

in today's lecture we will be looking at some example problems on using the method of work and energy but i'll start with explaining the concept of the term called power now in physics power has a specific meaning and what we mean by power is the rate at which work is done now work as we have seen is done by a particular force

so power is the rate at which work is done and what we know is that the amount of work done if a force  $f$  if under the force  $f$  influence of this force a particle moves a displacement  $\Delta r$  then the work done in this displacement is given as  $f \cdot \Delta r$  now if we look at the rate at which the work is done that will be  $\Delta w$  by  $\Delta t$  and this we can write it as  $f \cdot \Delta r$  by  $\Delta t$  and this will be nothing but  $f \cdot v$  if we are talking of when a force is being applied on a particle

so this is what we term as power and sometimes this is also called the instantaneous power because at time  $t$  when the force  $f$  is being applied and the velocity of the particle is  $v$  then because of this force  $f$  the power will be  $f \cdot v$  where  $v$  is the velocity of the particle now

so here  $v$  is the instantaneous velocity of the particle if we look at the units of power well first we realize that like work done power is also a scalar quantity and if we look at the to get the units of power we first look at the dimensions of power and the dimensions of power will be  $m \cdot l^2 \cdot t^{-3}$  to the power of minus 2 these were the dimensions of work done and then for power we have to divide it by one more  $t$

so this becomes equal to  $m \cdot l^2 \cdot t^{-3}$  to the power of two  $t$  to the power of minus three and if we look at in terms of  $SI$  units this will be joules per second and this is called as watt

so 1 watt is equal to 1 joule per second and often what you will find is you also will see a term called horsepower which is used for power and this is one horsepower is equal to 746 watts now this comes from british unit

so we just keep it because it's often very much used now one of the places where we see the use of power is when you get your electricity bill and in fact the electricity consumption this is in terms of energy and not in terms of energy per unit time

so what we have is the what we call as one unit of electricity which is what you see when you get your bills at home this is equal to 1 kilowatt hour which means if 1000 watts of power are consumed for 1 hr that is what gives us 1 unit of electricity and

so one can see this ah if you look at this unit one kilowatt hour one kilowatt hour this is one unit of electricity this is equal to ten to the power of 3 watts into 1 hr which is 3600 seconds

so this will be equal to  $3.6 \times 10^6$  joules that is the amount of energy used when we use one unit of electricity and

so you can then now compare how much you can now calculate how much energy is used when 100 watt bulb is lighted for 1hr etcetera and how many units it will mean

so this is in brief for definition of power now actually the power in a fundamental sense is used when we try to use this law of conservation of energy in a differential form but for our purposes we will not use it but one can differentiate the work kinetic energy equation with respect to time and then when we differentiate work we will get power on the right hand side other side we will get the change in kinetic energy when you differentiate that with respect to time that will give you  $dk$  by  $dt$  or the rate of change of kinetic energy by time

so we have seen the energy methods as a whole and let us try to sum up how how energy method help us in problem solving

