

in the last class we had started with newton's first law of motion and let us just recap that the first law stated that

So if a body is in such a state then the net force acting on the body is zero this is one way how the first law of motion can be stated now this law the first law sometimes is also referred to as the law of inertia because inertia we say is the tendency of the body to maintain its state of rest or uniform motion now we have seen when we did kinematics that a state of motion is a state which depends on the reference frame

So state of motion this is dependent on the reference frame whereas when we look at force forces are quantities which are frame independent that means whichever frame you measure a force in

So whether the frame is a frame at rest frame moving with constant velocity or a frame which is accelerating the forces if you measure they stay as same

So now when we look at newton's first law it is saying the net force is zero if a body is at rest or moves with uniform velocity that means something is missing because we are saying a frame independent quantity we are relating that to a frame dependent quantity and the catch to that is that newton's first law and for that matter even the second law which we will see later during the lecture today this is valid only if we are observing the state of motion from an inertial frame

So then the question which we then say is what is an inertial frame

So what we are saying is newton's law whether the first law or the second law that is valid only if the state of motion which is being observed is being observed with respect to an inertial frame now an inertial frame is a frame which is at rest that means the frame is not moving what we have also shown is that any frame which moves with a constant velocity with respect to an inertial frame is also inertial that means we are now widening the scope of the inertial frame

So first we say a frame which is at absolute state of rest is an inertial frame and then we also say that another frame which is moving at a constant velocity with respect to an inertial frame is also an inertial frame

So that means for newton's laws to be valid the frame of reference which you are looking at has to be either at rest and in that frame of reference you are measuring the velocity and the motion of the particle or the frame can move with a constant velocity now constant velocity means that two parts the speed has to be constant and the direction has to be the same

So that means

So let's explain this again

So the constant velocity this implies that there are two things the first thing is the speed is constant that means the frame should not be accelerating and the second thing is that the direction of movement should be the same that means the reference frame should be moving could move in a straight line at constant speeds

So if that is

So then the frame will be an inertial frame let us first give an example and then we will talk of inertial frames before we move to newton's second law

So let us say this is the carriage of a train and the train is at rest and we a person is standing on the train

So now when we look at the person who is standing on the train the person is not moving with respect to the train the train is at state of rest

So now let us say frame one we fix the axis on ground and frame two which i show with a different color i put this as small x y z

So frame two here we are fixing the x's on the train compartment and the person is standing it at a state of rest

So now when the train is at rest then both the frames in there we observe that the person is not moving

So both these frames at this time are inertial now what we do is let this frame let this let the train accelerate

So we have this train and now this is accelerating the person is still standing on the on the compartment is not moving

So here now when we observe with respect to frame one the person we observe is moving and has an acceleration a whereas with respect to frame two which is the frame on the train the person is still at rest now in these cases frame one is an inertial frame this is a frame which is at rest whereas frame two here we see this frame is mounting on the train this is accelerating

So this is not an inertial frame

So therefore for Newton's laws to be valid that means when we say when we look at this person with respect to an inertial frame the person is moving that means the person is accelerating

So therefore there has to be some force acting on this person which will cause it to accelerate had we looked at this person from frame two we would have said the person is not accelerating

So therefore there should be no force but there has to be a force because frame two is not an inertial frame of reference and just to clarify these things if we go ahead further let's say the train has moved for some time and the person the train is still moving but now acceleration is zero and velocity is constant let us say the track is straight

So now when we look at it what we have frame one which I have shown as capital XYZ frame two which is small x y z

So now if acceleration is zero and train moves in straight line with constant speed then both one and two are inertial frames

So this is how we define or we look at inertial frames of reference and Newton's first law which we have said the law of inertia and the second law which we will study they are valid only if the motion of the particle is being studied with respect to an inertial frame of reference now the question which one then asks is can we have an inertial frame of reference

So does an inertial frame exist and why do we ask this question ah we say ok I am observing I am standing here I observe this panel pen is moving I fix my frame on the ground here and the pen is moving

So why is this frame with respect to which which is connected to

So in most cases what we do is we fix the reference frame on surface of the earth and then we ask is this frame inertial or not well we do not observe when I am standing on the surface of that I do not see any motion

