so

So in the last class we discussed the concept of the centre of mass that is associated with the system of particles and rigid bodies and we had also seen a few illustrations and examples as to how to calculate the centre of mass some of the problems were straight forward and a particular problem where there was a continuous mass distribution was involved we had to make use of the an integration under

So please don't be scared of using mathematical techniques while solving physical problems before we move on to today's topic it is very instructive to work out a problem which was discussed in the last class which was ah sort of left in the middle this problem is like this

So we had four masses we had four masses which were distributed along the vertices at the vertices sorry of a square this is x axis this is y axis and here it is one kilo the vertex is ah minus one comma one and this here it is 2 kilos at this point at this vertex and this is 1 1 and here it is again 1 kilo and its vertex is x one and y minus one and at this vertex it is two kilograms and it is minus one comma minus one this problem we did

So just for the sake of ah what i can say recalling remember when i write a coordinate as one comma one this actually means a vector one times i plus one times j which is ah this is same as i plus j and sometimes unit vectors are also denoted as ah in this way i is denoted as e x times one plus one times unique vector along y direction therefore e x plus e y see both are same

So when you see a different notation when you see a different notation you should not get confused both these notations are used we did this problem and then we got the center of mass as our center of mass as 0 comma 0 that means it is at the origin this problem was after that we changed the problem the change problem is like this suppose it is a lamina this is the second one where the two electro the this is the this is the this this portion has two kilograms as weight and this this portion has one kilogram as weight but it is a lamina of uniform thickness ok this is two kilograms this is one kilogram this is one kilogram

So we want to calculate the center of mass of this system remember the center of mass of this is

So it is two kilograms the center of mass of this laminar portion remember its uniform lamina

So by symmetry considerations and geometry the center of mass should be located here what are the coordinates drop the x axis half and y coordinate is going to be this half right similarly the center of mass of this system would be from here it is half and if i drop here it will be minus half right

So let me write here it is two kilograms

So the center of mass of this is minus half comma minus half this is one kilogram mass lamina the center of mass would be minus half comma half

So you can check up whether you have done the calculations correctly now the center of mass of this system is equal to

So this whole mass is represented by this particular point that is where this 2 kilograms is concentrated

So it will be two into half comma half right plus we come here this one kilogram into it is located at this particular vector plus here it is two kilograms two into minus half comma minus half plus one kilogram sorry this one kilogram which is placed at the center of this ah squarish lamina right that is what the center of mass of each of this ok this whole thing divided by mass this is one three six kilograms now you can see that what is going to happen this is half this is minus half plus half minus of similarly half minus half will cancel and then you have minus half with plus half therefore this is again you get the answer as the origin ok origin is the center of mass see please do not be under impression in any problem where you are required to calculate center of mass it is going to be the origin ok i have chosen it for the sake of convenience

So for example if i am going to have one kilogram here and then two kilogram here obviously you will see that with respect to the mass distribution there is less mass on this portion one plus one two kilograms on this side there is more mass it is four kilograms obviously when you calculate the center of mass of the system will shift from the origin to the right and ok

So that you need to be careful now we will move on to the today's portion after having introduced the concept of center of mass yesterday now we have to move on to other topics

like what about the motion of center of mass how does it move this is something similar to the one dimensional problems and two dimensional problems which we had discussed earlier on a motion on a plane that is one thing and important concepts like linear momentum momentum conservation etcetera etcetera we have to deal

So you can see that there is with respect to the development of the subject there are close parallels between what you have studied in kinematic kinetics kinematics in one and two dimensions and now and right and now we shall proceed to study the this such systems

So just for the sake of convenience we will write the center of mass of a system then you have you have particles m one at r one and this is another mass m 2 which is at a position at r 2 etcetera and then i have here r n this is m n then the centre of mass of the system is given by which we have seen if you want to write as this is a rcm also you can write just for our reference is equal to what is the do

So each mass is multiplied by its corresponding position vector and this summation by m i i running from one to n here since we have italian number of particles and ok

So this is same as if total mass is m i running from 1 to little n m i r i divided by capital m where i can say that m 1 plus m 2 m n is capital m is total mass of the system of particles

