we will continue our discussion we will start with an example on work energy principle and then that will be the last example we will be doing and then we will move on to impulse momentum principle we will define the term impulse we will talk what is the impulse momentum principle and we will see how this leads to the concept of conservation of linear momentum

so that is what we will do in today's class we start with another example what we are given is that there is a table on the end of which there is a pulley and on the pulley through a string two masses a and b are connected so blocks a and b are connected by a light cable on a frictionless pulley and the pulley is also very light that means we can assume this is massless this is needed

so that we do not have to bother about the kinetic energy when this thing moves the pulley may rotate

so we do not have to bother about that because the pulley is given to be massless the system is released from state of rest and what we are asked to find is find the velocity of block a after it has moved 2 meters it is also given that the coefficient of friction mu k which means its the coefficient of kinetic friction between block a and the table is 0.25 block b is not in touch with the table

so the question of friction here does not arise

so this is the problem now if we did not know the work energy principle then we would have solved this problem by drawing a free body diagram of a drawing the free body diagram of b and then finding the acceleration of a and b which would be equal one will be in the horizontal direction the other will be vertically downwards

so using newton's second law we would have found the acceleration of block a and b and from acceleration we would have found the velocity of the block after it has moved to meters because we know that it is moving with a constant acceleration

so from there we would integrate that to get the velocity but if we use the work energy principle then we are saved of this intermediate step of finding the acceleration because here

so when we look at this problem what we realize is initial velocities are given as zero the system is starting from rest and the problem asks us to find final velocities

so therefore we think that possibly work energy principle would be a good way of doing the system where we will be saved of the intermediate step of finding the acceleration

so now when we have to do the work energy principle what we will do is we will first mentally draw the free body diagrams of bodies a and b

so when i draw the free body diagram of body a what i find is the string is pulling the body a with a force let us call this as t and this is what we have to realize when we have a light string which is connecting two bodies then if the force which the string applies on both the bodies will be the same

so if a force t is being applied to pull body a the string will pull body b with the same force t the force along the string stays the same

so we have this as long as the string is the same the force will be the same so we have a string force t here on the block a

so we are drawing the free body diagram of this block on this block what are the forces which are acting a string force its weight and the contact forces and the contact force will consist of a normal reaction and a friction force

so we show all these forces we have a string force a we have the weight a which we write as m a times g we have the normal reaction which is there let us call it as n sub a and the block is moving forwards

so here we have the friction force which is equal to mu k times n sub a
these are the forces acting on the body and here what we realize is if the if
the block
so this is friction is equal to mu k times n a now because acceleration in the
y direction is 0 this is our x direction this is the y direction since
acceleration in the y direction is equal to 0
so therefore n a is equal to m a times g
so the friction force is equal to mu k times m a times g and this we can
work out this will be equal to 0.25 into 200 into 9.8
so this works out to be equal to 490 newtons
so now what we see is the block is moving in the x direction there is a force
t there is a friction force f and the block is moving in the positive x
direction
so now when we apply if we apply the work energy principle on block a the
work energy principle tells us change in kinetic energy plus change in
potential energy is equal to work done by other forces now our state 1 is
the state of rest state two this is the final state this is let the speed of
the block be v
so we will call this as v and we know speed of both the blocks will be common
so i am not putting v a or v b they will be equal
so state 2 the speed is given as v now if we write
so therefore now we start calculating each of these quantities
so k 2 is equal to half m a v square k one is equal to zero
so this is for change in kinetic energy for potential energy both because the
block is moving in a horizontal plane due to gravitational potential energy v
one is equal to v two we call this as the reference state
so this implies change in potential energy is equal to zero now work done by
other forces what are the other forces which are acting on the block the
other forces in the x direction the block is moving in the x direction other
forces in the x direction are t minus mu k n a
so t minus mu k times m a g this will be the net force in the x direction
to this we have to multiply the distance moved in the x direction we call it
as s which is equal to in this case it has been given as ah the after the
block has moved two meters
so that means s is equal to two meters we say after the block has moved two
meters
so s is equal to two meters
so this will become equal to we worked it out t minus four ninety times two
is equal to half m a v square
so now there are two unknowns v and t that is the information we will get from
block one then we move to the block two let us call this as equation number
one now we move to block two if we draw the free body diagram of block two
we have its weight mbg acting like this and tension t acting like this and the
block is moving down
so this is the free body diagram of the block and even if you don't draw the
free body diagram when you apply the work energy principle make a mental note
you have to do this exercise now the work energy principle tells us delta k
plus delta v is equal to work done by other forces now delta k will be equal
to half mb into v square minus 0 delta v now potential energy will not talk
of work done by gravity will talk of work done by gravity as a change in
potential energy
so this will be equal to v 2 minus v 1 let us take the initial state as the
the initial state where the block was as the datum state
so where initial state if v 1 is equal to 0 then v 2 we know will be equal to