So it obviously looks to me that this is an inertial frame but what we know is that the earth is spinning about its center and

So suppose if I am at the equator then at this point on the surface of the earth the acceleration is equal to $r_e \times \omega^2$ where r_e is the radius of the earth and ω corresponds to earth spin which is one rotation in 24 hours

So ω will be equal to 2π radians divided by 24 into 3600 seconds

So if we work this out then what we find is that the acceleration when we do work this calculation the acceleration due to the spinning of the earth this is equal to $r_e \times \omega^2$ and if you look at these number this turns out to be 0.034 meters per second square

So now maybe for most purposes where we want to study the motion of a cricket ball motion of a tennis ball we can ignore this acceleration and if we can ignore this then it is fine and we can say that okay the reference frame connected to the surface of the earth is inertial but when we want to study the motion of currents and motion of winds then they are moving with respect to the surface of the earth and there this cannot be ignored

So then one says okay the earth's ah earth surface is not inertial let us go to something more

So what we do is then we say we fix a reference frame on center of the earth

So instead of fixing it on the surface I fix up a reference frame here now this frame will spin along with the earth's rotation

So therefore we can we say maybe this frame is is this an inertial frame now what we realize is that earth is in itself in an orbit around sun

So therefore there is some angular motion associated and there is acceleration

So this frame is again not fixed it is accelerating and if we try to work out the acceleration of the frame fixed to center of earth and we know it is actually an elliptical orbit but if we assume it to be a circular orbit then this will turn out to be if this radius is r_1 it will be $r_1 \times \omega_1^2$ where ω_1 is equal to 2π divided by 365 days

So if we let me write this clearly ω_1 will turn out to be one rotation in three sixty five days very small ω_1 but still it exists and if it cannot be ignored then it turns out that the acceleration of this frame which is equal to $r_1 \times \omega_1^2$ this turns out to be 0.006 meters per second square

So theoretically speaking even a frame connected to the center of the earth and rotating with the earth is not an inertial frame because the earth itself is revolving around the sun

So then what we say is if we fix

So we go one step ahead we fix a frame on center of sun but then what we see is that the sun is moving around the center of the galaxy and if we look at the acceleration of the sun towards the center of galaxy this turns out to be 3×10^{-10} meters per second square much slower but technically speaking then maybe even this cannot be inertial and then if we say you fix a frame to the center of the galaxy then this again the galaxy may be moving towards other galaxies and if that is

So then

So we are not sure if an inertial frame exists it cannot be answered does it mean that then all laws of newton are not valid certainly not

So because then what we do is we try to see in which frame do we find newton's laws to be valid one thing you realize we are using the word laws laws are like theory which means we have made some postulate and then we if that works then we use that theory

So therefore we do find that for most in some cases where we want to really account for the rotation of the earth then we use the frame fixed to center of earth is more accurate So normally in most problems of classical mechanics these two frames work as we said when we look at typical problems of a body moving vehicle moving and things like that the frame fixed to center of earth will suffice

So this was a bit of discussion which we had on inertial frames and with that then we move on to newton's second law now as we discussed in the last class we define a quantity called momentum which is the product of mass times the velocity and what the second law does is secondlaw relates the net force acting on a body to its rate of change of momentum

So let us first try to look at this quantity called momentum now we see momentum is defined as we have said as a product of mass which is a scalar and velocity which is a vector

So therefore momentum itself is a vector quantity now what we know is if we apply we observe this if we apply the same force on a light body or the same amount of force on a heavy body what difference do we observe in the motion what we observe is the light body this moves faster whereas the heavy body moves slower

So clearly force has a relation with mass at the same time we also realize that it is not only mass but speed is also another factor when we want to characterize the force let us think of a bullet a bullet fired from a gun when this hits a target it pierces through the target or it will get stuck in it if the target is very strong or very thick it gets stuck whereas the same bullet if i take it in a hand and i throw it on the wall it hits lightly and for example if i am standing uh and a bullet comes and hits me it will injure me whereas if you come and you throw the bullet on me i will not get injured

