

last class we looked at certain problems where we talked of forces on a single body and we saw how to apply newton's law to problems like that where we first drew a free body diagram and then used newton's second law on that problem now you could also encounter problems where there could be more than one body is involved example of this could be there is a mass m_1 tied to a string to another mass m_2 and this is being pulled with a force f maybe through a string or through just being pulled with the directly to a force f is being applied to m_2

so here we have a case where there is more than one body is involved so here generally the way when we solve these problems we may have to draw free body diagram of each body separately and then we apply after we draw the free body diagram then we apply newton's second law to each body but what we'll have to realize is when we do that whatever is the interconnection between these two bodies we have to make sure that we use newton's third law that is action and reaction of on the connecting body is there and so that has to that will give us a common link between these two bodies so common link will come from newton's third law and also what we will need is that if we look at kinematics then if these bodies are moving then accelerations of bodies one and two will be related and this relation will give us another equation to solve the equations for example let us say if these bodies m_1 and m_2 which i talked off they are lying on a table and they are being pulled in the horizontal direction if the string and this is a string which is there or a chord between these two bodies if now this cannot extend its length is constant then we can clearly say the acceleration of body one is equal to the acceleration of body two because the length of the string does not change

so these sort of relations between accelerations is what you will have to look for in problems in simple problems like this one the accelerations of m_1 and m_2 are equal but there may be more complex problems where acceleration of body one and body two will not be equal but there will be some relation and we will also look at some of these problems not today maybe in one or two classes from now

so

two things to keep in mind when you have multiple bodies one is kinematics and the second thing we keep in mind let us write that down is the relation between forces between the two bodies or the connecting element between these two bodies and for that we have to realize that we have to use newton's third law that if f_{ab} is equal to minus f_{ba}

so using these we can work out the problems

so now let us have a look at the general procedure which we have developed for solving these problems and mechanics and i am now going to do an analysis of the procedure which we have developed

so our procedure is that procedure for solving problems

so you have a body or a particle on which forces are acting and due to which the body accelerates and we have to work out one of these things

so now when we talk of forces as we said forces on the particle we have the weight which is due to gravity we have already seen this then we will have some contact forces and these contact forces we divided in two parts a normal reaction and friction if this is a solid in contact with another solid and we could have other contact forces like a string or a spring which is connected to a body connected to particle

so as we have seen for example lets see we had this block on a table which is block has a mass m and there is a string which is pulling this with a tension t

so if i draw the free body diagram let us say this angle is θ
 so if i draw the free body diagram of the block i will get its weight acting down there is a normal reaction there is a friction force
 so these two n and f are the contact forces and we have the tension t and because of which the body accelerates and due to the constraint of the problem that the body moves on a plane acceleration will be equal to acceleration as a scalar only in the x direction
 so on one side we draw the free body diagram on the other side we have kinematics
 so when you solve a problem you draw the free body diagram and you write the kinematics equation and then all what you have to do is sum of forces is equal to m times a equate the left hand side and the right hand side in the left hand side only the free body diagram comes we do not bother about the kinematics when we write the draw the free body diagram all we have to be careful about is that we are drawing the free body diagram with all the forces shown properly and the other thing we have to be careful about is that we should always show the direction of our coordinate axis
 so we show x and y and as we have seen maybe another student will solve the same problem by choosing this i am now using star he uses x along the string y perpendicular to the string
 so that can also be done
 so for these what we have to do is resolve depending on what my x and y are either resolve the forces or resolve the acceleration along the appropriate directions now what will happen is if we try to look at a physical problem the physical problem which i have told you is ok this is a block this is being pulled by a force and because of which it is lying on this table and it moves with an acceleration now physically when you look at this problem the mass of the block is something we will be able to know and that will be given to you in most problems the force t which is being applied either this will be known or this will be an unknown force unknown quantity and similarly the acceleration would either be given to you or this would be an unknown of course if the problem has to be solvable both of these cannot be unknown only one of them will be done now what we realize is when we draw the free body diagram then because of the contact force
 so this is what we have when we draw the free body diagram then we have a normal reaction we have a friction we have the weight which is acting this force because of the external body the string the tension like this and acceleration
 so now our number of equations from newton's law is two one in x direction one in the y direction okay and
 so now if we look at this normally n and f the normal reaction and the friction force physically would not be defined will not be able to know them a priority in a given data how do you know if i tell you this pen is there this is lying on the table then by the definition of the problem i will be able to measure the mass of this pen
 so i can say what is the weight of the pen but how much force is the table exert exerting on the pen will most likely not be known unless if in a contrived problem i give it to you
 so if we look at this then what we find is n and f these are two unknowns in addition to that what we have shown either t or a one of these will also be an unknown
 so now we have three things we have to find n f a or n f t if acceleration is known now acceleration may be given to you in terms of the block starts and stops in this way

