

In today's lecture we will consider impact and collision problems which means we will consider we have a body moving at some speed say v_1 and we have a body 2 moving at v_2 speed these bodies move towards each other and then they move towards each other. By touching, it is a body, a body of two stars they touch each other and we call it impact and the bodies separate again after impact

so this position we i and position just after impact we will use subscript f so what we have are two Body one and two mass m_1 and m_2 stars affect each other and go away after impact and this effect can also be called collision between two bodies

so what we will understand in today's class is the mechanics of this effect if we know that bodies are approaching each other, One body has a velocity v_1 and the other has a velocity v_2 I can find the ultimate velocity of these bodies using? The law of mechanics is therefore the hat we will see and what we will realize here is that the principle of emotional motion is the one we will use to solve these problems

so this effect is something that will be realized if we try to identify the effect. This effect occurs over a very short period of time and the impact forces that cause the collision are large

so the impact can be considered as an instantaneous emotional force and the analysis we are going to do will be the body analysis just before the impact.

The position we will specify just before the effect or pre-effect and we will use the subscript f for the position or configuration we have. We are analyzing this position when one body hits another during impact

so that the position will not change between these two bodies during impact so now let us define some parameters and see this analysis we draw body one and body two during collision

so clearly What we can define as t is the tangent of flat bodies

so when the bodies touch each other we refer to the tangents of these two bodies as t and when we analyze two dimensions t this will be a line if it is a 3d analysis then t will be a plane but in two dimensional analysis we look at two bodies touching each other and we draw a tangent for these bodies and we call it as T and I_m It should be very straightforward for each of you. What we do to get the next thing is perpendicular to t

so we have and we find out what is the perpendicular side of t in this plane we call it n direction and we call it as line of influence. These two things are very important for finding the tangent plane a . The perpendicular bisector of this tangent plane is a normal plane

so if we call it t then we can draw a perpendicular to it with t and we call it perpendicular to t and we use the symbol I_n . For this nn and we understand that it is very important to solve problems

so it means when we have two affecting bodies we identify what we call tangent plane or we can call it as contact plane and line of n . Impact Now we try to analyze some of the things that we will say about the effect that will be referred to as smooth effect and what we mean by smooth effect is that the impact ball is along the line of impact individually in each body. This line has either plus or minus direction but has a smooth effect along the force of impact

so if this body of ours touches this body then this tangent plane is the line of impact

so the collision force is just along the line of impact

so that if I move towards this body If this is a smooth effect, then the force of a body collision along this direction and the force of the call body to this point of contact is along the end,

so the force of collision is n ,
so for smooth effect with collision only n , why do we call it smooth effect
because the tangent plane. There is no force and the tangent plane is where the
friction will occur

so the effect or emotional force is only along the line of impact and there is
no force with the tangent plane. We have already written what i and f_i will
mean before. Impact configuration f will indicate all the quantities that are
post impact. Now we define another term. Define the head on the shaft as $v_1 i$
and $v_2 i$ v_1 is the velocity of the body one and v_2 before impact i v_2
the velocity of the body two before direct injury

so a head on a head on direct collision where $v_1 i$ if $v_1 i$ and $v_2 i$
If these vectors are only along the n direction then the collision is called a
head directly on the collision then how do we decide whether the collision is
on the head or straight and the easiest way to do this is to draw a tangent
plane or find the t direction and Once you draw t you get the direction n
which is perpendicular to t and then $v_1 i$ and $v_2 i$ If v is an i or v is
an element equal to t between two i which is not zero then the effect is not
direct or forward and we call this kind of effect diagonal effect

so it means we have decided and

so So now we've talked about the effects of the two bodies and we've decided
that what we're talking about is a direct collision or a head-on collision
which means that the problem of each collision that you see will be the
occurrence of a direct collision. Or a diagonal collision is a common problem
we usually see in a problem the initial velocity of the body is given the
initial velocity of a body two m_1 and m_2 and then we are given that these
bodies affect and they go after the effect and we $v_1 f$ And we want to find
the two f 's