so let's see problem solving using energy methods energy method i should say now energy method is useful and you will find it its often simpler to solve problems using energy method which i will explain why it is useful when we have two configurations or two positions of a body by two positions so for example i have a block which is travelling on the incline it is at position one it travels up to position two so or it could be something travelling a block moving on a circular track this is position one when it moves at the top most point this is position two of course there could be many more positions and we could apply energy method between any two positions and what will happen is that we will often be needed to will need to find either the force the velocity or the position at one of the two configurations in general what you will find is that when we use energy methods we will not be finding accelerations because when we see the energy method what we are doing is we are taking newton's law which is  $f$  is equal to  $m a$  and we are integrating this with respect to position with respect to  $d r$  we take  $f \cdot d r$  that is what gives us work done and on the other side we get that is equal to the change in kinetic energy so the work energy method is actually in some sense an integrated form of newton's law with newton's law when you apply second law you can get acceleration at each configuration not at one or two but when we integrate this then and we apply the integrated form starting from position 1 to position 2 that is where the work energy method turns out to be useful now why what are the advantages we will just see but when we start any problem where we want to use work energy method or we will be able to when you start you want to know whether you can use the work energy method will it be useful or not or should we use newton's law so the first thing which you do is that you physically and sometimes it may not be physically just or mentally you draw the free body diagram of the particle so the particle which is moving the block which is moving you draw the free body diagram and when you do when you draw the free body diagram what you will do is you will observe the forces acting on the particle this is what a free body diagram does it shows you all the forces acting on the particle so mentally you draw the free body diagram or physically you draw the free body diagram so for example when i say this block is moving up the incline i draw the free body diagram of the block so then what i will have is if the block is moving up the incline there is a weight  $mg$  there is a normal reaction and then there is a friction force and possibly because the block is moving up there has to be some sort of an external force which is pushing it up so we mentally draw the free body diagram if not physically at least have the picture of this free body diagram now the advantage of the work energy method is that if we use the work energy principle then we know the path of the particle any force which is perpendicular to the path of the particle does not contribute to the work done by external forces on the particle so this is the first simplification which comes let us look at this example in this case this particle is moving up along the  $x$  direction the normal reaction  $n$  is always perpendicular to  $x$  direction so if we use the work energy method in this problem we do not have to bother about  $n$  because we know the work done by  $n$  will always be zero in this problem there will be work done by  $mg$  work done by friction work done by force  $f$  which have to be accounted for but work done by  $n$  will not be there so this is where one simplification comes that when we draw the free body

diagram we will be able to realize that some of the forces do not do any work and this particularly will become more useful when we have more than one particles which are connected because then what we will find is that there are some interconnections between the two bodies and these interconnecting forces will do work on body one as well as body two but when we look at body one and two together as a system then these forces will not do any work and so the workless forces makes our job easier now what is the work energy principle work energy principle says that change in kinetic energy is equal to work done by all external forces so the body moves from configuration one to configuration two then we calculate the work done by all external forces as the body moves from one to two and the sum of all the work done is equal to the change in kinetic energy which means this is equal to kinetic energy at state two minus kinetic energy at state one now the second simplification which comes is that when we talk of work done by all external forces what we have seen is that there are some external forces where or whose work done can be written as minus the change in potential energy we have seen these forces in the last class so for some of the external forces the work done is equal to minus the change and we use the symbol  $v$  so let us say there is a force one which has a potential energy force two for which a potential energy can be defined then the work done by those forces can be written as minus the change in potential energy and these forces we call them as conservative forces so if we have any conservative forces in the system the work done by these forces will be given as minus the change in potential energy now the usefulness of this conservative forces is that the work done by conservative forces it only depends on state one and state two and it does not depend on the path taken between one and two so therefore if we take the work so what we can write is let's come back to the work energy equation  $\Delta K$  we write it as work done by conservative forces plus work done by non-conservative forces and the work done by conservative forces this can be written as minus  $\Delta V$  so this gives us  $\Delta K + \Delta V$  is equal to the work done by non conservative forces so this is the simplification which we get in case if there are no non conservative forces acting in the uh on the on the particle then change in kinetic energy plus change in potential energy is equal to zero and so we can find the ah find the potential energy or the kinetic energy at one of the two configurations and ah depending on whatever the unknown is we will be able to solve our problems so this is how one can then use work energy methods for ah for to help in problem solving now one thing we realize is and this you can see as a limitation of the work energy principle is that what we will be able to get at configuration 1 or configuration 2 we will be able to get the speed at one or the speed at two this we call as  $v_1$  or  $v_2$  assuming that one of them is unknown we will be able to get the speed but we will not get the velocity vector of course if the problem is of one dimensional motion then this also becomes you will get to know the velocity vector because velocity is along that line only but in case of two dimensional motion you will be only able to get the speed so does it mean that the information given to us which we get from speed is a useless information as far as accelerations go well the answer is no suppose if a particle is moving along a curved path then what we know is at any

position lets say one

so if this is position one the normal component of acceleration is equal to  $v^2/r$  where  $v$  is the speed of the particle and  $r$  is the radius of curvature

so if this is a circular path then  $r$  will be the radius of the circle and so  $a_n$  is equal to  $v^2/r$

so the speed is what we may get from energy methods but once we know the speed then we know the normal component of the acceleration is equal to  $v^2/r$  and

