we will continue our discussion on forces on bodies and towards the later part of today's lecture we will also we will see how to solve problems and we will look at one of the basic steps which is needed in all mechanics problems to solve problems and that is called the free body diagram

so let us start in the last class we ended with laws of friction and we talked of solid friction or friction between two solid bodies and we showed that friction force was proportional to the normal reaction if the bodies are slipping or impending slip is there and we had a detailed discussion about this now let just have a look at what happens if the contact of a solid body is with a fluid and by fluid i mean a liquid or a gas

so for example let us say this is a block which is moving in which is moving let us say with a velocity v and surrounding this is air or water in fact we could think of this as an aero plane which is moving

so now the surrounding liquid applies ah tangential force on the solid and the typical term which we use is for the fluid friction we call it as a drag force now there is no reason that the fluid should exert a force only in the opposing the direction of velocity the fluid could also exert a force in a direction perpendicular to the velocity and that force we will normally call that as the buoyancy force or a vertical force sometimes it is also called the lift force but when we talk of friction friction caused by the fluid is in a direction which opposes the velocity and what we empirically observe from experiments what we observe is that this drag force is a function of velocity now notice the difference between solid friction and fluid friction in the solid friction if the body was moving the friction force was related to the normal force but in case of a fluid friction the drag force is a function of the velocity of the body and this drag force is proportional to velocity if the speeds are very low sometimes when we talk of this for a sphere this is called stokes law of viscosity and if the speeds are relatively higher the drag force is proportional to the square of the velocity and and it could also be a function of v to the power of n where n could be between 1 and 2

so therefore in the general way we can say that the drag force is a function of velocity

so fluid friction when we talk of friction due to fluid on a solid body this depends on velocity and it opposes the motion of the body now in some cases it is possible that the fluid helps the body to move forward then the terminology we use there is thrust instead of drag but those cases are you'll probably come across them in detail later advanced courses of fluids now from here what we realize is if we look at the forces the forces which act on the body will look at some more forces but these forces i will just bring this in discussion right now since we have talked they could either

so external force on a body they could either be constant they could be a function of distance and this we saw in electrostatic forces gravity we saw all these were function of one over r square

so its a function of distance or the external forces could also be a function of velocity as we saw in the force due to fluid friction or sometimes these forces could be a function of time and in each of these cases the way we will solve the problems will depend on how these forces are acting because eventually finally what we will put is that sum of external forces on the body is equal to mass times acceleration

so this is newton's law but now the way we attack the problem will depend on if the force is constant a function of distance a function of velocity or a function of time and these details we will see as we come to different type of forces and what we will also realize is a constant force if we develop special techniques for when the force is a function of distance velocity or

time then the constant force could be used in all these techniques
so and this we will show for example we have seen we have talked of impulse
and we define impulse as integral of f times multiplication of force times
time
so when force will be a function of time then possibly impulse methods will be
the ones which we will use what we'll also see is when force is a function of
velocity then we will write this acceleration as dv by dt or vdv by ds and
then split the two parts when force is a function of distance then as we will
see in the next chapter we will develop work energy methods for solving the
problem and they often offer a convenient way of solving the problem and force
is a function of distance and when force is constant we could use either the
work energy method or the impulse method to solve the problems but before we
talk of uh these different methods let's continue our discussion on contact
forces we had talked of contact forces on a solid on a body and we first said
that if two bodies are in contact let's just sum the discussion up if two
bodies are in contact then
so let us say this is body a this is in contact with body b then on body a
the body b exerts a reaction force we called it force on a due to b and we
show it with the general direction now what we have also shown is that if
these bodies are in contact then f a b we divide into two parts a normal
reaction and friction force the generalized version of friction force would
be a tangential force and we find in most cases it is just the friction
force
so this is the contact force we divide that into two parts now we have some
special type of contact forces in addition to the bodies touching each other
and there are two special types which i would like to discuss the first one is
suppose we have a block or a particle and this is tied to a string and the
string is pulling the body
so this is body a this is a string and the string is pulling the body now in
this case the string exerts a force on the body in the direction of the
string and what we have is this is a block and this is a string the force
which the string exerts on the body we call that call this force as tension
and the string pulls the body with the force t and we give a special name
to this as tension now here what you will realize is that if first thing
is let's just see that this tension is in the direction of the string
secondly what we realize is if we have this block and if there is a string
like this but if i compress the string then the string will just fold and
it will not exert any force on the body
so this is very clear we have seen this if you have a body like this you tie it
 to a string
so let us say if this is a string if there is a body i pull it with a string
then a force is exerted on the body but if i if the body is lying like this
and i push the string then the string will just crumple it folds up and it
cannot exert any force
so therefore whenever we have strings we show the force due to the string on
the body
so if this is the string like this and if string is if this is a body string
is tied like this this force is what we call as a tension on the string and
in case what we can have what we know often have is problems like this where
we have
so let me show the string with a different color
so we have a string which ties two bodies now here if we make two we make
two assumptions one is if we say that the string is light which means its
mass is nearly equal to zero and secondly if the string length does not