so you use kinematics to find acceleration
 so that could be one way or it could be given to you directly but now we have only two equations and three unknowns
 so how do we get the third unknown from and that comes from the fact that this situation when the body is under this the x either the block could be at rest or it could be moving if the block is at rest then we have the third information that acceleration is equal to zero
 so then we have two unknowns n and f and we can solve the problem but if the block is moving then let us say the tension is given
 so acceleration is not known
 so then but there what we have is then f in this case is related to n by f is equal to $\mu_k n$ and
 so therefore this gives us that extra relation in case if the block is just about to slip for impending slip we will get f is equal to μ_s
 so depending on what type of motion
 so then friction is related to normal reaction and then we work out the problems
 so this is what we have to keep in mind now maybe there may be a problem where you do not know if the body is moving or not and we have discussed a very special simple case of this
 so in such a case how do we do
 so applied forces are given all the forces which are applied to applied forces maybe i will specify on the body or on the particle are given to you and you do not know what type of motion takes place whether the body moves or not or if it moves then without acceleration and in these problems particularly friction is involved then
 so and if here you have to find the value of friction at the contact surface so what we do is first we will assume that there is no motion we assume the system is at rest and then that means acceleration is equal to zero
 so when we do that now we can from our free body diagram and kinematics which we use that acceleration is zero what we will get is $\sum f_x$ is zero $\sum f_y$ is equal to zero using these we will find
 so using $\sum f_x$ is equal to 0 $\sum f_y$ is equal to 0 we find the value of f the problem is not complete then what we do is we we also will find the value of n and then we check if f is less than or equal to $\mu_s n$ or not we have the value of f we have the value of n we will do this check μ_s generally would be given to you
 so therefore you check this if this is satisfied assumption is ok and we have found the right solution but if we find if f turns out to be greater than $\mu_s n$ then the no slip assumption is incorrect the assumption we have made of no slip is incorrect because friction force which we are getting turns out to be greater than $\mu_s n$ as soon as it becomes equal to μ_s and the body will start to slip
 so that means now we revisit the problem now acceleration will not be zero the body will start to slip except of course in the case of impending slip when the body is just about to move
 so there and then what we will do is we will put the value of f we do not know the value of n till now but we will put f is equal to $\mu_s n$ and its direction has to be shown opposite of the relative slip between the body and the contact surface
 so the direction of the friction force has to be opposite and we put f is equal to $\mu_s n$
 so now acceleration will become the unknown for which problem will be solved

so this is how one tackles the problems of friction right where motion is not known a priori now in certain problems but it is specified that surface is smooth or frictionless if that is so then it is very obvious that in such cases friction force will be taken to be 0 and so probably then one of the forces or accelerations will be the unknown for which you will be solving

so this for this the typical word used is that the contact is smooth surface is smooth or frictionless would be given to you now this was about the friction side the other side is we have the acceleration force is equal to or sum of forces external on the acting outside the particle this is equal to m times a now let us have a brief discussion on acceleration if the particle moves in a straight line let us say the particle is moving like this so then if i choose x along the motion of the particle then acceleration vector will be equal to a times i or minus a times i depending on if the particle is moving up or down

so in this case then we have only one unknown for the acceleration but in a case like this if my x and y are chosen let us say i put it as star chosen like this then acceleration in the starred coordinate system will have components both along x and y and we will have to then find a relation between that and we know the net acceleration has to be along the plane so therefore we can find out a relation between acceleration in x star and y star directions this is if the particle moves in straight line and acceleration when particle moves in straight line acceleration is not equal to zero if velocity is changing in magnitude with time that means the speed of the particle has to change with time for acceleration not to be zero if speed of particle is constant when it moves in a straight line acceleration will be equal to zero