So therefore we realize from this that when it comes to causing an effect on my body the speed also makes a difference with the same mass bullet if it is coming to me with a very high speed it will injure me whereas the same bullet when it is thrown very with a low speed it will not injure me and the effect of these two quantities put together of the mass and the velocity and these two effects are put together in one way in form of momentum which we define as we have said as mass times the velocity now also what we said was that there is a directional effect and this comes because of the vector v and suppose lets think of it that there is a string there is a stone tied to a string and we we sling this stone

So i take a stone tie it to a string and move it in a circle and let us say the angular velocity is constant

So then what we realize is in this case the stone moves with a constant speed but its velocity is changing why is the velocity changing because the direction of the speed is changing

So therefore even though the stone is moving with a constant speed its velocity is changing and therefore its momentum is changing because uh the velocity is not constant its direction is changing and what we realize is if we are doing this exercise that we are taking a stone and we are moving it in a constant speed we do realize that there is a force in the string and that force has to be applied on on that string

So that the stone can move in a constant circle that means even though the stone moves at a constant speed a force has to be applied and these things these ideas which we

discussed they are quantified in newton's second law

So now let us see what is newton's second law and newton's second law says that rate of change of momentum of a particle is directly proportional to the applied force on the particle and the change of momentum being a vector quantity since momentum is a vector quantity it takes place in the direction in which the force acts now notice this force which we are talking of is applied on the body

So an external this is an external force on the body and we apply a force on the body and that causes a rate of change of momentum and the rate of change of momentum is directly proportional to the applied force and this is newton's second law

So now if we try to look at it quantitatively what we get is that if suppose a force f acts on a body for time Δt and let us say the body has a mass m and because of the action of this force it changes the velocity of the body from v to $v + \Delta v$ then what we find is the initial momentum of the particle was m times v the final momentum of the particle is m times $v + \Delta v$

So therefore the change in momentum which is Δp this is equal to $p_{\text{final}} - p_{\text{initial}}$ this will be equal to m times $v + \Delta v$ minus m times v

So this will be equal to m times Δv and what newton's law says is that the external force which is acting on the particle this is proportional to the rate of change of momentum

So change in momentum is Δp

So rate of change of momentum will be Δp by Δt and

So this will imply that force is equal to a constant k times Δp by Δt when a quantity is proportional to something else then this is equal to a constant times the thing with which it is proportional

So therefore what we do is in the limit Δt goes to 0 this becomes force is equal to k times dp by dt where dp by dt is equal to the rate of change of momentum now let us look at this quantity dp by dt this is equal to d by dt of m times v and if mass is fixed which we would expect for any particle if we are talking of a closed system then mass will be fixed

So then dp by dt becomes equal to m times dv by dt and this becomes equal to m times acceleration

So this is how we can relate the rate of change of momentum to the acceleration

So therefore what we get is since f is equal to k times dp by dt this will become equal to k times m times a now what we do is we choose our units of force in such a manner that k is equal to one for example we know that if we talk of SI units then we know mass is in kg acceleration is in meters per second square

So and the units for force this SI units are referred to as newtons and one newton is equal to one kilogram meter per second square

So then what we say is one newton is the force which one meter per second square

So therefore when we choose newton like this then k becomes equal to 1 and we get the formula f is equal to ma now some things to look about the first thing once again we would like to focus on is because we are talking of acceleration here or momentum or the rate of change of momentum when we talk of these have to be measured with respect to an inertial frame for newton's law to be valid we have to measure the acceleration or momentum whatever quantities related to the kinematics of the particle they have to be measured with respect to an inertial frame otherwise newton's law will not be valid because once again on the other side we have force we have a law like force is equal to ma a force does not depend on the reference frame acceleration depends on the reference frame

So for this law to be valid we have to specify the reference frame where acceleration is measured and that has to be an inertial frame about which we have discussed in detail earlier

So now if we try to look at some salient points about newton's second law the first thing we realize is if some of the external force is equal to zero then acceleration is equal to zero and if acceleration is equal to zero this will imply that velocity is equal to constant and that means this is what is newton's first law

So therefore some people even referred to as newton's first law as a special case of newton's second law whereas there is another school which says newton's first law helps us to define what is an inertial frame and newton's second law is f is equal to ma provided acceleration is measured in an inertial frame or some people even word the second law is saying there exists a frame I called the inertial frame where the relation

f is equal to dp by dt the rate of change of momentum is valid

So this is how different people look at it either you look at the first law as a special case or you say first law defines an inertial frame and in that frame the second law is valid the second feature let us look at it we have Newton's second law is a vector law ah what I mean by vector law probably not a standard term that both the quantities f or p these quantities are vectors