change

so length of the string stays constant when we apply force on the string or in as a reaction the string will apply force on the body

so now here let me call this m one let me call this m two

so that there is no confusion and what we will say is that

so now here when we look at it when we look at this body m one its weight w one which is equal to m one g acts down and the string offers a force on this which we call as tension let us call it t one now what will happen is if the string is light and if its length does not change then what we find is as we traverse along the length of the string the tension t the magnitude does not change and this is because when the string is light then even if we look at mass times acceleration that will be nearly equal to zero

so therefore there will be no force which will be provided internally by the string

so therefore the tension in each of these parts of an inextensible string string of which is light is the same tension t1 which goes on now when i look at this mass m2 and if i draw if i look at its uh show the forces on the mass m 2 then i will have its weight w 2 going down and i will have a tension let me first call it t 2 but because of what i have said t1 will be equal to t2

so it's the same tension which goes through the string and now if the in problems like this you will also have to realize we will be solving many of these but if there is a single string tying these and if this is inextensible then the acceleration of bodies one and two they will move by the same length if suppose body one is heavier this whole thing is moving down then this will move down but if this moves on by a distance x body two will move up by a distance x

so therefore the magnitudes of acceleration of bodies one and two will be identical and these sort of constraints you will have to work out when you solve the problems we will look at more details of these in specific cases but i am just pointing these out to you

so therefore this is how we draw we show the forces due to a string on bodies

so let us look at the second type of contact force by special bodies called spring springs you may have seen if you open up a ball point pen you see a small spring

so this is like a coiled wire now if you realize what we do is in a spring if i extend the spring

so if this is the spring and you pull it then you find a force is needed to pull the spring and not only can you pull it we can also compress the spring which means we make the same string and spring and we compress it and in this case also a force is needed to compress the spring

so if this now spring is tied to some body

so then since a force is needed to compress the spring the spring will ah will put an opposite force on the body

so if a body is connected to a compressed spring or to an extended spring then a force equal and opposite force will be applied by the spring on the body

so a spring is something which we can say that let me just give you the definition of a spring

so spring can be compressed or extended by an external force and it it generates ah restoring force which is what we are calling as the reaction which it applies on the body of contact and force on a spring what we find is this depends on the amount of compression or extension to compress the same spring by a bigger amount

so the if you want to make it smaller you will have to apply a larger force

and one of the ways how we mathematically write springs all springs may not behave like this but a lot of springs we have this relation that the force in the spring is equal to minus k times x where x is the change in length of the spring as compared to or with respect to its its unstretched length an unstretched spring we do not for it unstretched string x will be zero there will be no force required the minus sign tells us that indicates that force is opposite to the direction of displacement now this term k this we call the spring constant and as we said in a lot of springs we can assume that k is constant for that type of spring the units of k this will be equal to in s i units k will be obviously newtons per meter force per unit length so that will be the units of k

so this is how if we have a spring which is connected we write the expansion of force because of the spring which it will exert will be minus k times x

so now this is another case where the force is a function of the displacement of the body ok

so now we have seen different type of forces which act on the body now how do we go about solving a mechanics problem