minus mg into s is equal to minus 2 mg

so therefore change in potential energy will be minus 2 times mg and work done by other forces

so now we have t is acting upwards this is moving down

so work done by t on block b this will be equal to minus t times the distance moved which is equal to minus t times two

so let us write the equation for this

so we have half mb v square minus m b g times two is equal to minus t times two lets call this as or let me write it as minus two t this is equation number two

so now what we will realize is if we look at equation number one equation number one was half m a v square is equal to t minus four ninety times two equation number two is half mb vb square is equal minus mb g2 is equal to minus 2t what we realize is 2t and minus 2t will cancel out if we add the two equations

so we do 1 plus 2 and what we get is half m a v square plus half m b v square minus m b g times two is equal to minus two times 490 the tension component by adding these two cancels out and now we have everything else we have the value of m a we have the value of m b and we can put in all these and when we work this out we will get the answer v is equal to 4.427 meters per second or we can write it as 4.43 meters per second

so now what this principle what this problem also illustrates is that if we consider a and b together as a system and we apply the principle of kinetic energy to the system then what happens is the tension which acts on these bodies separately but because the work done by tension on body a is equal to minus the work done by tension on body b when we write this together as a system the work done by t cancels out and we are just left with a change in kinetic energy plus change in potential energy of both the bodies put together is equal to work done by external forces and t in this case happens to be an internal force whose work done cancels out

so therefore we just write it is equal to work done by friction and then we get our answer but there is a bit of a catch sometimes work done by internal forces may not cancel out and this would be particular because the reason is the forces which are acting may be equal and opposite but the bodies in some cases may not move by the same distance if they do not move by the same distance then the work done will not cancel out now there is one more thing which i would like to ah tell you before we wind up our discussion on ah the work energy principle that the principle of conservation of energy which we have derived and which we have written as change in kinetic energy plus change in potential energy is equal to work done by other forces and sometimes this is also referred to as principle of conservation of mechanical energy because we are talking the principle of conservation of energy is used in the sense of when we use the first law of thermodynamics in an overall way and

so therefore this is referred to as principle of conservation of mechanical energy now on from what equation have we derived this principle of conservation of mechanical energy this has been derived from newton's second law

so since it has been derived from newton's second law for this equation to be valid all the restrictions which hold on newton's second law have to be valid and the restriction on newton's second law is that whatever velocities we calculate velocity or displacements they have to be calculated with respect to n inertial frame that means the principle of conservation of mechanical energy will be valid only if we are taking the work done and change in kinetic energy or kinetic energy etcetera which are being measured are

measured with respect to an inertial frame the frame itself has zero acceleration that means it has to either be at rest or if it is moving it has to move with constant speed along a straight line which means it is moving with the constant velocity

so therefore the principle of mechanical energy will be valid only if the kinetic energy and the work done are being calculated with respect to an inertial frame of reference and this is very important

so this is the principle of conservation of energy now let us look at quantities with respect to or the newton second law with respect to quantity called momentum we have already discussed this we called use the symbol p for linear momentum now let us define we define a term we define impulse impulse is another vector and we define impulse as integral f d t from t one to t two now

so therefore we see there are certain things involved in definition of an impulse first of all we are defining impulse of a force

so if there is a force f acting on a particle from time t1 to t2

so in when we talk of impulse there are three things involved there is a force which is acting on a particle and it is acting on a particle on a particular time interval from t1 to t2 if that is

so then impulse of a force

so what we have is actually we should be calling it as impulse of a force f during time interval from t 1 to t 2.

