular displacement so you need to get this basic notions very clear in your mind this is what it is a fixed axis please remember that it is a fixed axis about this axis this rigid body is rotating and now there is this concept which is generally books do not care much about it but I think we should the what is called the

average angular velocity average angle over the interval over the interval
 Δt remember in the time Δt the infinite symbol time Δt the
point p moves from p goes from this particular position to p' and the
corresponding angular displacement $\Delta \theta$ and remember all these things
are happening in a plane perpendicular to this board you can see from the top
ok this is given by $\Delta \theta / \Delta t$ now you take the limit as $\Delta \theta / \Delta t$
as Δt tending to zero see this calculus notions are
very essential to understand this kind of problems this is given by the
derivative of θ with respect to time this is what is known as instantaneous
instantaneous angular velocity ω remember this velocity is a vector
quantity

so will be the angular velocity

so what is the direction of ω

so strictly speaking i should write like this is only the magnitude

so the direction of ω ok the direction of ω is such that yes now you
would have guessed what is going to happen because the rotation is happening
in a plane therefore the ω direction is it will be given specified by
the ah it will be specified by what is it the right handed screw

so when its right handed screw is ah this is how right so this is the
direction and

so when you rotate like this the right handed screw would advance above right
now we will ah what is the relation where is going to be the linear velocity
linear velocity will be here this will be tangential at the particular point p
it will be it is at the particular point it will be tangential to it now we
need to get various relationships ah this particular circle alone i will
amplify it and draw it here this is the top view when you are seeing from the
top you can see that this radius is let us say that i will take this radius as
 r and i will have an angle it goes by this amount this Δt this is $d\theta$
actually is joining this so r times $d\theta$ is what is ds

so r into $d\theta / dt$ is ds / dt now the limit as ah
 Δt to zero the limit as Δt tending to zero this relationship will
become r into $d\theta / dt$ is equal to ds / dt is the linear velocity
because ds is the displacement

so it will be $v = r\omega$ this is the standard relation $r\omega$ is equal to v which
one studies that when you consider the motion of a particle along a circle
and right now we need to consider this how v is a vector ω is a vector
look like a circle because we are viewing from side and this is the fixed axis
i consider a particular point o this is the point c this is the point p
so it is going to be ah this is the direction of linear velocity this is the
direction of angular velocity these things we have fixed it right now ah and
better

so i am going to join this point sorry

so this is the point origin this is the position vector r this is what
generally known as or perpendicular this is this also a vector from here to
here it is a vector i am just denoting the magnitude and right

so our now we consider $\omega \times r$ what is $\omega \times r$ $\omega \times r$ is same as ω crossed with ah oc ah this
vector is same as this vector plus this vector

so oc vector plus cp vector

so this will be $\omega \times (oc + cp)$ this vector product is distributive that means
this cross with this plus this crossed with this oc plus ω crossed with cp
you notice that the direction of ω and direction of oc are the same
therefore this will vanish this will go to this will become zero

so we will have $\omega \times r$ is equal to $\omega \times cp$ see $\omega \times cp$

is perpendicular $\omega \times c_p$ is perpendicular to both ω and c_p vector and what is the vector which is perpendicular to this as well as this that is this v vector ok

so let me repeat this this $\omega \times c_p$ is perpendicular to ω vector as well as c_p vector now we already have a vector or vector direction which is perpendicular to both these directions that is the direction of c_p and since ω and c_p are perpendicular this is perpendicular this is perpendicular therefore when I take the dot product I will simply have modulus of ω into modulus of c_p this is nothing but this is what is the length of c_p strength of c_p is what is called as r perpendicular or perpendicular

