so good morning all of you today is probably the last lecture on gravitation we have had a series of about six lectures

so what we did was to start with the basic conservation laws which underlie all interactions all fundamental interactions then we listed the fundamental forces and then we started looking at the most ancient and the most pervasive of the forces namely the gravitational force it was the first fundamental force to be discovered and it was the first application of newton's laws of motion which we have also enumerated the first law the second law and the third law after that we spent a lot of time discussing how ancient and medieval astronomers determine masses and distances of various planets and stars using that as an input we also studied how making use of tycho brahes observations kepler was able to formulate his famous three laws as i told you the most important thing about the three laws was that kepler did a great shift he did not try to understand the orbits of the planets in a frame fixed in the rest frame of the earth fixed in the earth but he shifted it to the sun

so that was a great shift because for a long time people believed especially europeans believed that the earth was at the center of the universe and the rest of the universe went around us because man was the highest creature created by god

so once the shift was done a very beautiful pattern emerged earlier kepler had been trying to fit the orbits into

so called platonic solids and things like that because people expected perfection in the spheres or in the motion of the planets in the celestial sphere but once he moved to the heliocentric description of the planetary orbits he got a very beautiful description in terms of elliptic orbits all planets moved in elliptic orbits and he discovered two more fundamental features equal areas swept in equal intervals of time and there was a definite ratio between the period and the radius distance from the sun t squared by r cubed is a constant

so in some sense observations and analysis spread over a few thousand years laid the ground for the formulation of universal law of gravitation newton exploited all this what was missing was actually the crucial concept of a force which can act at a distance until that time people believed that the only forces possible were contact forces if i want to push something then i better touch that object and push it even if i don't push it something else should push it like for example when there is a gust of air which i produce that may push an object for example when i exhale hair exhale air with a great force it is the air that comes in contact with a leaf or a sheet of paper and that starts moving that was the belief newton not only had to formulate the concept of force in a very precise manner that which causes acceleration in inertial frame he also was able to introduce a not

so easy concept the concept of action at a distance this was actually the basis for the concept of a field which came later

so once this was done newton was able to formulate the gravitational law and we are able to understand a whole lot of things galilean law of falling bodies the motion of the moon around the earth the motion of the earth around the sun motion of the planets around the sun motion of the satellites around planets not only that making use of these laws you can estimate masses you can estimate distance through many many ways and that is something that we discussed that great length we then looked at one of the

so called mysterious phenomena that used to take place on the earth namely the tidal phenomena and in the last lecture i worked out how the difference in the force between two different points on the earth in fact diametrically

opposite points on the earth will produce tidal forces water raises and i did not work it out completely but i showed how such a thing happens

so i hope the two people or a you students have spent enough time discussed among yourselves discuss with your teachers to understand more about the tights

so in some sense it was almost as if everything under the sun including the sun as far as gravitational forces and cosmological objects are concerned it was understood

so today what we are going to do is to look at gravitation from a slightly different view point all this time we have been looking at gravitation through the eye of forces through the lens of forces this has been fundamental what we will now do today is to switch gears and look at the same phenomenon from another view point from the top potential or potential energy when you put a body of mass m in a gravitational field i will come to that these two are not distinct from each other in fact they are equivalent but from the viewpoint of analysis they are complementary approaches what is simpler from the viewpoint of force can become more complicated from the viewpoint of potential energy and sometimes what is simple from the viewpoint of potential can become quite complicated from the viewpoint of forces i will give you some examples but before that we should carefully understand what is the meaning of gravitational potential newton himself did not make any extensive use of the idea of gravitational potential it was later mathematicians and physicists who put it into full use laplace especially when he wrote this equation and poisson poisson equations were written for the gravitational field i already gave a brief introduction to the concept of potential energy through spring mass system

so i said that the force f is equal to minus kx in a way implies a potential energy equal to half k x squared

so if i remember correctly what i did was to write the total energy the kinetic energy plus the potential energy i demanded that that quantity should be a constant kinetic energy plus potential energy should be a constant and we have obtained the equation of motion

so let us repeat that

so this approach is a utilitarian approach it tells you it is useful to employ potential energy but the idea of energy is much much deeper we have to understand that