so if we call this quantity as impulse and then this impulse is defined as integral of force with respect to time from t1 to t2 and if f is constant in many cases we have constant forces then impulse will be just equal to f times the time interval t two minus t one

so that is the definition of impulse to see how impulse can help us in solving certain problems is if we look at newton's second law newton's second law tells us sum of external forces acting on a particle is equal to the rate of change of momentum sum of forces on a particle and this is the right hand side is rate of change of linear momentum p

so here we take the dt on the other side

so we will get f times dt is equal to dp and then we integrate both sides

so we have integral f d t is equal to integral d p now t lets say goes from t one to t two and p we say will go from at time t 1 the linear momentum is equal to p 1 at time t 2 the linear momentum is equal to p 2.

so the right hand side now is integral d p this just becomes p2 minus p1 or this we can also write it as change in momentum and the left hand side here is nothing but impulse of force f on the particle from t1 to t2

so therefore what we get is this is what we can call as the impulse momentum principle that impulse on of a force is equal to the change in moment and where we know this impulse will be integral of f times dt

so the change in linear momentum of a particle is given by the impulse of the forces acting on the particle now the principle of impulse could be useful

so this principle you will find principle of impulse is useful if the force is a function of time if the force is a function of time then if we integrate this with respect to time what we get is the change in momentum now if we try to look at this what we realize is let us look at some salient features the first thing we see is that impulse is a vector quantity now because this is a vector equation we can write scalar components

so sometimes we may need only one component

so then we write for that component and what we will get here is the x component of impulse will be equal to the change in the x momentum of the particle the y component of impulse will be equal to the change in y component

of the linear momentum which means this we can write it as m times the x momentum of particle in state 2 minus x momentum momentum we can write as m times v

so this will give us m times the change in x velocity of the particle and this will give us m times the change in y velocity of the particle will be the impulse in the y direction second thing we note about impulses if we look at the units of impulse

so then the dimension of impulse is force force is equal to m times l by t square and we multiplied by t

so its dimension is m l by t and the s i units of impulse will be forces in newtons multiplied by time

so newton second now if we are talking of a single particle for a single particle impulse methods are useful if force is a function of time then impulse just gives us the change in momentum

so we can also write it in another way let's see impulse is equal to m times for a single particle v 2 minus v 1 where v 2 is the second state v 1 is the velocity in the first state v 2 is the velocity in that state

so here we can write this equation as m times v 2 is equal to m times v 1 plus i

so what we can say is this is the initial momentum and to that you add the impulse and that will give you the final moment

so if you have to find the velocity in the final state you have the initial momentum to that just at the impulse and that will give you the final moment

now sometimes graphically this could be useful if for example force in a particular direction is given as a function of time let us say like this then area under the f t curve gives impulse

so if graphically force is given as a function of time you for example in this case the area of this triangle if this is state one this is state two you find m times v one plus area of the triangle will give you m times v two where we assume force f is acting on the particle and there is no other force acting

so therefore then you can apply equation like this

so this is the concept of what we call as the impulse momentum principle now from here arises the con the principle of conservation of momentum what we have seen is impulse is equal to change in momentum

so therefore if impulse is equal to 0 then change in momentum is equal to 0 which means m times v 1 must be equal to m times v two and this is what we call as conservation of linear moment of momentum or as we will see there is another momentum we will define

so this is referred to as conservation of linear moment now for a single particle if we look at this this ah concept of conservation of momentum is not very useful because it is very obvious if no external force is acting then the momentum of the particle does not change

so if there is only one particle involved in the impulse principle may be useful if force is given as a function of time we integrate that to find the final momentum but if impulse is zero the conservation of momentum principle for a single particle the law is not very useful the conservation of momentum principle

so if we write the confirmation of momentum principle this is useful if we have more than one particles involved

so then our as we have seen the example for energy which we saw there were two blocks and if we consider both of them together as a system there are cases like that where we have two particles which are there and in there if these two particles are interacting then the principle of conservation of momentum