so therefore we have this relation $\omega \times r$ is equal to v now so for the relation what what is that we have done $\omega \times r$ is a vector of magnitude ωr perpendicular and is along the tangent to the circle so the linear velocity the linear velocity v at p has the same magnitude and direction that is what we have seen what is the linear velocity what is the magnitude of linear velocity of ω that is the calculation which are shown here $r \omega$ and linear velocity is perpendicular to r and ω therefore we say that $\omega \times r$ is equal to v vector and this may not be a very rigorous way of looking at things but the geometry gives us an insight what we have done is let me again repeat first we consider pure circular motion of a particle then we showed that $r \omega$ is equal to v that is what usually done then we consider here we take two vectors ω vector as well as r vector and consider the cross product and when we do that we realize that this $\omega \times r$ vector is same as $\omega \times c_p$ vector and its magnitude when we want this is not correct I will when I need to write a step here better I will do it here

so now $\omega \times c_p$ is equal to ωr I need the magnitude of this this is $\omega \times c_p$ vector this is same as ωr perpendicular

so $\omega \times r$ is perpendicular to both ω and vector r and its magnitude is given by ωr perpendicular and it is in the direction tangential to the circle at point p

so from this we identify both must be the same and write that $\omega \times r$ is same as v ok now we will be making use of things so for rotation about a fixed axis the direction of ω does not change if you are going to have a fixed axis and rotate ω direction will be always shown by the thumb in a general rotation ω may change from point to point etcetera angular acceleration next concept is angular axis $d\omega$ by dt I am going to the angular velocity vector differentiate it with respect to time that will be equal to the α which denote which is reserved for angular acceleration ok now we will we have v is equal to we have few important relations which are going to get we have starting from v is equal to $r \omega$ we will not do very rigorous derivation of this relations but we will prove we will I will give you an argument

so we have v is equal to $r \omega$ differentiating both sides with respect to time

so differentiating both sides with respect to t the time what do we get dv by dt is equal to dr by dt times ω plus and remember the particle is actually moving on this therefore I need to write only this rigid body therefore it is nothing but this ah this is what is dr by dt ah sorry dr by dt is 0 therefore I need to add the other term r plus $d\omega$ by dt this is not going to change for a rigid body therefore it is simply $r \alpha$ $d\omega$ by dt is α where α is nothing but the magnitude of where what is α α is the magnitude of the vector α ok now I consider a

quantity what is known as acceleration along the tangential direction this is you know the linear velocity is tangential circle at the particular point b therefore i can talk about dv/dt

so dv/dt is equal to that is what i have $r\alpha$ now yeah i can show that but we will indicate this is a vector a t is what this vector a t would be α cross with r how do i know this sir the tangential acceleration is α cross r what was the angular velocity we got when we wanted to have the velocity that is tangential it is ω cross r in the same way a t i write it as vector α cross vector r is for you to convince a little and we will give you a rational now ah

so i will only indicate that compare this with sometimes life is easy if you make use of analogies but we need to check v is ω cross r now i will call something else as radial acceleration radial acceleration first we will calculate this we know from the motion of a particular circle it is this acceleration is v^2/r what is v $r\omega$ whole squared by r ok this is equal to $r\omega$ times ω right it is better right the other way ω times $r\omega$

so i know this is ah now by analogy again what is a r maybe i will write it here you will be able to see by vector a r is equal to binology ω crossed with what is $r\omega$ what is $r\omega$ $r\omega$ a actually this magnitude is a vector which is obtained from ω cross r therefore it is ω crossed with r vector ok let me again give an argument this i can write it as the radial acceleration as ω times $r\omega$ by analogy with this angular velocity and tangential acceleration derivations similarly i can write this as this vector quantity i can it is ω cross ω cross r lambda ok

so this the tangential acceleration will be along this direction whereas the radial acceleration will be along this direction and right

so at the whole timer oh ok then fine 1340 minutes i can conveniently come here now ah we will proceed further

so now only we are studying rigid dynamics earlier we would have we have studied already it had been introduced to you motion of a particle in two dimensions

so a simple a single particle can go around an object its something like planets going around sun etcetera

so we will derive relations for that and from that we will see that how these relations actually gel with that of rigid dynamics and it is fairly simple you need to be attentive ok

so now i consider a particle moving in dimension particle in two dimensional motion ok

so i have here this is x axis i will take for some convenience y axis

so this is a point this is the r direction position vector know therefore r direction