so if we know the path generally in a lot of problems it will be a circular path then it will be just  $v^2/r$  here  $v$  is of course the speed so speed square divided by  $r$  will give us the normal acceleration

so we will get this value of  $v$  from energy methods by using principle of energy and then we can use this to have the normal component of acceleration and using normal component of acceleration we may also get the normal component of the force and this will come from newton's second law

so having seen this now let us look at some of the problems which we can solve using work energy principle let me just go back to this particular

example which we talked of when we said we use if we want to use work energy principle to solve to look at the motion of the block from one to two then what we realize is there are four forces acting normal reaction weight

friction and the force  $f$  which is being applied on the block which is pushing it up on the incline now here what will happen is you realize the work done by gravity this we can calculate by minus the change in potential energy

because for gravity which is a constant force the acceleration is constant we have defined the potential energy is  $mg$  times the height where we choose any

reference datum choose that as zero work done by  $n$  is zero and the work done by friction force and capital  $f$  will have to be calculated and if we have to find that we may have to use newton's second law to find these things

so this is a sort of brief description of how this can be used now let us come to specific problems

so let us look at example one a block of mass 5 kg is fired up a 30 degree incline with a speed  $v$  zero is equal to five meters per second it travels a distance  $d$  is equal to two meters up the incline ah it comes to rest for a

moment and then slides back to the bottom and we want to find the velocity of the block when it comes down at its starting position now here one thing what will what is given to us is the since the block is starting with some this thing it travels only a distance up to two meters and then it starts to come down

so here lets first look at the friction force between the block and the ground and we will assume this friction force is not equal to zero it is not given in the problem

so we will start by assuming that the friction force is there it is not equal to zero

so if we mentally or physically draw the free body diagram of this block

so the picture is something like this this block is here it is given it starts it is given a push we do not maintain the force on it we just give it a push and leave it and because of this push the velocity is equal to 5 meters per second it travels the distance  $d$  which is equal to 2 meter it goes up the ramp and this angle is given to be 30 degrees and

so and after that what we want to find is it stops there and then it comes down and when it comes down when it comes back to this position we wish to find the speed the velocity of the block

so let us when we look at this when i draw the free body diagram of the block

i have when it is moving up

so let us look at the block moving up there is normal reaction there is weight and there is a friction force these are the only three forces acting on the block

so external forces are friction  $f$  to the weight  $mg$  which is acting vertically down and three the normal reaction now friction force because the block is sliding this will be equal to  $\mu_k$  times  $n$  it is a case of sliding friction

so there is relative motion

so friction is equal to  $\mu_k$  times  $n$  now when we do our this thing we look at it  $n$  is equal to  $mg \cos \theta$  that is the if i take if let me call this as the  $x$  direction let me call this as the  $y$  direction

so this is comes from some of the forces in the  $y$  direction is equal to  $0$  because the there is no acceleration and if we look at the  $x$  direction then we will get in the  $x$  direction i will get the force is minus  $mg \sin \theta$  minus  $f$  is equal to mass times acceleration in the  $x$  direction and because both of these are negative we realize acceleration in the  $x$  direction is negative so now one can solve this here but what is given to us we are given the distance of two meters

so if we solve if we try to start solving the problem in this method it will it will we can solve the problem but we will have to first find acceleration related to the distance  $d$  and then solve it but one thing we realize when we write these equations is because  $n$  is equal to  $mg \cos \theta$  and none of this changes and friction is equal to  $\mu n$

so that means the friction force is a constant force it is not changing as the particle is moving up the block well when it moves down it will be a separate story the friction will again be equal to  $\mu kn$  but its direction will change right now it is in the minus  $x$  direction when the block is moving up so what we do is instead of solving this equation finding the acceleration and then relating it to distance  $d$  we use the work energy method and when we use the work energy method let us call the starting point as one and the point where the block stops we call this point as the final position

so now work energy method tells us  $\Delta K$  plus  $\Delta V$  is equal to work done by non conservative forces

so lets start to see when we look at our external forces this is why you need to draw the free body diagram as i said it may be a mental exercise not a not a physical drawing of the free body diagram we see there are three forces the work done by  $n$

so when we come to this the work done by  $n$  is equal to  $0$  the work done by  $mg$  this will come in the potential energy due to gravity and the work done by friction is what will come in the work done by non conservative forces