so now in a typical problem what will happen is

so lets now come to problem solving using laws of mechanics and what we have is essentially we will be using newton's second law as the governing principle and that is some of the external forces on the particle will be equal to its mass times acceleration now this is a vector equation we will write it in the general form like this now what will happen in the problem will be that either the acceleration or one of the forces which are acting on the problem will be unknown and by applying this equation we will be able to get that unknown

so and now because this is a vector equation if we talk of two dimensional situations that means we can write it in the

so this is vector equation and for two d we will have two components the x component and the y component

so that means in a problem we will have two unknowns now the unknown could be either a force or an acceleration but also we realize when we solve problems like this for example there is a force of friction which comes then we may have to use the other relations which we know if the body is slipping then force of friction we know is equal to mu k times n

so therefore friction is related to the normal reaction

so that will come in as a third equation in addition to the two components of newton's law in x and y direction

so we may have to use the third equation f is equal to mu n in some cases f is equal to mu n will not be used if it is a case of no slip and then we will have only two equations and two unknowns

so now what we do is when we solve the problem the first step is what we call as drawing a free body diagram of the system i am using the word system so that we can generalize it later in current context we will be drawing free body diagram of the single body in which we are interested sometimes there may be two bodies let us first talk of free body diagram of a single body what do we mean by a free body diagram

so what we do is we first talk of a single body

so we isolate the body in question and then we show on notional diagram of the body and if we are not interested in rotation we are talking of the body then this will be treated as a particle then we can even show it as a point and there what we then what we do is we show all external forces acting on the body and this is what is a free body diagram

so let us say let us take a very simple case there is ground on which i have

been taking this example again again and again a block is kept on the ground and here we want to now draw this block is at rest it is just kept on the ground

so the case as we saw yesterday of let us say this pin which is let it come to rest

so this is lying on the ground i want to draw the free body diagram of the pen

so here i am just replacing it by the block

so first what i will do is i will show the block now notice in the free body diagram the ground will not come in when i say free body diagram of the block only the block comes and then what i do is i show all the external forces

now when i show the external forces there are two types of external forces one are forces acting from a distance

so on this block which force is acting from a distance and what we find is the only force which acts from the distance is gravity and this will be the force due to that will be represented by its weight

so the first force we show on the body on this block is its weight w and sometimes if we want either we can show it as w or we can write it as mg where m is the mass of the block and we may have to use w is equal to mg in the problem now

so this is forces acting at a distance secondly we will look at the contact forces

so now for the contact forces let us show this was our problem this is the block we have to draw the free body diagram of this

so and what we have done is we have isolated the block

so first step is isolation of the body we show forces at a distance we show the force w now what we should do is in a general problem mentally make a trip just go all around the block where the block is in contact with anything else or not and what we find is this block is in contact with the ground at the bottom at this place the block is in contact with the ground

so now we have to show the effect of ground on the block and the effect of ground as we said in general would be ah a normal force and a friction force

so therefore we show the normal force and the friction force maybe you could have also shown this as w and a single reaction force at an unknown angle actually this is the way we should show it but then we know we resolve this reaction into a normal component and friction

so we show it as n and f and as of now we do the block is not moving

so this is the free body diagram of the block actually the free body diagram of the block is w and a reaction force at some arbitrary angle phi we do not know what phi is or we could show it as w and n and f

so this is the free body diagram of the block now let us make it slightly more let us add things to this suppose this is the block lying on the ground and there is a string which is tied to this which is at an angle theta which is constant and the string is pulling the block with the force t which we call the tension t and now this block has a mass m and we are supposed to draw the free body diagram of the block now

so now when i draw the free body diagram of the block first again i only show the block showing the block and the ground in the free body diagram is incorrect only that body has to be shown on which the the forces are acting or which you want to analyze and then we show the weight w we have a normal reaction we show a friction force now in this case ah we will have to analyze but friction force could be in a tangential direction it could be forward or backward when we once draw we just show it as f and then we will analyze and

get the direction of the friction force and then also we have the tension t and if we treat this block as a particle what we could do is instead of showing this as a full block we could have just shown this free body diagram treating this as a particle

so therefore we will have the weight w there is a normal reaction there is a friction force and there is a tension t which is at an angle theta