so what is it that we did we defined the potential energy to be half k kx square remember this x is the displacement from the equilibrium position equilibrium position then of course we have the kinetic energy half mv square

so if we stipulate maybe i should use yeah kinetic energy if we stipulate that the total energy is t plus v equal to constant in time

so we know that when a pendulum oscillates or when a spring is oscillating about its equilibrium position it has zero velocity at the turning points it has the maximum velocity at the equilibrium position when it passes through the center

so what we are saying is that the change in the kinetic energy is compensated by a corresponding change in the potential energy you raise your kinetic energy the potential energy goes down you raise your potential energy the kinetic energy goes down such that this sum is a constant

so we do not have to go from force to this energy one way of doing that is to simply take this as a hypothesis and demand this must be equal to zero

so what do we get this tells you that mv dv by dt which is equal to the acceleration this plus kx equal to zero k x v equal to zero

so i have used my chain rule of differentiation d of x squared by dt that will give me a 2x and dx by dt and the 2's cancel this should be valid for all velocities

so what we do is to cancel this and sure enough we get the hook law mv equal to minus kx

so as a mathematical step it might appear to be very simple but conceptually there is a leap in our understanding of the nature of energy and we should remember that this has played an extraordinary role in our understanding of so many

so many phenomena of nature including human physiology it is not just physics one way to look at kinetic energy is to say that ok kinetic energy is lost for example when the particle is moving away from its equilibrium position and suddenly it starts gaining kinetic energy when it comes

so there is a continuous loss and gain but that kind of a picture does not account for what happened to the kinetic energy that is lost

so it is not good book keeping that is what we have but the minute we say that there is another form of energy namely the potential energy and this is perfectly natural because if i take a spring compress it i apply a whole lot of force and then i put a block

so that it does not move everyone knows that there is enormous energy stored there that is how we store the energy and then the minute i remove this top it springs back or for example if i take a kettle of water cover it fully that is the famous experiment of observation of james what lot of people had done that but james what put it to good use

so if you start heating you are supplying lot of energy because you are heating and the water molecules are gaining kinetic energy

so at some point the energy will increase

so much that the lid will blow off until that time the energy is stored somewhere okay within that

so that is not probably a very good example but there what we are looking is the conversion of some heat into kinetic energy here we are looking at heat into potential and potential to heat therefore this broadens our horizon or our understanding of what energy is and this is but a simple example and this principle of conservation of energy as i told you has played a very very important role and i will make some statements at the appropriate time the oscillation of a spring or the oscillation of a pendulum is not very different for example from the motion of a ball which is thrown up

so law of falling body

so what do i do this is my ground i take a ball i give it a certain velocity and i throw it up

so all of you know how to solve this problem because we assume that gravitational field is a constant therefore the acceleration is a constant it reaches a maximum height h this reaches the maximum height h and then what happens after it reaches a height h there it has zero velocity and therefore it becomes the turning point

so it goes there and it comes down and when it comes down very close to the surface of the earth if you were to make careful measurement of the velocity you will find provided that you know air resistance is small or ignorable or you do it in lattice in evacuated chamber the speed with which it strikes the earth is the same as the speed with which it was sent up

so there is a conservation of kinetic energy when it reached back there was nothing that was lost is that ok

so we can take the analogy by the same account and we should say that when it is going up potential energy is increases when the ball raises and

decreases when it falls

so potential means it is not visible but it is hidden inside that is what we say a person has a potential to do this we have not yet seen that

so in some sense potential energy is always the stored energy it is stored in some particular way at least in the context of mechanics and then when it falls down whatever is stored is brought out and we see it as the kinetic energy kinetic means motion

so you are seeing it as the energy of the motion therefore what we have to do is to obtain a general form for potential energy why did i say general form because when you look at the freely falling body in the earth's gravitational field we make the assumption that this height is very very small compared to the radius of the earth and therefore we take the gravitational force to be practically constant the acceleration due to gravity is a constant it does not change with the height at which it is from the surface of the earth

so we want a general form we are not going to get into the mathematical details although it is easy to work out it is more important to get the concept

so one observation from hooke's law is let us go back to hooke's law because that is what you are more familiar with if f is equal to minus kx then your potential energy is nothing but minus f dx i am not going to get into the nitty gritty details of very general motion we will assume only one dimensional motion