so unit vector e_r would be like this along this now i can have the two directions here like this let us see what happens r is equal to now i am going to use what is called as a circular polar coordinates very simple what is it x is equal to x coordinate when i drop the perpendicular here this quantity is $x = r \cos \theta$ and whereas y is $r \sin \theta$ because this is the θ direction $\cos \theta$ $\sin \theta$ it is fairly simple and ok now if i consider this vector to be ah this magnitude of this vector to be 1 simply i have x is equal to $\cos \theta$ unit vector then y is equal to $\sin \theta$ etcetera

so i have such a vector

so $d r/d t$ is equal to magnitude of the vector times neat vector right

so i have this is $\frac{dr}{dt}$ times unit vector \hat{r} plus r times $\frac{d\hat{e}_r}{dt}$
 this is the ah time derivative of the unit vector along the radial direction is
 it right that is what i need to calculate and it is very simple ah this i
 know what is $\frac{dr}{dt}$ the $\frac{dr}{dt}$ is what we will call the \dot{r} the \dot{r}
 dot denotes this dot denotes first derivative one derivative taking
 derivative this again a very standard notation and
 so this is \dot{r} times \hat{e}_r this quantity is what i have here plus r times $\frac{d\hat{e}_r}{dt}$
 i have to calculate $\frac{d\hat{e}_r}{dt}$ i have to calculate
 so i know my unit vector \hat{e}_r as i said here is nothing but $\cos\theta$ times \hat{e}_x
 this unit vector is $\cos\theta$ times \hat{e}_x unit vector along this plus $\sin\theta$
 times unit \hat{e}_y
 so therefore $\frac{d\hat{e}_r}{dt}$ of \hat{e}_r is equal to $-\sin\theta \dot{\theta} \hat{e}_x$ plus $\cos\theta \dot{\theta} \hat{e}_y$
 now if you look at this vector that times ah there is one
 time derivative i have forgotten i should write this $\cos\theta \dot{\theta} \hat{e}_y$
 similarly here when i differentiate ah $-\sin\theta \dot{\theta} \hat{e}_x$ now
 what is this vector look at this vector this vector is perpendicular to this
 vector if i take the dot product between \hat{e}_r and then the resulting vector
 it is $-\cos\theta \sin\theta \dot{\theta} + \sin\theta \cos\theta \dot{\theta}$
 times $\dot{\theta}$ therefore the dot product between \hat{e}_r and then the resulting
 vector is zero therefore this vector is the direction of \hat{e}_θ this is one
 way of looking at things so i have here ah i have here $\frac{dr}{dt}$ sorry i
 forgot to write nobody noticed this $\frac{dr}{dt}$ is equal to r times $\frac{d\theta}{dt}$
 $\frac{d\theta}{dt}$ is $\dot{\theta}$ i will keep it here some cosmetics i have to do
 so that i earn space please remember what is $\dot{\theta}$ $\dot{\theta}$ is $\frac{d\theta}{dt}$ angular velocity magnitude of
 that here that is what we are not now in a
 rigid motion what happens when i have a disk and i have the point at a
 particular position whose position vector is \vec{r} then what happens to this $\vec{r} \cdot \vec{r}$
 $\frac{d}{dt}(\vec{r} \cdot \vec{r})$ is $\dot{\vec{r}} \cdot \vec{r} + \vec{r} \cdot \dot{\vec{r}}$ therefore $\dot{\vec{r}} \cdot \vec{r}$ is zero i will have only $\vec{v} \cdot \vec{v}$
 is equal to v^2 these are these in general we call this as for rigid body is zero in
 general we call it as this is radial component times \hat{e}_r plus this is the
 angular component of velocity times \hat{e}_θ now for a rigid body r is fixed
 because we are at a particular point even when the body rotates this r is
 not going to change therefore this is zero for rigid body but rigid body
 motion we have the important relation this is zero we will simply have v is
 equal to v is equal to v $v \hat{e}_\theta$ and $v \hat{e}_\theta$ is we know is $r \omega$
 what happened when ah ah $r \dot{\theta}$ $r \dot{\theta}$ is yeah $r \dot{\theta}$ $\dot{\theta}$ is ω
 times \hat{e}_θ now ah this is for rigid body $\dot{\vec{r}}$ is zero that is why this
 relationship is coming from here or ω now i will differentiate it
 further ah yeah i need that space over time ok then continue ok now i will
 calculate acceleration acceleration is equal to $\frac{d\vec{v}}{dt}$ this will be $\frac{d}{dt}$
 $\frac{d}{dt}(r \omega \hat{e}_\theta)$ this will be $\frac{d}{dt}(r \dot{\theta} \hat{e}_\theta)$ will start from here $\dot{r} \hat{e}_r$ plus $r \dot{\theta} \hat{e}_\theta$
 $\dot{\theta} \hat{e}_\theta$
 so this will be first i will differentiate this they will have $\ddot{r} \hat{e}_r$
 plus then i differentiate keep this \dot{r} constant $\frac{d}{dt}(r \dot{\theta} \hat{e}_\theta)$ plus
 next $\frac{d}{dt}(r \dot{\theta} \hat{e}_\theta)$ plus $r \dot{\theta} \dot{\hat{e}}_\theta$ when i differentiate this i
 will get $\ddot{\theta} \hat{e}_\theta$ plus ok
 so we have this expression for $\ddot{\vec{r}}$ here even though i didn't write it it was
 very clear $-\sin\theta \dot{\theta} \hat{e}_x$ plus $\cos\theta \dot{\theta} \hat{e}_y$ ok therefore this is equal
 to $\ddot{r} \hat{e}_r$ this term will be there \dot{r} into $\frac{d}{dt}$ of \hat{e}_r we
 have calculated we have here $\hat{e}_\theta \dot{\theta}$ plus $\frac{dr}{dt}$ is v ah
 sorry $\frac{dr}{dt}$ is \dot{r} $\dot{\theta} \hat{e}_\theta$ unit vector great vector i
 forgot right here plus here i have \ddot{r} or $\ddot{\theta} \hat{e}_\theta$
 plus what is the time derivative of this $\frac{d}{dt}$ of \hat{e}_θ is equal to $-\sin\theta \dot{\theta} \hat{e}_x$
 sorry if i differentiate $\sin\theta$ i will get $\cos\theta \dot{\theta} \hat{e}_x$ plus