So ah

So what do we do this we can write it as i can bring the m to this side i have m times r is equal to m one r one plus m two r two vector ortho rather plus m sub n times r n vector now what i do i differentiate on both this looks like a momentum conservation this is what some of the momentum on the of all the particles

So each moment is being multiplied by and this is nothing but ah this also represents the momentum of center of mass of total mass m rather ok now i will differentiate on both side with respect to time let us remember that the masses are not changing in time and

So masses are all constants and time constant masses and therefore differentiate with respect to differentiate with respect to time on both sides remember the particles are going to move masses are not going to change therefore the position vectors of this particles are all functions of time therefore there is meaning in talking about differentiating the positions vector with the position vectors with respect to time

So what do we get m into d r by d t is equal to m one d r one vector by d t plus m two times d r two vector by d t etcetera all the way up to m sub n d r n vector by d t

So we know that what is this d r one by d t is nothing but the velocity vector of the first particle similarly this is this quantity d r two by d t is the velocity vector of the second particle therefore i can write m into velocity vector i will call by little v one m one plus m two times v two etcetera plus m sub n v n now this d r by d t i will call it as capital r capital v sorry

So this is now what it means this is the velocity of centre of mass the whole the whole masses ah the all these masses are represented by capital m therefore this v is the centre of mass velocity

So v is we will write here ah

So here v is the velocity of center of mass ok

So so velocity of the center this is the expression for the velocity of center of mass

So if you want we can bring this m here and have a nice expression this is good enough now what i will do i will again ah differentiate this with respect to time because velocities also keep changing therefore m into we will call this as ok is equal to m one a one plus m two a two etcetera plus m n n where a is what is the acceleration of the centre of mass therefore it is d v by d t and a sub n is the acceleration vector of the nth particle therefore it is given by d v n by d t

So so far

So good ah of the i t particle i can do y only doing for nth particle ok now this is same as m into a is equal to what is m one into a one that is the force external force that is acting on the first particle that is the exactly the force term which is responsible for causing an acceleration of a one on the first particle therefore it is f one plus this is second is f two force plus f sub n

So what is what does it mean the vector sum of forces acting on the particles is exactly equal to the mass of the system of particles multiplied by the acceleration of the centre of mass

So this is mass of this ah the centre of mass times the acceleration of center of mass is equal to all the external forces acting on the system

So this is you call it as you call this term as force external we should call this as

force external and

So the center of mass of a system of particles moves as if the entire mass of the system is concentrated at the center of mass and all the forces where all the external forces were applied at that particular point

So what you have is this situation let us say that we have here a scenario this is ah this is what we ah maybe i can use the same diagram ok

So this is m 1 here m 2 here m i here etcetera right

So external forces are acting on it various external forces acting on is f one here this is f two here i am not denoting the actual force vectors now the whole picture can be replaced by the center of let us say the center of mass of the system by a different colored chalk i have here

So this is the centre of mass

So all these white dots can be sort of replaced by this m and it is going to move ah and it is going to move with an acceleration ah a i will denote it by a here

So all these particles and the external force acting on it can be replaced by simply this particular one ah center of mass moving with an acceleration a this is a very nice way of looking at things and ok now this is the governing equation rather m into a is equal to f external we will explain the moment what are the external forces this is the sort of governing equation this is the what you call it as the governing equation for a system of particles on one hand you have forces external forces each force will cause an acceleration on the left hand side you have the whole scenario can be replaced by one mass namely capital m and it is moving with an acceleration capital a and we have specifically used the term f x f sub external what do you mean by external cell forces are divided into ah forces can be divided into we will come over here

So right now the kind of problems we are dealing with where the forces are involved they are divided into external forces external forces external and another one is internal what are external forces for example we have let us say several particles of masses m one empty etcetera they are all falling under gravity

So gravity is the external force it does not take into account the kind of interaction between the different particles and again suppose i have some charges i place these charges in an electric field or a magnetic field this this

So these fields will in turn cause ah some forces this forces will make this charges move in a particular way this is external forces right you know that a charged particle moving in a electric and magnetic field it is given by the

So called the lorentz force term and

So these are external forces what are internal forces internal forces are something like tension tension in a string compression torsion shear and suppose i take in a gas the van der waals gas etcetera there is a kind of attraction or repulsion at various orders of length these are all internal forces this internal forces largely they they do not contribute they do not contribute for the dynamics of the system rather they contribute with structure how the whole system is going to look like what kind of structure it will form and