so the motion is always either along the force or opposite the force if you have a more general case for example a particle is free to move in two dimensions or three dimensions then what will happen the force may be acting in one direction and the motion may be in another direction

so we do not want to do that

so what are we saying basically we want to write it as minus x dx by dt into dt for example i want to follow what is happening is that right

so this is what i have in my mind

so in rectilinear motion there is no problem at all because force is in some direction dx is there

so if i now write minus f dx dx is of course always a positive increment you find that this quantity is nothing but half kx squared in fact f dx you will learn is nothing but the work done by the system it is nothing but the work done by the system that is something that you would have done in your lectures on work energy

so what i am asking you it is to remember whatever you have been taught the work energy theorem in your mechanics and make good use of it in the context of gravitation and i am repeating this only for your benefit

so that you will recollect you will recall whatever has been taught there this is not a systematic exposition of the concept of potential energy

so it works very well in a similar manner what we can do is to start with the gravitational force f is equal to minus g m m by r square and do an integration along the line of force do an integration integrate along the line of force line means direction of the force everybody knows how to do the integration all integrations need some reference point that is very very important because in your class in your calculus class you will be taught or probably have already been taught that whenever you evaluate an indefinite integral that indefinite integral is unique up to a constant because when i differentiate that constant goes away we need to do that

so what i will do is i will give you the answer and then i will tell you what kind of a constant that i have employed

so that there is no ambiguity that is what i want to do

so we will leave it to you as an exercise to verify
so let me leave it give it to your exercise verify that my v equal to minus
g m m by r is a good candidate for potential energy it is convenient
although it is not compulsory to assume at this particular point that the
body of this small m mass small m is moving in the field of a body of this
capital m of course whenever we write a capital m and small m we mean m much
much less than m
so that we do not have to worry about the motion of the larger mass
so we are having in our mind for example the motion of blocks of stone in the
gravitational field of the earth or the motion of the moon in the
gravitational field of the earth or the motion of the earth moon system in the
gravitational field of the sun mass of the earth i think is about 10 to the
power of 22 kgs and mass of this one is 10 to the power of 30 kgs
so we have a factor of 10 to the power of 8 which is 100 million
so that is the difference
so that is the picture that we have this does not take away the generality of
this expression but for our purposes it is enough because if you allow the
large mass and the small mass to be comparable and if you want to write down
the equation of motion it becomes a little bit more complicated we will have
to introduce the concept of a reduced mass and at this stage it is not
required
so let us work within this approximation that is what i have
so when i am writing v equal to minus g m m by r i told you i could have always
 put a constant and it is very easy for you people to verify that if i were
to differentiate then i should get the force
so what is my force my force is minus dv by dr that is what we are saying
into r hat i am not writing that
so when i do minus dv this becomes d m m by r and when i differentiate it will
get another minus sign which will tell you that it is an attractive force
so r is the unit vector that connects the body of larger mass to the smaller
mass
so it is pushing towards it the bigger mass is pushing the smaller mass
towards it
so that is what you have and this is the force acted upon by the larger mass
on the smaller mass
so i can actually employ my usual notation
so i have my larger mass i have my smaller mass and i am denoting the unit
vector like this that is what we have now all of you are proficient
completely capable of doing the differentiation i am not even asking you to do
the integration you can verify that this will do the job minus gmm by r
so what about the constant
so in order to understand the constant what we are saying is that look at the
limit r going to infinity
so we are asking what would happen to the potential energy of the particle
so this is the small mass the big mass this is my small mass this is the
distance between them and i am asking what will happen through the potential
energy as r goes to infinity now the expression for the force tells me that
the force goes to zero it is the inverse square law now imagine that a
particle is at rest far far away from the mass
so for example we see all the great stars in the sky we see the milky way in
the sky they contain bodies of enormous mass some of them are many many many
hundreds of times larger than they are heavier than the sun but we don't
experience any force from those stars because we are
so far away hundreds of light years away thousands of light years away we

don't experience any force

so for example if a body is moving or if you look at our solar system itself if it is moving we do not care about the potential energy we will worry only about the kinetic energy and the potential energy of the neighboring forces we do not worry about that because that is redundant