minus $\sin \theta$ times unit vector y times $a_h \theta$ dot term remember sine is a function of θ θ is a function of time therefore $\dot{\theta}$ has to come this we know what it is $\cos \theta$ a_x if i remove this minus sign i will have minus of $\cos \theta$ e_x plus $\sin \theta$ e_y therefore i will have a_h this relationship d by $d\theta$ of e_θ is equal to e_r is equal to minus u i will have minus a_h minus $\dot{\theta}$ e_r minus $\dot{\theta}$ v_r so i have here minus $\dot{\theta}$ e_r so i have here $r \ddot{\theta}$ minus $r \dot{\theta}^2$ times e_r i am clubbing this term and this term and then i will have here plus or this is these two these two terms are the same i will have $r \ddot{\theta}$ plus two $r \dot{\theta}$ $\dot{\theta}$ both of them are in the this direction uh you will see that $r \ddot{\theta}$ is zero so this i will call it as radial acceleration this is radial a_h this is $a_r e_r$ plus $a_\theta e_\theta$ so the acceleration will have both radial components as well as a_h θ components now you see that for rigid body r is fixed therefore i will have for a rigid body or any point on the rigid body that is what i mean is it body therefore i will have the radial acceleration is equal to minus $r \dot{\theta}^2$ squared and angular acceleration is equal again this will be $\theta \ddot{\theta}$ or θ double dot so from this it is very clear that what for a particle on a rigid body there is no radial velocity but accident but radial acceleration will be there whereas both $r \dot{\theta}$ and $r \theta$ components will be there for the acceleration ok so tomorrow we will calculate this quantities and then see that whether they gel with the results thank you so you