So generally external forces are what one calls them as low applied loads this is a terminology which is generally used and now ah we will move on to the next concept the linear momentum of a system of particles and right momentum of a particle linear momentum of a system of particles right

So we know the basic definition of a momentum if a particle is moving with a velocity v then its momentum is given by m times v and and a newton's second law is stated in this very familiar form the rate of change of momentum is what we call it as force or the other way you want to write force is defined as the rate of change of momentum ok

So the linear momentum of a system of particles is already defined as capital p which is same as m into b remember this is the terminology i guess we have used it here in this equation in fact this equation is ah momentum conservation this equation represents the momentum conservation just for the sake of clarity again i am writing this is nothing but m 1 v 1 is p 1 vector plus p 2 vector plus p n vector right ok

So the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of center of mass

So i had said it earlier the picture of that n individual particle switch which a particular mass moving can be replaced by the motion of the center of mass and it has got a mass the center of the mass of the the center of mass is capital m sum of all the masses and right and ok now what happens if external force is zero if there are no

external forces

So from this equation i can write from this equation i can get d p by d t this is the So called equation of motion for the center of mass now we can get very relevant and useful information from this suppose suppose f external is zero there are no external forces acting on the system then what will happen then automatically dp by dt is equal to zero this implies p is equal to some constant vector

So this means the this means what the this means that the linear momentum of the system remains the same this means this means the linear momentum of the system of particles remains the same as time progresses and ok this is what you call it as linear momentum is a constant of motion

So this implies for a system of particles which are not subjected to external forces the centre of mass moves such that the the linear momentum capital p is a constant of motion in a technical language you say that the the capital p is conserved its a technical language better you learn this that means it remains the same ok now what happens this now p is same as mass times v therefore therefore the velocity of the centre of mass remains constant this remains that the velocity of centre of mass remains constant that means it is not going to change its direction because it is a vector quantity this is true only if there are no external forces acting on the system and we will consider an illustration of this particular one let me consider an illustration you can treat this as illustration or a simple problem whatever way let us say that a particle is moving along a straight line this capital a thats a mass thats not important what we are trying to do here its moving along a straight line there are no external forces ok now due to some reason it explodes it is something like a bomb or something it explodes and it goes it becomes two pieces let us say

So one of the pieces goes like this other piece goes like this let us say right now what can you say about the center of mass and since external forces are zero the velocity of center of mass remains constant therefore it has to move along it has to move along the same straight line further where is going to be the the center of mass this has to be somewhere here such that you can join it has to be in a straight line let us say ok

So if external forces are not there then we can if any such explosion happens it can get split into y two it may be three or four or several particles then we can say for sure that what is the direction of center of mass and that will be same as the earlier one ok now we will work out ah simple problem you can treat as an illustration and before that will make few comments

So when a system of particles is separated into ah sorry a system of particles represented by the center of mass we have the following scenario

So the motion of the various parts is such that it is separate into center of mass and what about the motion of this particles this particles b and c move with respect to the center of mass i will just call this accent of mass let me repeat forget about this portion what we have had we have here now this two particle system b and c which are moving now the center of mass is somewhere here it is located at this particular point

So the motion of b and c can be studied in such a way that the the linear motion of centre of mass and the motion of b and c with respect to the center of mass this is what you call it as the the motion of various parts of a system is the technical language you use the word separated that means it is split is separated you can use r split you split into one the motion of centre of mass and two motion about the center of mass about the center of mass please notice the words which are being used the whole motion of this system is separated into motion of centre of mass plus the motion about the center of mass ok this is the advantage of using this now we can we can ask various questions what about the total kinetic energy of this system this two particle system whenever or what about the total kinetic energy of the several particle systems relative to the center of mass ok that we will calculate i will consider a simple example we will consider some more harder examples involving intricacies little later

So the question we have is what about what about the total kinetic energy what about the total kinetic energy of a two particle system what about the total kinetic energy of a two particle system relative to center of mass this is an important question now we will consider the situation i have two particles this m one is the mass second one m two and its velocity is v one this velocity of this is v two with respect to a suitable coordinate system let us say that then first thing is what is the velocity of centre of mass velocity of center of mass is ah we know this definition we have been using it again m one v one plus m two v two divided by m one plus m 2 i do not need this any longer

again have the same situation as there are no external forces only this is a two particle system