so this is the free body diagram of the block on the ground with a string attached all the forces are to be shown now you may ask the question what about the accelerations well we will show all the forces and then when we solve the problem then we will say sum of all the forces must be equal to mass times the acceleration

so in a free body diagram we only show the forces and then we analyze the free body diagram to get the solution of the problem by putting f is equal to m a and we will just see that now typically once we have these at least for your level you will have two dimensional problems

so one thing which you should do is because the if the problem is two d or even if it is one d in one d you will have only one direction in two d there will be two directions

so we should show x and y x is on the problem on the free body diagram

so somewhere next to the free body diagram for example i had drawn the free body diagram normal reaction w friction force t i should show my x and y coordinates on the problem the reason is because eventually i will have to do a vector balance i will have sum of forces in the x direction is equal to mass times acceleration in the x direction now if you have shown x like this then anything which is in the opposite direction will come with a minus sign

so therefore drawing the directions is very important that is one thing and secondly x and y need not be along horizontal or vertical they can be along an incline or at an angle only thing we have to ensure is that x and y have to be perpendicular but other than

so for example if we have a problem of a block lying on an inclined plane which is at an angle theta from the horizontal

so we could choose for example x like this y like this

so only thing we require is that they have to be mutually perpendicular other than that and maybe i can choose x like this y like this even that would work all i have to ensure is that i make perpendicular axis

so these angles have to be 90 degrees otherwise i can choose x and y as any two perpendicular directions along any orientations and then we will stick to whatever is along the positive x what we have chosen will be positive whatever is along the negative x or negative y will be negative

so this is what we do then let us now

so after we put after we draw the free body diagram

so we will draw the free body diagram show all the forces on the body and then we will put f is equal to m a

so in a two dimensional problem we will resolve f along x and y now this f is a sum of all the forces acting on the body

so for example let us go back to this block to this problem we have drawn the free body diagram here of the block there is this normal n there is w which is equal to mg there is a force of friction there is tension and in this problem suppose we choose x and y like this

so now when i apply newton's law to this body

so i will put sum of forces in the x direction is equal to mass times acceleration in the x direction sum of forces in the y direction is equal to mass times acceleration in the y direction now

so here now when we look at the left hand side the left hand side comes from

the free body diagram the right hand side will be the kinematics of the problem which we will analyze
so for example we are talking of a block which is tied with a string and this is on the ground like this and we want to find its motion
so let us say suppose the tension t is given and we want to find the acceleration of the block
so if t is given and we want to find acceleration of the block
so here what we will do is we draw the free body diagram
so that means it is given to us in this problem that the block accelerates to the right with acceleration a and we want to find this
so now when we draw the free body diagram we have drawn this again and again we have t we have n we have w and we have the force of friction lets solve this problem now when we start counting the number of unknowns
so the normal reaction is not known this is one unknown the friction force is the other unknown number two the tension t is given to us acceleration is unknown number three
so that means we have three unknowns in this problem and we have to solve one of them that is the acceleration
so after we draw the free body diagram let us then we write forces in the x direction
so this is x this is y
so force is in the x direction if this angle is theta then we have because of this tension t the force in the x direction will be t cos theta
so its t cos theta minus f is equal to m times a this left hand side comes from the free body diagram t cos theta minus f and this is equal to mass times acceleration of the block then we also go to forces in the y direction
so what we will get is n plus t sin theta minus w is equal to mass times in fact let us call it as m a x and m a x is equal to m a this is equal to mass times acceleration in the y direction and we know that this block is moving only along the x direction
so therefore this acceleration is equal to zero
so what we get is n plus t sin theta is equal to w which we can also write as mg
so this gives us the second equation we need a third boundary third equation again and the third equation which we will get will come from the force of friction now here we know this is a case of the block is slipping on the ground and in fact that is why because it is slipping in the forward direction
so therefore friction was shown as acting in the backward direction which is the correct direction and because the block is slipping friction will be equal to mu k times n
so friction is equal to mu k times n this gives us the third relation and now we can solve the three equations and three unknowns and we can get the value of acceleration if we want to actually do this
so what we have is n is equal to let me work it out n is equal to mg minus t sin theta mass of the block is given to us
so therefore we know the normal reaction now and from there what we get is friction is equal to mu k times n
so is equal to mu k times mg minus t sin theta and then what we get is m times a is equal to t cos theta minus f
so that will be equal to t cos theta minus mu k times mg minus t sin theta we know the values of ah mu k will be given to us mass of the block is given tension is given the angle theta is given
so from here we can get the value of a
so this is how one uses a free body diagram to solve the problems once again