can be useful and we will see how it is done but for that to be useful what we need is newton's third law which tells us the forces between these particles are equal and opposite

so when we consider both these particles together as a system then the internal forces between them will not matter they will cancel out and we will just talk of external forces acting on the total system but before we do that we introduce another concept we introduce a concept of instantaneous impulse and instantaneous impulse means if a very large force acts on a particle for a very small time then the impulse of this force is said to be an instantaneous impulse and the imp this instantaneous this force sometimes is referred to as an impulsive force

so now let just ah try to see mathematically what this means

so what we are now defining is we have already defined this quantity impulse let us say inst which shows an instantaneous impulse this will be integral from t 1 to t 1 plus epsilon of integral f d t where when we take the limit epsilon tends to be zero and f tends to be very large which means we can idealize it to be going to infinity and if the average value is of this force is f average which is large and we are multiplying it by t 1 into t 1 plus epsilon which is basically epsilon this delta t is nothing but epsilon which is very small

so this product is like something like infinity being multiplied by 0 this will be a finite product and this is what we call as an impulsive force and the impulse due to this force is what we call as an instantaneous impulse is this just a theoretical concept or is it a practical concept and let us give a couple of examples both of which are very similar the first one let us take the case of roger federer hitting a tennis ball let's say nadal is serving the ball comes and federer hits the ball with his racket now the time of contact between the ball and the racket is very small and the contact force is very large

so such a force is an example of an impulsive force ah force which acts for a very small time but the force is very large and what is the effect of this force let's see the ball is coming from this side till it hits the racket and so when the ball is coming it is coming with some momentum and the racket applies a force on the ball and what is the net effect of that

so this force plus the impulse will be equal to the final momentum and the final momentum is nothing but the final velocity of the ball multiplied by its mass

so therefore whatever force federer applies on the ball with this racket is what gives the new velocity to the ball and it has a twin effect first effect is its coming in an opposite direction first it stops it and then it makes it go with a very high velocity on the other side and

so this contact period and this contact force is what we are calling as an impulsive force and the impulse is what we are calling as an instantaneous impulse and the second example of this could be another very similar example when virat kohli hits the cricket ball with his back once again the ball is coming the bat changes its direction bat applies a force because of which the ball changes its direction

so what we see is the effect of the impulsive force is one the direction of the particle could be changed and secondly the speed of the particle is also changed

so either one or both of these effects could take place and this is what the impulsive force or any force would do on the particle now when an impulsive force acts on a particle other finite forces may also be acting during this interval when in impulsive force acts its possible other forces are also

acting for example when the tennis ball is coming and is being hit by the
racket gravity is also acting during this time but because the impulsive
force is very large and this force is acting from t 1 to t 1 plus epsilon a
very small period of time

so during this time period epsilon we neglect the effect of the other
finite forces notice the effect of these other finite forces is only being
neglected during the time period epsilon before time t1 the trajectory of the
ball is governed by the finite forces for example the gravity acting on the
ball and after the time period t1 plus epsilon once again the impulsive force
is not acting

so the motion of the ball will be governed by its initial state and also of
the finite forces but during this interval while the finite forces act we
sort we neglect their effect and this graphically we can show very easily
suppose if we draw force versus time let us say this is gravity force which is
acting on the ball there is some other finite force it may be some other force
which is of the same order and the impulsive force which is which is acting
it will be zero up till time t one and at time t one a very large force acts
and it stops

so this is this time is t 1 plus epsilon

so what we are saying is during this time period from t 1 to t 1 plus epsilon
 only the effect of the impulsive force is counted we do not count the other
effect and this is very clear that if we look at the change in momentum it is
due to impulse

so this area will be much larger during this interval epsilon then the areas
by gravity and always and as epsilon goes to zero these other force the
impulse will go to zero

so therefore we just ah use this now sometimes in the problem you are asked
to find the average contact force f average and if the time of contact is
given as delta t

so then what we have is f average times delta t now when we talk of f f
average that means what we are saying is this average force is acting over the
entire period delta t