so by the same token if i imagine that this mass were to be at rest far far away from this small mass it is not experiencing any force therefore it does not even recognize the existence of this body as far as force is concerned i may see a brilliant object but it has nothing to do with me and therefore if it is at rest i will say the total energy is equal to zero and for example if i connect it to a spring i will say its kinetic energy is half k x square i do not have to worry about whatever this residual potential energy is because it is

so far away because the force is not to be worried about therefore if i write my total kinetic energy is equal to half mv square minus gm by r

so i should not use the word notation t i should use the notation e because we have used t for kinetic energy what we are saying is that we have employed the constant such that at r equal to infinity body at rest has zero energy so this is the constant that we have employed otherwise you could have jolly well added a constant v naught and this v naught is an unnecessary baggage everywhere it shifts the energy by a overall constant

so this essentially restates what you already know and what is that all energies are measurable only up to a constant only energy differences make sense absolute energy is of no interest to us that is something that we should remember and this is what we have employed

so now i am given the total energy in a gravitational field to be let me repeat my half mv square minus gm m by r

so i invite you although i told you what the result is to verify please verify that d a by d t equal to zero implies newtonian law implies newtonian law of gravitation and therefore it is very very convenient to use this concept of energy because as people often put it while force is a vector with three components energy is a scalar and it is always easier to deal with scalars than with vectors and when i do the differentiation and if i know how to keep track of directions you will learn about that later you can always get what the forces

so far i introduce the concept of a potential energy of a smaller mass in the field of a larger mass it is really not required because if you go back and if you look at this force equation now where is that

so here it is if you look at this force i wrote the force exerted by the larger mass on the smaller mass maybe it is not very clear

so let me write big letters larger mass on the smaller mass if i want it on the the force on the larger mass by the smaller mass what will i do i will again write the equation but the direction of r will be unchanged

so this unit vector r always gives the radius vector from the object to the object on which the force is acting

so if you remember that the potential energy is the same it is only way the way we define the unit vector is it from a to b or from b to a that is what tells me whether i am looking at the force acted on a by b or b by a you are getting the point right that is what we are doing if we did that what i can now do is to consider collection of masses collection of masses and see what we get

so in discussing this particular problem i am going to make use of one more very very important principle which you will encounter even in electricity and magnetism and what is that that is the principle of superposition the

forces add up i will make a few more statements about that in a while
so what we shall do is to look at various masses bodies okay this is my
coordinate system
so this is a mass m one this is the mass m two m three mk and let me call
this as i am going in this direction mn
so how many mass bodies are there n bodies of respective masses m1 m2 m3 mk
mn
so if i write mk k goes from 1 to n now each of these bodies interact with
every other body through the gravitational force that is the principle of
superposition
so if i were to ask you to write down all possible forces how many equations
would we write there would be n equations of motion
so i am interested in m one
so i will write m one d v one by dt that is the force is equal to minus gm1
m2 by r12 squared unit vector m1 m3 etcetera and the right hand side will have
 n minus 1 terms this is my fourth equation n equations with n minus 1 terms
no body will act upon itself it is always acted upon by other bodies that is
of assumption and this statement that n equations of motion is what we have is
also a misnomer because each equation is actually a collection of three
equations because it is a vector equation therefore let us be more honest and
let us write three n equations of motion
so you have to keep track of all that and you should not make any mistake and
you should write that how would it look like if i were to write the same thing
in the language of potential energy and kinetic energy
so let me repeat that again
so i have let us say these bodies i have shown one two three four five six
so this is m m1 this is m2 this is m3 this is m4 m5 m6 then what i will
do i will connect this i will say the distance between them is r12 the
distance between m4 and m5 will be r45 r12 is equal to r21 r45 equal to r54
or more generally distance between m i and m j i not equal to j is denoted
by r i j the principle of superposition of forces translates into the
principle of superposition of potential energies
so that is what we have therefore how will i write my total energy now if
there are n bodies of masses m1 mn then my total energy e t is half i
equal to 1 up to n m i v i squared is what i have if there are two bodies
there is only one potential energy term i should not repeat that because it is
a potential energy between those two and depending on how i differentiate i
get the force that is on two because of one or one because of two that is
something that we have to remember in a similar manner if there are three
bodies how many such pairs are there one two two three three one there will be
three such pairs and more generally if there are n bodies there will be n
choose two such pairs we should not over count and we know how to do that
there are many many ways of writing that is that okay no i am going to write
my potential energy
so maybe i can pull the g out but that is not required
so one way to write that is put i not equal to j and put a half this half
takes care of double counting one two two one they are both the same therefore
i divided by equal to two or another way of writing is to simply write i less
than j there is no problem at all
so if i start with 1 i will get 1 2 1 3 1 4 1 n then 2 will start with 2 3
therefore 2 n will not be counted either way it can be written and this will
be g m i m j divided by r i j this is how i write
so please remember r i j is the distance between m i and m j at that
particular instant now of course if i take these bodies and leave them even