so now to calculate these terms  $\Delta K$  is equal to  $K_2$  minus  $K_1$  now  $K_2$  is equal to half  $m$  times zero square because the block is stopped at point two  $K_1$  is minus half  $m v_0$  square where  $v_0$  is given to us it is given as five meters per second

so we know  $K_2$  minus  $K_1$  change in potential energy this is equal to  $v_2$  minus  $v_1$  let us take the highest position

so this is the block this distance it moves is  $d$  this angle is  $\theta$

so we take this highest position let us take the potential energy at the starting position to be zero

so  $v_1$  we take it as zero  $v_2$  will be the height of this point from with respect to this

so this height will be  $d \sin \theta$

so  $v_2$  will be equal to  $mg$  times  $d \sin \theta$

so what we have is  $v_2^2 - v_1^2$  this is  $mgd \sin \theta - 0$   
 so write each of these terms in one term separately each of these terms is quite easy when we look at the full problem the problem may look complex but we divide it into parts write each of these parts and each of these parts  $\Delta K$  is very simple  $\Delta K$  at  $K_2$  is equal to zero  $K_1$  is  $\frac{1}{2}mv_1^2$  zero square similarly potential energy when we come to it  $v_2$  is equal to  $mgd \sin \theta$   $v_1$  is equal to zero now work done by non conservative forces the friction force is acting in this direction the block is moving in the upward direction  
 so work done by friction can be written as minus of  $f$  times  $d$  because friction as we have seen is a constant force  
 so therefore this will be minus  $f$  times  $d$  and because these two are in opposite direction we get the minus sign  
 so now once we do this then we combine everything what we get is  $\frac{1}{2}mv_2^2 + mgd \sin \theta = \frac{1}{2}mv_1^2 - fd$  and from here what we will get is  $f$  is equal to  $\frac{m(v_1^2 - v_2^2)}{2d} - mg \sin \theta$   
 so we get the value of friction force now to find this friction force we know if the block has to move up has to be always positive and this will give us the condition on  $v_0$  for this block to move up or up till what height can it go now after that what we have is let us look at the what we have to find is when the block comes down the speed at its starting point  
 so what we will do is now let us call this position when it returns to the starting point we call this as three now physically one and three are the same point but what has happened is the block starts from one moves up to two and after that it is coming back down and it comes back to three  
 so what we can do is let us apply the work energy principle between one and three now what we will what we are we will exploit is that friction force has the same magnitude we have seen seen this that magnitude of friction force is equal to  $\mu_k n$  and whether the particle is moving up or moving down  $n$  is equal to  $mg \cos \theta$   
 so magnitude of friction force is equal to  $mg \cos \theta$  but what happens is when the particle moves up then friction force is in the downward direction the particle is moving up  
 so this is when the particle moves from one to two and when the particle moves from two to three it is coming down  
 so now the friction force is in this direction and this is the displacement  
 so once again the work done by friction from 2 to 3 will be equal to minus  $f$  times  $d$  and the work done by friction during its movement from 1 to 2 will be minus  $f$  times  $d$   
 so that means when the particle moves from 1 to 3 the work done by friction is equal to minus  $f d$  plus minus  $f d$  which is equal to minus two times  $f d$  and we have our principle  $\Delta K + \Delta V$  is equal to work done by friction and this is now on the entire journey from one to three  $\Delta K$  will be equal to  $\frac{1}{2}mv_3^2 - \frac{1}{2}mv_1^2$  because  $K_3$  is  $\frac{1}{2}mv_3^2$  this  $v_3$  is the unknown we want to find out  $v_1$  is given to us now what about  $\Delta V$  what is potential energy at three and potential energy at one both of them are equal to zero because the particle is at the same location where the datum we have taken as zero  
 so therefore from here we straight away get  $\frac{1}{2}mv_3^2 - \frac{1}{2}mv_1^2 = -2fd$  and  $f$  we have calculated earlier  
 so now we can put everything and we will get our answer and when we work this out we get  $v_3$  is equal to 3.77 meters per second and we realize the particle comes back with a slower speed because there is work done against friction