so we have two bodies. These come into effect. The basic conditions are given.
We have to find the $v_1 f$ and $v_2 f$. We say that we have a body, this and we
have a body, two and such a collision as we have shown here, it is a head on
the collision. I will see an example of a diagonal collision a little later
so we have this body now when one body is affecting two. What we get in a
collision is an energy f and an impulsive force. I look at the two bodies then
the pressure on the two bodies will be equal minus integral $f dt$ s Because
the action and reaction between body one and body two are equal and opposite,
so the impulse of body two will be equal to the subtraction of eye. We show
the action of the emotion as i and initially the momentum is $m_1 v_1 i$ and
finally the momentum is equal to $m_1 v_1 f$ then what the momentum momentum
principle tells us is that the momentum momentum principle tells us $m_1 v_1 i$
plus net impulse We have to take with the correct sign that $m_1 v_1$ is equal to
 $v_1 f$ or it can also be written that the impulse is equal to the change of
momentum

so now if it happens with one of the body it will have the effect of collision
or a head on direct collision then what we know is $v_1 i$ is only with n

so all we have to do is first show n the direction n along the horizontal and
if $v_1 i$ is only along n then what we get means that the tangent to $v_1 i$ is θ .
Now impulse is also only along n because it has a smooth effect

so these two will only be along n $v_1 f$ only along n

so if $v_1 i$ is only along n then it is a phenomenon of collision which means v_1
 i 's element is equal to zero and we What we get is that n can be added or
subtracted along with the emotion,

so v is equal to f and n and v is equal to t is equal to zero,

so what will happen now at the head of the collision? The body is one of two
bodies and suppose the velocity of one of the bodies is now at an angle at the

time of injury.

But it has n component and that is that both of these components in the component are not equal to zero and it will be in the case of a diagonal effect

After this why the material will not change Will not change because the emotion on one of the body is only equal to n

so what I get if I write v_{1t} then it will be equal to v_{1t} final

so the tangent element of velocity one which means this element now if I draw it it is v_{1t} This is because the tangent element of v_{1n} body 1 will be

the same immediately after the effect regardless of the element. In direct collision but in a diagonal direct collision v is equal to one zero

so v_{1t} is equal to zero and the same thing holds the same argument for two bodies

so for both bodies I will have subtraction of impulses which works here emotion works on one body

so subtraction i_x on body two and then we have v_{2i} was the initial velocity v_{2f} is the final velocity

so what happens again since the impulse is n direction it t direction tends only n what we get is v_{2t} is equal to the initial v_{2t} and Normal elements change

so now when we equate If we assume that it was a direct collision, let's first look at a direct collision which means that when we solve our unknowns, we have v_{1f} and v_{2f} which is equal to n . Now there is no t element in direct collision and

so we There are two unknowns v_{1f} and v_{2f} and if I write the equation of a body then what we will have is that it is a body it was traveling with v_{1i} and then there is an emotional force like this subtraction which

Inspirational force is equal to $m_1 v_{1f} - v_{1i}$ known m_1 is known impulse unknown In fact this emotion is also a scalar because it is only along the i direction

so my vector mark should be removed assuming that it is only n and an unknown with v_{1f} So there are two unknowns. Now we write this equation. In Body Two, our passion for Body Two works like this. So we have M_2 and Body Two. The initial momentum of the two is $m_2 v_{2y} + i$ is equal $m_2 v_{2f}$ all unknown we will take them along the positive side Now notice I wrote these equations with the right hand n holding positive and

so I have now given a minus sign here if I calculate these equations then v_{2f} and an unknown

so Now when you look at these equations I have already calculated that we have an unknown i second unknown v_{1f} and v_{2f} third unknown there are only two equations and that means another way to summarize an equation is we have momentum in both directions n Write the equation we consider a system

so corpse ing body one and body two together

so this is the alternative way if we put both bodies together then what we get is the initial motion

so we are talking about a case it is bringing v_{1i} it v_{1i} And these corpses are hitting each other and then they go. We call it a positive end and v_{1f} and v_{2f} we take it along the positive n if we get something with a minus sign which means it's along minus n if we start Take along the momentum But if we put the directions together for both the bodies, it would be equal to $m_1 v_{1i} - m_2 v_{2i}$ along n and the emotion for both the bodies together would be equal to one system zero because there is an emotion of i on the body. Adding an emotion of a minus i to two bodies you add these two is equal to zero