if they are at rest what will happen they will start moving because they will start all attracting each other

so what should i write i should put a t here and i should put a t here

so at any given time my total kinetic energy is a function of time because the velocities of each of these particles is changing as a function of time because the velocities change the distances between them change

so that becomes a function of time because the distances between them change the force changes therefore my velocity changes that is how the cycle closes by itself

so this is a function of time this is a function of time and what are we asserting we are asserting that this is independent of time this is independent of time and this is the first quote unquote non-trivial statement of the conservation of energy in a dynamical system where we have a very very large number of particles and this is a very important principle we are going to put it to many many practical users when we look at escape velocity satellites

so on and

so forth but before i do anything i want to show how this is a very good book keeper conservation of energy energy law of conservation of energy is a good book keeper in fact this was recognized even more by people who do thermodynamics than people who do mechanics or cosmology at that particular time because if you remember the first law of thermodynamics what does it state it essentially states that the total energy is a conserved quantity and there of course the energy is much more than kinetic or potential it can be any energy internal energy chemical energy depending on what the system is it encompasses all kinds of energy but even for us it is a very good bookkeeper even in this limited context

so let me explain as i told you if you give me two masses then and let us say i know the initial velocity of mass m one and the initial velocity of mass m two and the initial distance between the two of them then the subsequent motion of m1 and m2 can be completely solved in newtonian gravitation that is the famous two body problem in your examples what we did was to make one of them infinitely heavy but even otherwise it can be completely solved as i told you by using what is called as the reduced mass

so please take my word at that or go and look up a good mechanics book you will understand in about ten fifteen minutes now and very interesting question occurs what happens if i have three masses m1 is free to move m2 is free to move and m3 is also free to move there is no condition on them i can there is no condition on the motion there is no condition on initial velocity there is no condition on initial position

so that is what i have

so what will i do the initial separation is r13 this separation is r23 of course i have written it in a plane they need not lie in a plane they may lie anywhere and this mass 3 m3 has a velocity v3 and everyone knows that it is child's play to set up the equations of motion because i told you how to do it for n bodies what can be done for n bodies can be specialized for three bodies setting up an equation is one thing solving the equation of motion is another thing after all even in this limited context of two bodies you looked at only circular orbits whereas the capillary and orbits are all elliptic we still do not know how to do it

so a big question arises as to how to solve the motion of three bodies

so imagine that i have the sun the earth and also the contribution coming from venus venus is the nearest planet as far as the earth is concerned suppose i want to solve this problem this problem is what is called as a three body

problem and mathematicians and physicists spent centuries trying to solve this problem they tried many many techniques whatever you do you are not able to get solutions which are stable that is if you make a small change in your initial position or velocity you would not be able to predict what is going to happen later this was a big problem until the end of the 19th century when poincare actually conclusively showed that it is impossible to solve this problem when i say that it is impossible to solve this problem you should understand what i mean by that by that i mean you cannot get close form solutions you can always solve it numerically but your numerical solutions will be valid only up to a certain time t you should understand it it is valid   only up to a certain time after that your approximation will break down then you will have to make a even a more refined numerical calculation even that will break down

so on and

so forth basically we do not have a robust mechanism numerical method either of solving this particular problem