So that means our in when mass is constant we get Newton's second law as f is equal to $m a$ So we can write the or we can write it as dp by dt

So therefore we can write the scalar components of this this was our first law but we can write scalar components

So the x component we can write it as f_x is equal to change of x momentum by t rate of change of this which is equal to m times the x component of acceleration the y component of force is equal to the rate of change of y component of momentum

So we can write f_y is equal to m times $a_{sub y}$ where $a_{sub y}$ is the y component of acceleration and similarly the z for z component of force is equal to the rate of change of the z component of momentum or mass times acceleration in the z direction

So independently we can apply these three scalar equations

So we are talking of a three scalar equations they are equivalent to the one vector equation f is equal to $m a$ and sometimes in problem solving this may help because we will apply these equations maybe only along one or two components not all along all the three components now the third thing which we see is that Newton's law in the form in which we have seen this is valid for a point particle a particle which exists whose motion is there and it is occupying a very small region in space now the law can be extended for a finite body in particular for a rigid body but when we apply it on a finite body one thing we have to keep in mind is that the forces which we talk of they have to be external to the body the forces which are internal to various points on the body will not be considered when we apply Newton's law for a rigid body and the second thing which will come in here is that the acceleration when we apply to a finite body it will turn out to be the acceleration of a special point on the body not of all points and this point we will see this later we will call it the center of mass

So this discussion of how to apply Newton's second law for a rigid body we will leave till we talk about rotation and rigid bodies but it can be extended and sometimes Newton's second law applied to a rigid body is also referred to by some people as Euler's first axiom

So more details on this will follow when we talk of rigid bodies but the other thing what we realize is that this relation f is equal to $m a$ is a local relation which means the force is being exerted at time t on a body and it causes acceleration at that time t So when we write f is equal to $m a$ the force is applied at a time t immediately it causes an acceleration at that time

So therefore in f is equal to $m a$ there is no sense of history of motion of the particle of course if the f force f is constant is acting for a long time then maybe we can integrate it on time but as such given this relation f is equal to $m a$ it is just a local relation at that time whatever force is being applied is equal to the rate of change of velocity which is what is acceleration multiplied by m for the particle at that time itself now it also let us look at this relation from the momentum view point we have f is equal to rate of change of momentum

So this we can write it as $f dt$ is equal to dp and now if we integrate on left hand side we are integrating with respect to time

So let us say we integrate for time t_1 to t_2 and on the right hand side we have a dp

So let us do it again integral $f dt$ we integrate this from time t_1 to t_2 and this is equal to dp

So this will be momentum at time t is equal to t_1 and this will be momentum at time t is equal to t_2 and we have a special name for this quantity integral $f dt$ which means we integrate the force with respect to time this is referred to as impulse and if we look at the right hand side it is equal to integral dp which will become p at t_2 minus p at t_1

So this is also written as ah we can define we have impulse is equal to p at t_2 minus p at t_1

So we can say impulse of a force acting on a particle from time t_1 to t_2 this is equal to the change in momentum of the particle during this time interval

So this is another way of writing Newton's second law and sometimes if f is constant then

integral $f dt$ will just become equal to f times Δt and even if the force is not constant sometimes we use an average force and we say $f_{\text{average}} \Delta t$ and this is what we refer to as the impulse and the usage of impulse comes because impulse if we know this is equal to the change in momentum of a particle during the time interval on which it is acting

So this in effect is essentially Newton's second law now this impulse relation is particularly useful when we talk of a system of particles or two or three particles which collide now when there is an interaction between two or more particles then the interaction between two particles suppose I have this hand it comes and hits the second hand

So there is a force which is transmitted now there is a relation let us say this is body one my right hand my left hand is body two my left hand is fixed here body two comes hits this body