so we treat f as a constant and this must be equal to the change in momentum
so if we know p one the initial momentum and the final momentum p two this p
one just to recall is m v one this is m v two

so if we know these then we can find the average force

so if you want to find how much force does coley's bat apply on the ball for
that you need to know the initial momentum of the ball the final momentum of
the ball just after he has hit the shot and the difference of these two will
tell you how much force and if you have an estimate of the time of contact
during which the time for which the ball has been in contact then you can find
the total force ok

so now let us see how this principle of momentum can ah be used when we
have more than one particle and this is what we said that this is where we
will have the we could possibly use not the impulse principle but the
principle of conservation of momentum now what are the type of problems where
we will have more than one particle where such things could work out one of
the very typical problems which we have are what we term as collision problems
 that means we have a body of mass m one traveling with the velocity v one
and we have a body body of mass m two traveling with the velocity v two these
two this is what we can call as the pre collision stage they touch each other
so this is at a velocity v one this is at a velocity v two they hit each other
and this is what we will call as the collision stage and after they have hit
each other this goes with v two prime this goes with v one prime these are the

final states and this is what we can call as the post collision

so this could be one state where we have ah more than one particles involved

the second type of problem is when we have one body which is moving and it suddenly breaks into two or more parts

so its something like a splinter which is moving and then it breaks into two parts

so this is m a and this breaks into two parts b and c now this breaking up will be because of internal forces and then we can let see how we can apply principle of conservation of momentum now in these every treat

so that means we have more than one particle if we treat both the particles

so let us take a case of two particles and we treat both the particles as one system and let me first state the principle if no external force acts on the two particles then the momentum of the two particles as a system is conserved

so first i have stated this ah principle if no external force acts on the two particles now by external force fourth which is external to one and two

so we will just show how this works but the principle says that the momentum of both these particles together as a system is conserved

so let us say we have a particle a and on which an external force f a is acting and we have a particle b on which a force fb is acting and these particles may be they interact with each other

so let me show them close to each other and maybe they are hitting on this external force fa is acting on particle b force fb is acting now if i draw the free body diagram of particle a then free body diagram of particle a will show me f a now i have to show all forces external to particle a now particle b suppose these are touching each other will exert a force on particle a and let me call this as f a b this is the force exerted on particle a by particle b now let me also now draw the free body diagram of b free body diagram of b shows the force f and then on this i will have fba this is the force exerted on b by particle a now when we add up this two systems and what do i mean by adding up the two systems is let us add up the forces acting on the two systems because we are considering both of these systems together then what will happen is we realize we have newton's third law which is there and newton's third law tells me f a b is equal to minus of f b a

so the internal forces which exist among these two particles they will cancel out

so now let us write the let us write the impulse momentum principle

so what we will have is the impulse momentum principle for particle a that will tell us ah integral f a d t plus integral f a b d t is equal to change in momentum of particle a and impulse momentum principle for particle b gives me integral f b d t plus integral f b a d t is equal to change in momentum of particle b and when we add these two what we will get is integral f a plus fb dt plus 0 is equal to change in momentum of particle a plus change in momentum of particle b and if

so this is the total impulse momentum principle applied to each particle and added up and if f a and f b both are equal to zero this is what we said in the law of conservation of momentum that external forces to the system now we have particle a and b external forces to the system are f a and f b now these could be because of any external things if this is equal to 0 then the if f a plus f b is equal to 0

so if these forces are equal to 0 then the change in momentum of particle a plus change in momentum of particle b is equal to zero and this i can write it as m a v a plus m b v b at state one is equal to m a v a plus m b v b

at state two and this is what we call as the law of conservation of moment and this as we said this could be used if we have impact problems collision problems then if external forces are zero then we can use this and as we have seen in some cases external forces may not be zero but in a collision let us say when we talk of a collision problem during the collision period the collision forces are much bigger than the other finite forces

so for the period of collision since if both particles treated as a system then during this interval the if both the particles are treated as a system then the momentum of the system is conserved that is we get the initial momentum is equal to the final moment