so we do not know how to solve this problem

so what is the role of the conservation of energy

so now i will say these three masses are given at certain time t equal to zero

so there is a velocity v one v two v 3 at time t equal to 0

so you have r 1 2 r 2 3 r 3 1 we can at least ask the question if at a later time this configuration is possible let us say

so i will draw these lines

so my mass 3 is here my 1 is here and 2 is here

so what happens

so i will again give velocities v one bar v three bar v one bar v two bar and this will become r one two bar 1 2 yeah r 2 3 bar and this will be r 1 3 bar this is the new distance at a later time at least i can find out that the total energy in this configuration is the same as total energy in this configuration if they are not there if they are not the same even though i do not know how to solve the equation i can be confident that this configuration is impossible

so conservation of energy at least allows us to rule out impossible geometries impossible states now what if the total energy here is the same as the total energy there now you can do a little bit more analysis and then you can try to find out whether the solution is possible or not but at least we have reduced the total number that is the number of trajectories or the number of configurations that are possible in a similar manner you can calculate the total angle or momentum that will put one more condition because the total angular momentum is a comes out quantity you can look at the total momentum of this three body system you can look at the total momentum of the three body system

so how many constraints are we getting total energy three components of total momentum three components of total angle or momentum you put all these constraints you can try to solve for the equation for lesser number of coordinates is that right

so that is something that you can do with a very very great efficiency you don't have to solve for all nine components r one has three components r two has three components r three has three components you do not have to solve for them you can eliminate a large number of variables by these constraints go to smaller number of coordinates this in fact is the idea behind many many problems that you solve in mechanics later therefore conservation of energy is an excellent bookkeeper because it tells you what is not possible even if it

cannot tell you whether something is possible or not
so this is one of the great advantages of this now as a second application
what i shall do is to look at a very very favorite and familiar problem for
all of us and that is escape velocity which is a stuff made of dreams of
human civilization for ages human beings not only wanted to fly high in the
air like the birds dominici made many many models in order to fly but human
beings also imagine what it would be if you could actually escape from the
earth there are
so many myths
so the greek mythology has the myth of ikarus i don't know how many of you know

so ikarus was a very powerful king
so he made himself wings okay which could flap very hard okay those days
people did not know how far the atmosphere extended is that okay
so and then he started flapping the wings those artificial wings and it
started going higher up and higher up he escaped away from the earth i think
he had an engineer or a designer who made the wings for him you had told him
don't go too far away from the earth don't go to near the sun but icarus was
very very ambitious
so he kept on going near and nearer the sun
so the heat of the sun melted the wax and the wings collapsed and this guy
came crushing down in our own country we have the story of jatayu and sampati
in ramadan ramayana they were brothers
so the two brothers started flying high up in the air and they escaped from the
 earth and when they were reaching the sun the scorching heat of the sun
started burning them the elder brother who was like a father figure to his
younger brother protected the younger brother
so the elder brother sampathi collapsed and he had lost the power of his wings
but jata you survived and he later on played a very important role in the
story of rama and asu people no
so the big question that we want to ask is suppose i want to throw an object
at a certain height what should be the membrane velocity right because i want
to send a plane higher up and then it won't i want it to go in a circular
orbit or i want to launch a rocket or i want to send some other thing which
will look for alien human beings maybe in far away galaxies and stars
so basically that is the idea behind escape velocity and the principle of
conservation of energy tells you very simple answer and let us see how it
works out
so let us take the surface of the earth and then there is a body here and it
wants to escape away to infinity
so here what is the total energy according to what we have written the
kinetic energy plus gravitational energy which is minus g mass of the earth
mass of the object divided by the radius of the earth that is what i have now
what is the minimum energy that i should supply this body
so that it can reach here after that you can supply excess energy what is the
minimum energy that i can supply if it had excess energy that means even at
infinity it would have had more energy than being at rest
so the minimum energy that is required is that for escaping is that this is
equal to zero because we said that an object at rest at infinity will have
zero energy
so the bare minimum energy that is required for the object to escape from the
surface of the earth will be given by half m i will put the notation escape
is equal to minus gm by r
so if we substitute what are we going to get we are going to get psk square

is nothing but $2 g m_e$ by r_e which you see is a highly non start alien expression we think that if you want to throw an elephant up you should have higher velocity rather than a for example a small piece of ball that is not true the escape velocity is the same but the energies are different for the same v_s an elephant would have much more kinetic energy than the small ball so this escape velocity is independent of the mass