we the basic equations to be used are sum of forces in the x direction is
equal to mass times acceleration in the x direction sum of forces in the y
direction is equal to mass times acceleration in the y direction and in
addition we may have to use friction force is equal to mu k n or mu s n to
solve the problem but if the bodies are not slipping this is only in case of
slip or impending slip in case of no slip friction will be an unknown like
other forces and will not be able to solve for friction directly it will come
from the solution of the equations let us come back to the example of the
block lying on a table

so now we draw the free body diagram of the block we showed the block like
this or maybe we show it as a particle and then we have the weight the
normal reaction and the friction force now if the block is at rest that
means there is no slip its just moving

so acceleration of the block is equal to 0 now here when we apply sigma fx is
equal to 0 because acceleration is 0 and sigma fy is equal to 0

so this gives us n is equal to w and this one gives us friction force is equal
to zero sorry the other way round f i will give you this f x will give you
friction force is equal to zero

so in this case there is no friction force acting on the block now lets
consider a case when the block is on the table and the table accelerates
with an acceleration a in the right direction and the block is not slipping
on the table

so in this case we would like to know if the acceleration a is given what is
the friction force acting on the block

so acceleration a is given and we want to know what is the friction force on
the block

so what we do is again we draw a free body diagram of the block and which i
show as a particle i will have its weight normal reaction and then as usual
i show the friction force like this

so i have drawn the free body diagram this is a case of block not slipping i
since everything is moving to the right i show the friction force on the block
as in the left direction

so this is my free body diagram n w f and then i put sigma f y is equal to
0 there is no acceleration of the block is equal to a in the i direction
because it is moving to the right with an acceleration a it is not slipping
on the surface of table

so sigma fy is equal to 0 gives me n is equal to w and when i put sigma fx
is equal to m times a this gives me minus f is equal to m a now we know a is
positive

so it gives us f is equal to minus m a which means that friction is in the
right direction

so therefore the correct free body diagram finally turns out to be this and
friction force on the block is acting in the forward direction and this is
equal to m times a

so this is the friction force acting on the block and it acts in the forward
direction somehow ah intuitively we always feel that if the block is moving
in the forward direction from a ground frame of reference friction on this
should be acting in the backward direction but actually here it is the
friction force which is providing the acceleration to the block in the x
direction and now is it counterintuitive let us try to think about this the
block was it a state of rest when the table started to move now the table has
suddenly started moving the block is not slipping

so now something has to provide an acceleration to the block in the positive x
direction and that something is nothing but the contact force which it has

with the table and whose tangential component is nothing but the friction force

so therefore the friction force on the block is acting in the forward direction another way to look at it is that the inertia of the block tends to hold it on the table that means the block tends to stick on the table but it is forced to move forward

so therefore the relative motion of the block with respect to the table will be in the minus i direction and

so a friction force has to oppose this

so it will be in the plus i direction

so now if you have to find the maximum value of acceleration for which the block will not slip

so if you have to find the maximum value of a for which block does not slip on the table

so then clearly we know till the block starts to slip f is equal to m a now the maximum value of f before which slip takes place will be equal to mu s times n and this must be equal to m a and n from the balance of forces in the y direction turns out to be equal to w

so mu s times w is equal to m a and w can be written as mg

so therefore what we will get is if we put this in we will get the value of mu s is equal to a by g and

so if

so a at which slip takes place will be equal to mu s times g and if the acceleration exceeds this value of mu s times g that means the block will start to slip

so if acceleration exceeds musg the block slips and then once the block slips what we will have is the free body diagram we will have n we will have w we will have friction and this will be equal to mu k times n and this will become mu k times mg and then ah we can apply the relations ah n is equal to w has already been used and the forward acceleration of the block m times a will be equal to f

so now the block is slipping this value of acceleration this is acceleration of the block in the ground frame