So as we had seen this problem yesterday the simplest one is a two particle system let us work it out for this what is the question we asked it is we need to calculate the total kinetic energy of this system with respect to the center of mass right

So velocity of m one velocity of m1 with respect to center of mass see this is a notation you have to very careful velocity of one with respect to centre of mass ok

So this is velocity of the first particle with respect to the center of mass now it is like a relative velocity its a relative velocity it is nothing but by definition v 1 minus velocity of centre of mass velocity of center of mass we have the standard notation right this we can calculate this is nothing but v one vector minus m one v one plus m two v two divided by m one plus m two i can multiply on both sides i will have sorry ah i do do the simplification rather not multiplication by both sides v one minus m one v one plus m two v one minus m two v two divided by i will do this calculation of the first particle second particle is simple right what i have done is that multiplied this m one plus m two by v one that is what m 1 v 1 plus m 2 v 1 and these 2 terms i will write as they are

So these 2 terms will get cancelled i will have m 2 into v 1 minus v 2 by m 1 plus m 2 this is the v 1 centre of mass velocity of the first particle relative to the center of mass ok now you can write down what is the velocity of center of mass for of the second particle which i will write here velocity of the center of mass relative to the sorry velocity of the second particle related to the center of mass ok this again i will do the same thing ah instead i will write here v two minus v center of mass this is equal to v two minus again i will do this m one v one plus m two v two divided by m one plus m two right

So i know what i will get ah v two ah v two m two is going to get cancelled with this i will have m one v two minus v one this divided by m one plus m two this is the velocity of the second particle with respect to the center of mass right

So you can see the symmetry there was velocity of the first particle with respect to center of mass consulted m two and then v one minus v two here velocity of the second particle with respect center of max cancels m one here and then v two minus v one right and ok now what i need to do is i need to calculate the kinetic energy of the kinetic energy of i will use a notation kinetic energy of the first particle relative to the center of mass this straight line just says where i am calculating this quantity kinetic energy of the first particle relative to the center of mass this will be by definition will be half m one into the square of that m two v one two incidentally what is this quantity the relative velocity of the first particle with respect to the second particle that is what we denote by v one two what is v one two the relative velocity of the first particle with respect to the second particle that is nothing but v one minus v two these are all fairly standard things similarly this would be what is this quantity the quantity here this will be v two minus v these two are magnitude was same but they are in opposite directions ok that divided by m one plus m two whole square

So i can do this calculation what i will get half of m one then m two squared then magnitude of v one two squared that divided by m one plus m to the whole square similarly i can calculate the kinetic energy of the second mass relative to this center of mass ok this is half of m two m one v two one by m one plus m two the whole square this would be half of m two squared sorry m two m one squared this will be since it is a magnitude i can write whichever way i want either i can write v one comma two or v two comma one divided by m one plus m to the whole square lambda ok now when i ah now i want to calculate total kinetic mass of the system i need some space but i cannot erase there therefore

So kinetic energy of the system of the system that is both the particles with respect to the center of mass z equal to i need to add this terms ah it is very easy here half i have m one and m 2 i can take out common and then mod v 1 comma 2 whole squared here i will have an m 2 here i will have an additional m 1 coming here that divided by m one plus m two whole squared

So one power will get cancelled i will simply have kinetic energy of the system with respect to the centre of mass is half of m one m two by m one plus c m two times the relative velocity of one of them with respect to other this is a very important quantity there is a very specific reason why i have chosen this particular illustration to be done and you see that this particular quantity is what is known as reduced mass reduce mass of

m one and m two this is ah denoted by mu generally it is m one m two by m one plus m two it has the dimensions of mass very clear because there is a m square here in the denominator m therefore the reduced mass has dimensions of this mass

So therefore we see that the the two particle system moves this two particle system moves as if it has a reduced mass and its value is given by m one multiplied by m two divided by m one plus 2 and what is the velocity with which this moves this is the relative velocity with which it is moving and okay now this is about the kinetic energy of system we will we will make some comments about the center of mass little later i will erase this this i will keep other expressions again