so my v escape is given by root two g m_e by r_e it is more convenient to express this expression like in your text books in terms of a quantity that you are all familiar with and that is g acceleration due to gravity

so how do we get that you write m g is equal to g m_e m by r_e squared

so m cancels

so j m a by r_e is r_e into g therefore v escape is equal to root $2 g r_e$

this is my escape velocity you could also do it from the force expression but it would not be conceptually straight forward you have the concept of energy and you are conserving it

so here the total energy was shared between the kinetic energy and the potential energy it is

so shared that the particle is at rest at infinity but at infinity the force is equal to zero therefore the particle has escaped

so strictly speaking we should write v escape is root three g r_e plus some small epsilon some small velocity otherwise this velocity is critical if it is nicely less than that it will come back if it is slightly greater than that it will not come back that is something that we have to remember

so therefore we will add a small epsilon it is small increment greater than that it is like when you know when an object is on a maxima it is an unstable equilibrium even the smallest perturbation will do that that is what we said that is what we have

so that is what we have established this is your escape velocity and this was the great challenge because we will see how large it is

so there are some numbers

so let us say g is something like 10 meters per second square and radius of the earth is 6400 kilometers which is like 6400 into 10 cubed meters

so if you plug it in your v escape turns out to be something like 11.6 kilometers per second 11.6 kilometers per second is what you are going to get that is your escape velocity

so if you want to know how big a number this is please multiply it by 3600 because that is what we are used to when we look at vehicles that move on the earth are aeroplanes that move high up in the sky

so we are speaking of 11.6 into 3600 kilometers per hour

so that is close to something like 40 000 i don't know kilometers per hour 36 into 11 36 10 is 360 plus 36 396 there is another six actually greater than forty thousand kilometers per hour that is what we have whereas the fastest of the airplanes at least the usual airplanes that we take move with the speed of about how much 700 kilometers per hour 800 kilometers per hour that is the kind of speeds that they have there are planes which move with the speed greater than that of speed up to 7 mark the speed of sound in air is about 300 meters per second

so it goes with a speed of 300 meters into 7 let us say 2100 meters per second that is only 2 kilometers or two point three kilometers per hour these very very fast planes which create shock waves and all that even that is very short compared to this 11.6 kilometers per second in fact one speed that is comparable to this speed is the speed with which the earth is going around the sun it is about 30 kilometers per second but then there is a different matter it is a freely falling body in the field of the sun

so this is not easy to achieve and that is the reason why we had to wait for a
long long time as far as technology is concerned in order to attain these
speeds and this is what we have this was not calculated under any other any
assumption because i simply equated the total initial energy with the total
filing energy but there are some corrections and what are these corrections
these corrections are whether i launch the rocket high up in the air
perpendicular to the surface or at an angle or at a tangent there are
differences you may wonder why there should be a difference because when i
wrote this equation although i indicated the distance in a direction
perpendicular this rea doesn't care for in which direction i went i could have
written it this way this way or whatever in whichever direction you go to
infinity my total kinetic energy is always your potential energy is always dmm
by re irrespective of the direction therefore you must be wondering why i am
showing different directions it is small but not insignificant and that is
because the earth oh that is how it is rotates about its axis for simplicity
let us not make a distinction between the axis of rotation and the geometric
north pole and the south pole whatever it is
so let us assume that it is rotating because of the rotation there is a
centrifugal force i should be extraordinarily careful there is a strength
centrifugal force in the frame fixed to the earth and centrifugal forces
radially outwards because it is radially outwards it is acting in a direction
opposite to the direction of the gravitation and therefore depending on in
which direction you shoot your body the escape velocity is going to change
if i shoot it perpendicularly upwards there will be the maximum decrease in
the escape velocity if i shoot it tangentially then there is no decrease at
all we will discuss that in the next class we have run out of time and i will
conclude the lecture this was not to be the last lecture on gravitation by
giving applications to various artificial satellites and india is one of the
countries which is a leader you know in satellite launching and it uses
incredibly intelligent and sophisticated technology probably will spend some
time on that also until the time please revise have a good you