which can which is not restorable we do not get it back the work done by gravity is restored back in form of kinetic energy and that is why it is a conservative force but work done by friction is not and sometimes here you may see books will say that any constant force we can express a potential energy with respect to that we can write an expression for potential energy here in this example we see friction the magnitude is constant but still it cannot be expressed as a potential energy because its direction changes so therefore the work done by friction cannot be restored back once the particle comes down we do not get it back in the kinetic energy and that is one thing which will happen with all non-conservative forces okay now also here one can possibly we know the value of friction so we can work out what should be the value of  $\mu$   $\mu_k$  is not given in the problem

so if you are asked to find the value of  $\mu_k$  then you can find that out you know the friction and the  $f$  is equal to  $\mu_k$  times  $n$  is equal to  $\mu_k$  times  $mg \cos \theta$  we have worked out the value of  $f$

so you can find  $\mu_k$  and the other thing which we realize is because the block is falling down that means  $\mu_k$  has to be greater than  $\tan \theta$  otherwise the block will stay there only now let us look at another class of problems here and this is where we use work energy principle quite effectively and this is motion in a vertical circle two types of problems are very common one is we have a block or a particle which is moving in a circular path and here by saying its a vertical circle means gravity is acting vertically downward its something like a ring which is kept vertical and on that ah block or particle or an insect is moving on the ring the second case of this is a case of a particle tied to a string which is almost weightless and then the part and then the particle is of performing a circular motion so this is like a pendulum in a pendulum we give a small oscillation so it keeps on oscillating but here we do not restrict to a small oscillation let us say at this bottom point it has a velocity  $v$  does it complete the circle does it oscillate what happens to it what should be the value of  $v$  is all what we are going to analyze now

so this is the second case is a mass  $m$  tied to a string of length  $l$  in a with

so let us write the second case fully

so we have a mass  $m$  tied to a string of length  $l$  and is rotated in a vertical circle with the center of circle being fixed to the other end of the string

so this is a string of length  $l$  or radial length  $r$  that will be the radius of the circle and ah this block is moving now in both these cases if we draw the free body diagram of the particle let us say for this one i draw the free body diagram of the block what i will see is that there will be a weight  $w$  and when the particle is here there is a normal reaction  $n$  similarly when i and then of course we will assume this is a frictionless path otherwise there will be a force of friction also similarly in this case of a string what we will have is if i draw the free body diagram of this mass  $m$  we have the weight  $mg$  acting downwards and a string applies a tension force which is equal to  $t$  now this is when these when the particle is at the bottom what happens when the particle is at the top

so assuming it and goes undergoes the full circle when it is at the top what you will see is if i draw the free body diagram the weight will be acting here and if i look at the normal reaction then if the particle has to maintain contact the normal reaction has to act downwards here because the particle will exert an upward force on the circular block

so the weight on the particle will be end downwards here and in this case also if we look at the tension  $t$  the tension the string has to pull the particle down and that force we will call as  $t$  of course maybe i should call it  $t_2$  call it  $n_2$  because these will not be the same the weight will be the same but the tensions will be different

so this is how we draw we if you draw the free body diagram we will get pictures like this now when we analyze this once again this is a mental picture we have to make in mind when we analyze the whole motion we may not draw the free body diagrams but mentally we will have to keep this in mind now one of the things which we ask in these problems is often what is the minimum velocity which the particle should have at the bottom

so that it can execute the full circle now what do we mean by the condition that it can execute the full circle and for that what is critical is the velocity at the top point and to get that velocity at the top point what will be the condition how much should be the velocity at the top point

so that it is just able to complete the circle and the condition for completing the circle will not come from velocity it first has to come from the normal reaction or the tension  $t$  if the particle has to complete the circle then normal reaction  $n_2$  has to be just positive that means either this  $n_2$  has to be there and the limited condition will give us  $n_2$  is equal to  $0$  and similarly the limiting condition for the particle for this pendulum to reach the top part and come back will be that the tension at this point will just be zero it has to be positive for this that means it has to be downwards positive as shown for this to be able to complete the circle