So does it represent the total kinetic energy system no remember once you are having the kinetic energy of this of this particles m1 and m2 then what about the velocity of the center of mass

So that i need to take into account therefore total kinetic energy of the system is equal to half of mu times v relative whole squared plus we cannot afford to forget the center of mass that is somewhere sitting cm center of mass that would be what is the mass center of my mass of center of mass capital m m one plus m two then its velocity c m square c m then i have to square this ok

So therefore i will have half of mu times v relative whole squared plus half of m one plus m two and we know what is the expression for center of mass m one is moving with the velocity v one and m two is moving with the velocity v two therefore velocity of the centre of mass is this quantity and i want to calculate the total kinetic energy of this system ok right this is the total kinetic energy of the system it looks little ah causing a fear but there is nothing it is a very straight forward expression right now ah what is that we have done the whole motion we are considering as if this the motion of center of mass and then the relative motion part

So this is the kinetic energy of the relative motion part this is the kinetic energy of the center of mass therefore total energy should be this let us check up whether this is correct what all things we need to do is substitute for in this particular expression substitute for ah relative and then add those things let us see what we get right we have the expressions and all here therefore i need not have this ok

So i am going to write i will call this as right hand side this is what i am going to calculate here the right hand side expression is equal to half of reduced mass is m one times m two divided by m one plus m two into this is relative velocity is v one minus v two the whole square correct

So this would be half of m one m two divided by m one plus m two multiplied by v one square plus v two squared minus two v one dot v two this dot is very important because these are two vectors there is a dot product involved when you take a square whereas here they become simply scalars ok plus oh sorry sorry this is one half of the expression other half is this expression is what i have written this expression i have to write

So that i will write it as plus half of m 1 plus m 2 times the quantity over there in the denominator i will have m one plus m two the whole squared times i will have here m one squared v one squared plus m two squared v two squared plus two m one m two little bit of space adjustment on the board v one dotted with v two ok

So one term will disappear and this i will oh sorry let me not do it here let me not do it here this will be plus half of this will disappear i will have 1 over m one plus m two whole squared there is ah and i will have here m 1 square v 1 square plus m 2 square v 2 square plus 2 m 1 m two v one dot v two now i can add this things take term by term i will see that now let us look at this term and then let us look at ah

So two is not there please because that is gone i will have half m one m two by m one plus m two then v one dot v two here i have again same m one m two times v one dot v two by m one plus m two therefore this term and this term they will cancel

So remaining i need to add these two terms i will have here yes half of m one v one squared plus half of m two v two squared is the total kinetic energy of the system

So we had we have a two particle system which is moving with velocities v one and v two its total kinetic energy is this is the kinetic energy total if we are looking the same system with respect to the center of mass then we need then these two particles then the reduced mass will have a kinetic energy

So this two particle system has a total kinetic energy this much expression half m one v one square plus half m two v two squared the same thing can be viewed the centre of man and the centre of mass language in the following way what is it it says that the reduced mass of the system namely m1 and m2 it has a velocity that is v relative velocity

therefore the energy corresponding to this is this much plus the centre of mass center of mass always has this much it will also have kinetic energy you add these two things you will exactly get to be the same

So you need to do a little bit of algebra in simplifying it i hope i can leave it

So now we come to the ah we will do little more problems later we have time 15 minutes ok four minutes ah yeah now we will make some comments about the center of mass how is the center of mass of a system defined center of mass of a system is defined as m one m two by m one plus m two ah now we invert it

So i will have one by m is equal to one by m one plus one by m two ok ah the centre of mass is also we call it as mu better i always use this symbol you know it if you have a fraction equal to sum of two fractions then what you can say from this mu is certainly less than or equal to m one and also mu is certainly less than or equal to m two ok

So the reduced mass is always less than or equal to mass of the each body let us write it here the reduced mass of a system is always less than or equal to e the mass of each body equal to mass of each body you will find that this particular technique is extensively used whenever we have multi particles in particular the simplest multi particle system is hydrogen atom where the nucleus consists of a proton and you have an electron this is a simple two body system where you where this problem is studied using this particular techniques in the coming class we will discuss maybe one or two additional illustrations and then move further and we have to move on to other topics like torque angular momentum angular momentum conservation etcetera and i will stop at this stage

So do you