so it means that the momentum of both the bodies along the n direction must be

preserved because there is no emotion

so what we get is $m_1 v_{1i} - m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ and again we see that it has only one equation and two unknowns so these two are unknown and there is an equation

so we have an equation smaller usually in these problems when we shorten an equation Well then we need some extra information and this extra information actually comes in that either we will say and we will generalize it but most of the time when you start doing it we say that we are talking about collision is an elastic collision or completely non-elastic collision and The general expression for this is e We can express this and I am going to do this. This is how we express it in terms of an empirical quantity and this quantity is called the recovery coefficient and the symbol we will use for this is e so we will talk in terms of how things are in terms of e . Would be a common expression but when we talk about an elastic collision it would mean that the value of e which I am going to explain now would be completely non-elastic or one for a plastic faucet would be equal to lision e zero which would mean both bodies after collision Will move with the same velocity that occurs when e is equal to zero

so let's try to explain this term but what you get is when we talk about elastic collision in an elastic collision which is equal to e which says the same thing that two kinetic energy Are preserved together before and after the collision

so we can see the meaning and mathematical inherent of this ah but if the collision is not elastic then the kinetic energy after the collision is two bodies before the collision Is less than the kinetic energy and this excess or lost energy is lost energy which is not in elastic collision is converted into sound or internal energy of two bodies and may appear as heat

so kinetic energy is not saved in a non-elastic collision and this lost energy Can come in this form. Is not lost it is now saved as we said in the case of a completely elastic collision we talked about t_i without explaining e is equal to one e is equal to zero what i will do in a moment but if it is a completely elastic collision which in a case When e is equal to 0 and at this point v_1 is towards f and v is towards 2 f_n ,

so what are these when we look at the n elements of v_{1f} and v_{2f} ? One f body velocity one post effect v_{2f} body velocity two post effect n direction they are equal when e is equal to zero then v_{1fn} element of one f is equal to n element of two ffs means we are immediately after the effect Speaking of these two velocity components are equal and this is equal to zero in the case of e .

This will be given to you. It depends on the two surfaces in which the effect occurs and the way we write e is equal to e , let us understand this very well it will be subtracted. How to do it Let us say this very carefully first when we talk about separation and procedure

so separation will mean when the corpse is gone

so this separation method is when the corpses come together for the case which I did

so separation is always after effect and pre-effect of procedure

so This is what we mean by separation and approach but another thing we mean separation and method when we talk about separation and method we will only talk about the n component of the velocity of the dots which are affecting each other

so there is a dot point when I speak I van I look at v_{bn}

so n component

so it's t direction it's n direction here the point of contact is v_a and v_b

so v_{an} and v_{bn} they will give me separation and approach velocities when i . Of course we talk about the coefficient of retrieval. Of course when we talk about the body of the translator, v_a is equal to v_1 and v_b is equal to v . Whole body but now v_a is v_1 v_b is v_2 When we talk about separation I mean when we talk about v_{an} final and when I talk about approach I mean v_{an} initial and similarly v_{bn} now there is one more word that I have written here relative speed Now we also understand when I say the relative velocity of separation let us understand this

so it is a point it is b point and let's say it after the effect and what we have said is the post effect we have assumed if it is the n side everything is positive

so now When I am talking about the relative velocity of separation, what I am saying is that v_a is the final n element minus v_b is the final n element. Let us understand that this relative velocity always means v_a minus v_b or v_b minus v

so I have taken it as a f . There are directions along which we assume because we do not know their direction

so the relative velocity of the division will be v_{afn} minus the v_{bf} component. Their final and because we are talking about relative velocity which means we take one of them first after the second and we put a minus sign

so this is the relative velocity of separation for the problem that we have now. So let's do this for the given problem which we had which means our mass m_1 is coming v_1 with i mass m_2 is coming v_2 with i it was v_1 f unknown it was v_2 f unknown

so it means now we Here we write the relative velocity of the division is equal to we write v_1 f minus v_2 f now we write the relative velocity of the method now the relative velocity of the method will be because we took one in the first and two seconds d we will follow the same thing