so the limiting condition will be  $t^2$  is equal to  $0$  or  $n_2$  is equal to  $0$  not  $v^2$  is equal to  $0$  in these cases because what will happen is the place at which  $v^2$  becomes  $0$  will not be some before  $v^2$  becomes  $0$  what you will find is the tension somewhere will become  $0$  before  $v^2$  becomes  $0$  and once the tension becomes zero then the string which is carrying this once this tension is zero the particle will just fall freely from that position and similarly here once normal reaction becomes zero then the particle will just lose contact

so if we want the particle to travel the full circle at the top position that is where the normal reaction or the tension must be zero

so once we understand this then actually what we want to find is we want to find two conditions we want to find the minimum velocity at the top

so the particle can rotate in a full circle and if that is

so then we want to find the minimum velocity at bottom for a to happen and typically the problems will say in effect you have to find the velocity at the bottom

so that it can encounter the full circle

so find the problem will not give you part a it will just say find  $v$  at the bottom

so that the particle undergoes a full circle motion

so doing this is quite straight forward as we said if i le to find the velocity at the top

so this is the circle let's if i draw the free body diagram here as i have said we have  $mg$  and then we have this tension  $t$  which is acting these are the two forces acting at the top and

so we get  $mg$  plus  $t$  this is equal to total forces in the radial direction and this must be equal to  $m$  times the velocity at the top divided by  $r$  and  $r$  in this case is nothing but the length of the string

so  $m$  times  $v_{top}^2$  upon  $l$  and

so now the condition to complete circular loop is that  $t$  must be greater than or equal to zero

so here what we have is  $t$  is equal to from here what we get is  $m$  times  $v_{top}$  square by  $l$  minus  $mg$  and  
 so  $t$  greater than or equal to zero when we put this in this will imply that  $v_{top}$  square is greater than or equal to  $l$  times  $g$   
 so that means minimum velocity at the top must be equal to square root of  $l$  times  $g$  or square root of  $r$  times  $g$  where  $r$  is the radius of the circular loop  
 now to find the velocity at the bottom we use work energy principle with one being the top and two being the bottom  
 so  $k_2$  is equal to  $\frac{1}{2} m v_b^2$   $k_1$  is equal to  $\frac{1}{2} m v_t^2$  square and which will be equal to  $\frac{1}{2} m v_t^2$  will be equal to  $l$  times  $d$  ok  
 and then lets what we see is the only force is acting as the particle is moving now as the particle moves when its in a general position we have the weight  $mg$  acting we have this tension and we have the normal reaction and or tension or the normal reaction which will be acting that is it  
 so therefore the work done by  $t$  or  $n$  this will be equal to zero  
 so  $\Delta k$  plus  $\Delta v$  is equal to zero then we have  $v_2$  is equal to  $mg$  times  $2l$  or  $m g$  times  $2r$  and  $v_1$  is equal to zero  
 so we have taken the datum as the bottom  
 so with respect to that the vertical height of the top point is  $2r$   
 so this is what we have for  $v_2$  and  $v_1$  and  $\Delta k$  plus  $\Delta v$  is equal to zero  
 so this implies this gives us  $\frac{1}{2} m v_b^2$  minus  $v_t^2$  square minus  $mg$  times  $2l$  is equal to zero  
 so we get velocity at the bottom simplify this you will get it as equal to root of five  $g l$   
 so now one thing what we realize is if you look at this motion this is not a uniform circular motion why is it not uniform because the velocity is changing  
 so that means the speed is not constant throughout  
 so we cannot use formulas for uniform circular motion it is changing also  
 what we realize in this problem is that the tension  $t$  is changing with position tension  $t$  or the normal reaction  $n$  in depending on the class of problem will be equivalent they change with position but because we are using the work energy formulation the work done by  $t$  work done by  $n$  is equal to 0  
 so we do not have to bother how what is the value of  $t$  at each location and this is the sort of the power of the work energy principle otherwise if we were to do a newton's law at each point then we would have to find  $t$  at each point and we would not have been able to work out things  
 so easily but what we can do is if we know velocity at any location now this we can find even at any  $\theta$  that means at any angular location we can find the velocity how can we find this by work energy principle we will use just ah for this of course we will need to know velocity at one location and then we can find velocity at any location on the circle we use the work energy principle  
 so simply using this we can get the velocity at any location  $\theta$  and once we know that then we can draw a free body diagram and use newton's second law along the  $r$  direction to get the value of  $t$  or  $n$  at that  $\theta$  because what will happen is ah because this is circular motion  
 so along the  $r$  direction towards the center the acceleration will be  $\frac{m v^2}{r}$  and  $v$  is the speed  
 so at any location actually ah the  $i$  had used velocity  $i$  should have used here speed speed at any location can be found out and  
 so once we find the speed then we can find the ah we use this and we can find  $\frac{m v^2}{r}$  by  $r$  we can use that to find  $t$  or  $n$