so this was the principle of conservation of linear momentum let us now also define another quantity which we will call as angular momentum or moment of momentum and lets define this if we have a point o

so there is a point o which is fixed and we have a particle moving with the velocity v which is at this position p

so here now if we write o is a fixed point the particle is moving along some path its current position is given by p

so if the if the position vector of the particle which we write as op and like it lets write it as r or i could even call it r o which means its r with respect to o which is a fixed point then we define angular momentum or we will also call it as moment of momentum of the particle about point o now this is important angular momentum is always about some point and this we will define it as the vector r cross with m v this is the linear momentum or the momentum and whenever we cross a quantity with r we call it as moment of that quantity

so r cross m v is called the moment of momentum or linear momentum and we use the symbol capital h and o represents it is moment of momentum about point o so this is how we define angular momentum of a particle

so let us say if a particle is moving along this path this is a 0.0 let us say this is one position

so we draw a position vector at this position 1 and its velocity is like this now this need not be perpendicular to each other they may be perpendicular if it is a circular path about o

so then if this is v at position one

so h about o at position one will be equal to let me call this as r one r one cross m times v one similarly at this position if the vector is r two the velocity here is v 2 and then we have angular momentum at this position 2 will be equal to r 2 cross m times v 2.

so this is the definition of angular momentum now what we realize is angular momentum once again this is a vector and we define it as r cross m v

so that means it is perpendicular to r and it is perpendicular to v because it is a cross product

so this is how the angular momentum goes now let us look at this quantity

so we have defined h zero is equal to r cross m v where r is the position vector of the point from a fixed point now let us define this quantity let us take a derivative with respect to time of both sides

so this will become d by d t of r cross m v this we can write it as d r by d t cross m v plus r cross assuming m is a constant m times d v by d t when we are writing for a particle obviously mass can be taken as constant can be taken as a constant now d r by d t is nothing but the velocity vector the change in position vector with respect to time ah from a fixed origin is the velocity

so this first term becomes v cross m times v

so therefore this becomes equal to 0 and this becomes equal to the second term

becomes r cross m times d v by d t is nothing but a
so therefore this becomes equal to a
so what we get is dho by dt is equal to r cross m a and if we are measuring
things in an inertial frame of reference m times a can be written as the
external force on the particle
so let us write it as f
so therefore what we get is dho by dt rate of change of angular momentum is
equal to r cross f
so if a force f is acting on the particle its the rate of change of angular
momentum will be given as r cross f and this is as we will see when we do
rotational mechanics this is also written as moment of the force about point
o just as we have write written moment of momentum r cross f is called the
moment of the force
so therefore we have this moment of the force and
so this we can write therefore if moment of a force can be written as m sub o
what we have got is dho by dt rate of change of angular momentum is equal to
m sub o and if moment about o is equal to 0 this leads to the condition
that dho by dt is equal to 0 which implies h o is equal to constant and h is
the angular momentum
so therefore this gives us if we are talking of two states one and state two
h about o at state one is equal to h about two at state two and this is what
we can call as the law of conservation of angular momentum
so this is law of conservation of angular momentum and for this law to be
valid what we are saying is if moment about o is equal to zero then h about
o is conserved and the place where this becomes particularly useful is when
we talk of planetary motion when we talk of motion of a satellite about a
planet or motion of planet around the sun then the force acting on the
satellite is just the gravitational force which is given as minus g times m1
m2 up upon r square if this is distance r and it acts towards the center of
the planet
so if we call the center of the planet as o then what we will realize is that
for the motion of satellite about the planet its angular momentum about o
will be constant
so in problems like this if we have a force which is always acting towards the
point o then the particle is under the motion of a force
so if particle is under the motion of a force which always towards a
fixed point o and this is the only force on the particle then r cross m v at
position one will be equal to r cross m v at position two for the particle
where and r is the position vector with respect to o
so this is the law of conservation of angular momentum for a single particle
so today we have seen the principle of conservation of linear momentum and
principle of conservation of angular momentum in the next class we will
specifically look at the collision problem where two particles come and
collide and then they move away how do we solve problem like this how do we
apply the conservation of momentum is that enough or do we need something else
 you