so we will first write v_1 relative velocity and then What is the normal velocity of the approach for body 2 minus v_2 i

so I get v_1 i minus minus v_2 i it is clear and when we do this we will finally write e is equal to minus relative velocity divided by the relative velocity of the divisor method

so e will be equal now we have a velocity of division

so first we put the subtraction of v_1

so the velocity of division v_1 f minus v_2 f

so v_1 minus the subtraction of two f is divided by the relative velocities of the two f methods

so what we get from here That is, if we multiply e by v_1 i , v is equal to two i v minus two f minus v_1 f where both v is two f and v_1 f is assumed to be positive

so for this problem we got where we started using the recovery coefficient

so once we have it

so let's write this equation now our equation was m_1 v_1 i minus m_2 v_2 i it was The initial momentum is equal to m_1 v_1 f plus m_2 v_2 f this was ours and now we add the second equation we add e is equal to two f minus v_1 f divisible by two i plus v_1 i

so now it is our The second relation gives now we have two equations equations one is two equations and we have two unknown ones v_1 f and v_2 f

so all we can do is we have this relation

so what we can write from here is v_2 f equals v_1 f plus e times v_2 i plus v_1 i It comes from the number two equation and we can substitute it for one if we substitute for one then we get one v_1 i minus m_2 v_2 i equal m_1 v_1 f plus m_2 v_2 f plus e v_2 i plus e v_1 i and now we

can do this when we do this we get We work differently. We simplify these expressions. We get $v_1 f$ is equal to $m_1 v_1 i$ and then we have to subtract $m_2 v_2 i$ plus e times $v_2 y$ plus e times divided by $v_1 i$. m_2 and $v_2 f$ is equal to one multiplication $v_1 i$ plus e multiply $v_1 i$ plus e multiple v_2 minus $m_2 v_2 i$ plus m_1 divided by two you don't need to memorize these formulas you just apply these equations

And then you can solve these different kinds of problems. Now you get a lot of simplifications. Ah you will have a lot of simplification when you work on these problems. You only have two m_1 and m_2 equal so you get two at the denominator. That's fine. You can talk about an elastic collision which is equal to an elastic collision. $v_2 i$ plus $v_1 i$ it will be two $v_1 i$ plus $v_2 i$ minus $m_2 v_2 i$ etc. Bong if it is a plastic collision or a completely unstable collision then you put t equal to 0 if you keep $e = 0$ then you get $m_1 v_1 i$ minus $m_2 v_2 i$ and it is $m_1 v_1 i$ minus $m_2 v_2 i$ So we will get it and actually if there is a collision of plastics you can use the direct momentum equation because there your initial equation was $m_1 v_1 i$ minus $m_2 v_2 i$ now because both bodies are for it so since both velocities are equal after collision So m will be equal to m plus two times v is equal to two or v is equal to f . There is a minus sign because $v_2 i$ was waiting for the opposite direction if both bodies move in the same direction then it would be a plus sign equal m_1 plus m_2 times $v_2 f$ or $v_1 f$ because both are equal

so it is for direct plastic collision or elastic completely Elastic collision gives you direct origin So you don't even have to worry about writing the coefficient of recovery relationship as I told you that one can show that e equals one equal to one half $v_1 i$ square plus half $m_2 v_2 i$ square equal half $m_1 v_1 f$ Square plus half $m_2 v_2 f$ square head collision which we have just done and it can be used as second equation instead of e is equal to these two equations now identical now you will find one more thing which is done in many text books and which A smart way to do what they do is they are talking about this analysis which we have done at the head of the collision of this analysis. The v is assumed that $w_0 y$ is equal to zero so this means it can be a case. First we have a ball hitting with $v_1 i$ and for $v_2 y$ equal to 0 it is obviously a case that is being dealt with but Even if $v_2 i = 0$ is not equal to what we have done, we can change, we can change the reference frame and study the motion of a frame moving with the constant velocity of $v_2 i$,

so if we study the motion in a frame which is v_2 With the velocity of i running in the new frame $v_1 i$ is equal to $v_1 i$ minus $v_2 i$ but the advantage is that $v_2 y$ becomes 0 . Now the question you can ask is whether this reference frame will hold Newton's formula. The question is very logical. If we have Newton's formula then we have to answer this wish because Newton's formula is valid only in an inertial frame and where are we using Newton's formula?