so now lets beat this try to find some more things up from this problem we have this string a mass of  $m$  mass  $m$  being tied here and what we want to find is what is the speed at  $o$  so that the mass  $m$  just reaches  $a$  and then what we have is half  $m v$  zero square minus  $v$  a square plus zero minus  $m g l$  is equal to zero now this i have written it directly this change in kinetic energy change in potential energy this is point one this is  $0.2$  so at  $0.2$  the potential energy if i take it as  $0$  then the potential energy here will be minus  $mgl$  so this is how i have taken the datum at point  $a$  so because this point is down so minus  $mgl$  is the potential energy at position  $u_h$  at this is the position potential energy here potential energy at  $a$  has been taken as zero so from there what we can get is velocity at zero is equal to square root of two  $g l$  now what we have seen is that if what we saw here was if velocity at the bottom was root five  $g l$  then the part then this pendulum completes the full circle when the velocity is root  $2 g l$  then it just reaches  $a$  after that it cannot move so then it starts to come down so if the velocity at the bottom is equal to root  $2 g l$  then it oscillates in a semi circle about point  $o$  if the velocity is less than at bottom is less than root two  $g l$  then it will oscillate but it will not be able to go up to  $a$  it will be able to go up to some intermediate point  $b$  so it will oscillate with some angular  $\theta$  oscillation where this angle  $\theta$  will be less than ninety the limiting case will come when velocity  $v$  zero at the bottom is root  $2 g l$  then it will be able to go up to  $a$  and when velocity at bottom here so that means if  $v_0$  is less than root  $2 g l$  the particle the pendulum oscillates about  $o$  and if or less than or equal to root  $2 g l$  if  $v$  zero is greater than root five  $g l$  then the pendulum undergoes a full circle motion now what happens if  $v$  zero lies between root two  $g l$  and root five  $g l$  so if  $v$  zero lies between these two values so the particle with star let's say it starts from velocity  $o$  it moves here because the velocity is greater than root two  $g l$  it will go past the point  $a$  i am just showing this full circle but then what will happen is depending on the magnitude of  $v_0$  somewhere here at whatever this angle  $\theta$  is the particle will encounter a position encounter  $t$  is equal to  $0$  and at that point it leaves the circular path it will leave the circular path and then it will move like a projectile under influence of gravity because then the string will fold tension will be zero so once it leaves the circular path it will move like a projectile and it will move so once  $t$  is zero  $0$  so moves like a projectile in a parabolic path so this is how one can solve these problems the angle  $\theta$  at which the particle leaves can be found out and in fact after that because this particle is moving like a projectile so then you can apply the equation of the projectile to find the height the particle will take with respect to this position add that to the initial height that will give you the final height which will be attained by this particle when it is starting from  $0$  with the velocity between root two  $g l$  and root five  $g l$  so this is how one can solve such problems now in some other problems you may find springs being connected to the particle if a spring is connected then the

only

so let just look at this if a spring is connected to a particle the only change which will come is in the potential energy and potential energy is equal to half  $k \Delta^2$  where  $\Delta$  is the compression or the extension of the spring

so rest all stays the same you will have a new force called the spring force and will have the potential energy in the potential energy term we will add the potential energy due to the spring force which will be half  $k \Delta^2$  and other terms if there is a gravity is changing place to then you have to also account for change in gravity and sum of kinetic sum of change of kinetic  $\Delta K + \Delta V$  some of these must be equal to work done by non conservative forces non conservative forces typically will be forces like friction or a constant force  $f$  being applied in a problem

so with this we have seen some examples in energy motion in the next class we will take up the principle of conservation of momentum and conservation of angular momentum how they come from newton's second law for a single particle you