so therefore this acceleration a will be equal to acceleration of the table minus acceleration of the block with respect to table

so we will get m times a minus acceleration of the block with respect to table is equal to mu k times mg and from here we can work out the value of acceleration of the block with respect to table

so m a minus m

so m will cancel out from all these

so what we will get is acceleration minus mu k times g will be equal to acceleration of the block with respect to the table and this will be of course in the minus i direction

so this is how we can work these problems ah similar example comes and this is often used to find how do we work out the value of mu s and lets look at it suppose we have a block or a coin kept on an inclined plane and the angle is theta what we do is we initially keep theta is equal to zero degrees and we raise the value of theta

so we initially the block would be at an ah horizontal level and slowly we raise the angle of the incline and what we find is at an angle theta s lets call it the block starts to slip

so what we want to find is find the friction force when theta is less than theta slip when theta is equal to theta slip

so let us do the analysis of the problem for all these three cases now when a

problem like this is given and we ask you to find the friction force one of
the most common errors we encounter is students writing that friction is
equal to mu times n they will use either the value of mu s or mu k and this
clearly is incorrect because if we are in the range when theta is less than
theta s then friction force we cannot relate it to normal reaction except by
saying friction has to be less than mu s times n
so block is here it is kept in incline this is the angle theta lets draw
so for let us look at the first case when theta is less than theta slip and
lets draw the free body diagram of the block
so this is we represent the block by a point and in this example because we
have things at an incline we can choose x axis like this let us choose y axis
like this
so we have choosing and we realize this x axis this angle is theta
so when i draw the free body diagram what i have is the weight w which is
acting vertically down and then we will have a normal reaction and because
this is tending to slip down the friction force we show as up we will analyze
the case as it comes turns out to be
so that means these are the forces which are acting on the particle a vertical
downward phase mg the force mg ah normal reaction n perpendicular to the
block
so this is this is the direction of n this is the direction of f along the
block and these are the forces now in this particular case and this we should
learn very fast how to do these things that if this is at an angle theta with
the horizontal then what we find is that the component of w along this if
so this is w and this makes an angle theta with the y axis because x was
making an angle theta uh along with the the direction x makes an angle theta
with the horizontal w is vertical
so it will make an angle theta with the direction which is perpendicular to
the given x and
so if this is w then we can resolve w as w cos theta and w sin theta like
this
so along the x direction the component of w will be w sin theta and along the
y direction the magnitude is w cos theta
so if i once again this is my incline i draw the free body diagram of the
block what i can do is i can write this as w cos theta w sin theta i have
done this and then we have the normal reaction and we have the friction
so now when theta is less than theta s block is at rest
so in this case n is equal to w cos theta and f is equal to w sin theta
because a x and a y are zero
so we have this relation
so friction force in this example when we are not when the body is not
slipping is just at rest is equal to w sin theta ah ok and the normal
reaction is w cos theta now these this free body diagram is valid for all
theta
so
this free rule let us write this this is valid for all theta what is going to
change will be when theta is equal to theta s then friction force will
become equal to mu s times n which will become equal to mu s times w cos
theta and this must be equal to w sin theta
so therefore what we get is mu s is equal to sine theta upon cos theta which
is equal to tan theta and this is how one determines the coefficient of
static friction in cases like this that you slowly increase the inclination of
the block and the angle at which the body which is lying on the incline starts
to slip that gives you the coefficient of friction between these two bodies

that is tangent of that angle gives you the coefficient of friction and to measure tangent either you can measure the angle or you can look at the x distance and the y distance and take a ratio of y distance to x distance that will give you the angle theta and what happens if theta is greater than theta s if theta is larger than theta s then the block accelerates down with the incline with acceleration a and now this acceleration becomes an unknown the free body diagram stays the same as we have done n f but now this f becomes equal to mu k times n w cos theta w sin theta

so we will get n is equal to w cos theta w sin theta minus mu k times w cos theta is equal to m times a and this w is nothing but m times g and you can use this to find the acceleration of the block

so with this we have seen simple systems how to draw free body diagrams and how to solve problems in the next class we will continue we will look at more system of two bodies or multiple bodies and also circular motion thank you

you