Impulses use Newton's law in momentum relation

so if Newton's law is valid then the answer to whether our equations are valid now or not Newton's law is that we are talking about a frame which has a constant velocity

so this frame is also an inertial frame Some common things that we can see would be valid. For example let's say we are looking at a frame where v is equal to two y zeros and let's see a diagonal elastic collision between two bodies of equal mass

so it is equal to body one m_1 and m_2

so maybe my it I need to draw better I have to draw the same size

so we have body one and two bodies hitting the body one velocity $v_1 i$

tangent plane it is a normal plane it is a diagonal collision because v_{1i} is not equal to zero now if we write and we are equal Speaking of the elastic collision of the body of mass and we write it in a frame where the initial velocity v_{2i} is equal to zero

so now if we write the energy saving equation what we get is $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ It comes from energy conservation which follows because e is equal to one and because m_1 and m_2 are equal they can be discarded and what we get is $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

so for example in this case it was equal to v_{1i} then v_{1i}^2

so now after the collision if v_{1f} and v_{2f} are two velocities of two bodies then suppose if ball one moves like this If it goes v_{1f} then what we know because actually it would be a wrong way let me draw again because it must not work because $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ means these three structures ah right angled triangle So if it is v_{1i} and if it is v_{1f} then v_{1f}^2 and v_{2f}^2 squares sum of these two is equal to v_{1i}^2

so it is a right angled triangle which means v_{1f} is perpendicular to v_{2f} and

so on What we can get from this is that two velocities v_1 and v_2 must be perpendicular to each other after impact

so if there is an elastic effect an arrow The y effect is two objects of equal mass,

so what we have shown here is that the post effect two velocities v_{1f} and v_{2f} must be perpendicular to each other

so one can draw this kind of conclusion and one can come here

so now we have to save the effect and the collision. Now let's look at one more thing and the last thing I want to show in these equations is suppose if body m_2 is very large it means m_2 is much larger than m_1 if m_2 is greater than m_1 Where would we have such a big case? This is what happens when two bodies are something which means two bodies can be earth. We are talking about throwing a ball on the surface of the earth. Now come here. We see these equations. So what we do is whenever it is we have this relationship. Let's show that it's equal to. I will do this for one body. I will do this. $\frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$

so now what we do here is m_1 is the numerator and the denominator is divisible by m_2

so what we get here is $\frac{m_1}{m_2} v_{1i} + v_{2i} = \frac{m_1}{m_2} v_{1f} + v_{2f}$ plus v_{2i} plus v_{2i} divided by $m_1 + m_2$ and because m_2 is too big then this word $\frac{m_1}{m_2}$ will be a very small number it can be ignored and what we get is v_{1f} is equal to subtraction of v_{2i} plus v_{2i} plus v_{2i} times v_{1i} and suppose if v_{2i} is equal to 0 then what we get is v_{1f} is equal to the subtraction of v_{2i}

so if v_{2i} is equal to 0 When that happens What we get when a ball hits the surface of the earth is $v_{1f} - v_{2i}$ is equal to v_{1i} and similarly when we see the equation of v_{2f} which will be equal to the subtraction of v_{2i}

so v_{2f} will be v_{2i} equals subtraction of v_{1i} and if it is zero then of course it does not matter otherwise subtraction of v_{2f} will be equal to v_{2i}

so one can do these equations here again which all I have done is m_1 divided by m_2 So that we can get our relationships this way and what we are going to do in the next class is we will look at some of these problems involving a single particle where we will use different methods where we have seen that our energy

conservation method equals kinetic energy and potential energy. We have seen the law of conservation of momentum and how the initial momentum and the final momentum of the motion and the combination of these are used to solve the complex problem of single particle mechanics that we will solve. Some examples of rotation problems will be discussed later in the discourse on what we will do and what we will conceptually do. What is a rigid body and the mechanics of a rigid body which is referred to as rotation and rotation problems thank you

Prutor@MITK