So now when it is hitting there is a force which is transmitted from body two to body one and at the same time body one transmits another force on body two what is the relation between these two forces and the relation between two forces when two bodies interact this is given by Newton's third law and what Newton's third law says is when two bodies interact then f_{ab} the force that body b exerts on body a is equal and opposite to f_{ba} which is the force that body a exerts on body b the two bodies when they interact and forces are being exchanged between them we represent this interaction with the forces the forces which are transferred between the two bodies are equal and opposite now what Newton had stated was the Newton's third law if we look at some classical texts this states that to every action there is an equal and opposite reaction this is the classical statement of Newton's law but in this statement when we write it like this action here means force exerted on body a by body b and the reaction is represented as f_{ba} but because this has been put in words like action and reaction this has led historically to a lot of confusion and misconception the misconception as we see that basically this gives a sense of feeling that when the body hits the body a is doing some effect on body b body b is reacting and giving an opposite effect and actually the they are just what we can see is if we look in Newton's language then action and reaction we just have a mutual pair of forces between bodies and these pairs are actually this pair acts at the same time

So there is no cause effect relation now the other thing one can see is that if we consider motion of a and b separately that means I am looking at this body a and there is a body which is very close to it it touches it it applies a force when I look at this motion separately then on body a f_{ab} the force on body a due to b this is an external force if we are looking at only body a

So if I

So for example let me draw this is body a and I have f_{ab} acting because b is coming in hitting the body I look at body b then I have f_{ba} acting as an external force this is external force on b this is external force on a but now if we study a and b as one system So that means now I am talking of body a and body b now f_{ab} and f_{ba} these forces are internal to the system my system consists of both a and b together

So these two forces are internal and as you see because they are equal and opposite they cancel out and now when I have to study the dynamics of a and b together as a system then I will say there is no force acting these two forces will cancel out if other external forces are acting on a and b let us say the weight or the reaction because of ground then they have to be counted but the interaction between a and b will cancel out

So therefore they are not to be counted they are internal forces and in some sense then Newton's third law tells us that internal forces always are occurring in pairs

So they act in pairs and they act on bodies and they cancel away in pairs

So this is Newton's third law now another thing which will happen when we talk of rigid bodies is which we have not discussed here we have said this is f_{ab} f_{ba} ah Newton's third law when we take it in a bigger sense when we extend it to from particles to rigid bodies will also tell us that f_{ab} and f_{ba} they are equal and opposite and they act along the same line of action and the reason why this gives us is because when we combine these two then the total effect of these forces has to cancel if they do not act along the same line of action then we will realize that there can be a rotational effect of these forces if they act on different lines

So therefore these internal forces which we have said they act on different bodies they and their effect is they are equal in magnitude opposite in direction and they act along

the same line

So this is what is essentially newton's third law

So newton's third law we will find will be particularly useful when we study system of particles which means that in our subject matter of study there is not one particle but two particles or three particles and there the when we consider the entire system as our unit of study then the internal forces between various particles will cancel out

So we will not even talk of these forces and that is where we will find the major use of newton's third law now what we have seen is let us sum up we have seen newton's first law newton's second law and newton's third law newton's first law was the law of inertia and what we can see is essentially newton's first law leads us to laws of statics that is if a particle is not moving then sum of forces on the particle must be equal to zero

So newton's second law which we have studied essentially we will see it being written generally as f is equal to $m a$ this is just one form actually the law says f is proportional to the rate of change of momentum mainly this will be used when we talk of a single particle or more particles or rigid body where we will use this f is equal to $m a$ and when we have more than one particles then we have to keep in mind the force we are talking of is the force which is external to the system and this comes because of third law which says that mutually the forces between particles they cancel out they are equal and opposite

So ah in a sense newton's second law is used when we talk of dynamics of a single particle when we talk of a system of particles or a rigid body then we are effectively using both newton's second law and third law together with the third law canceling out the effect of internal forces internal particles and

So therefore what we have to consider when we consider a total system of bodies is only the external forces

So these are the three laws and the other thing which we have to keep in mind is that when we talk of motion of a body then this motion has to be seen with respect to an inertial frame only only then will newton's laws be valid if the motion is given with respect to a frame which is not inertial then we will first have to convert it how this motion looks with respect to an inertial frame and then apply f is equal to $m a$ in the next class we will continue ahead we will look at different forms of newton's second law depending on whether f is constant a function of time a function of space or a function of velocity and

So we will have different formulations which will come one of which we have seen the impulse momentum formulation and this will also lead to