so we can in fact we have already written this we wrote the magnitude of acceleration is equal to dv by dt where v without a vector sign is the speed now this changes we have seen this before when the particle moves in a curved path or when particle moves on a curved path in that case what we find is that the particle is moving along the curve path acceleration has got two components and acceleration we can write it as equal to i will just explain these two unit vectors e_t is the direction which is tangent to the path and e_n is a vector which is normal to the path pointing towards the center of the curved path and by center we mean if you assume the motion to be locally in a circle then it is pointing towards the center of the circle and let us look at this expression again we have seen this before when we did kinematics but let us now understand this because this will have a lot of repercussions

so acceleration has two parts when the particle is not moving on a straight line there is a part which is equal to the rate of change of speed tangent to the path this is the same as what you get for motion in a straight line but now an additional component of acceleration comes in when the particle is moving on a curved path and that component of acceleration is perpendicular to the tangent to the path it points towards the center and this is given as v square upon r and if you recall r is what we call as the radius of curvature of the path for a general case now we will look at this in a special case when the particle is moving along a circular path but before that what we realize is that even if the speed is constant then there is a component of acceleration v square upon r which is non-zero this component did not exist when the particle was moving in a straight line and we can also see it from this equation in the fact that when the particle moves in a straight line then

the radius of curvature is infinity
 so therefore v^2 upon r becomes zero but by virtue of the fact that the particle is moving along a curved path and if its speed is not zero which will be there because it is moving then the acceleration has to have a component normal to the path and this becomes very important when we apply Newton's law because sometimes particles may be moving with a constant speed but there has to be a component of acceleration normal to the path which means a force has to be there because f is equal to m times a
 so a force has to be there acting on the particle which will cause this acceleration to take place and this force as we see from Newton's law has to be an external force when the particle is moving
 so how can it come it will either come because this is in the tangential in the normal direction this will either come through something which is contacting the particle for example if I take a stone tie it to a string and move the stone I rotate the string
 so that the stone moves in a circle then the tension in the string will provide this uh force which will cause this acceleration if it is sometimes it is friction force which will do that as we will see
 so therefore this force will have to be applied to the particle it has to come from outside the particle
 so now let us look at the special case of circular motion of bodies which is a special case of the curved path when a body moves in a circular path then the first thing is the radius of curvature are this is nothing but the radius of the circle at all times now let's take the case of uniform circular motion first in case of uniform circular motion if you recall what we had shown was that
 so if there is a circle a body moving on a circular path at this instant this is the center
 so what we have is there is a component of x because uniform circular motion means speed is constant
 so acceleration of this particle its acceleration is equal to v^2 upon r pointing toward the center and we realize as the particle moves in the circle the direction of acceleration keeps on changing and because the speed is constant we do not have the tangential component
 so we have an acceleration and
 so the component of the acceleration towards the center of the circle is called centripetal acceleration and this is equal to v^2 upon r towards the center of the circle
 so now from ah this is from the kinematics view point from the dynamic view point as we discussed there has to be some force which has to be equal to mass times acceleration and this force has to be external to the body now if circular motion is not uniform then acceleration has two components the first component is as we have discussed the centripetal acceleration towards the center and this we said is equal to v^2 upon r and the second component is a tangential component and this will be equal to the rate of change of speed and this is tangent to the circle at that instant in the right direction now also what we have seen is that we have defined angular velocity v can be written as ω times r and this ω we wrote as the angular velocity
 so therefore now the centripetal acceleration can be written as v^2 upon r
 so that will be equal to $\omega^2 r^2$ upon r this will be equal to $\omega^2 r$ and the tangential component this is $d v$ by $d t$
 so this becomes equal to $d \omega$ by $d t$ of ω times r r is a constant

so therefore this becomes equal to $d\omega$ by dt times r and $d\omega$ by dt what we have written here this is called the angular acceleration rate of change of angular velocity this called the angular acceleration and so what we have is the tangential component is equal to r times α where α is equal to $d\omega$ by dt the angular acceleration so we have to take care of this when we have circular motion even when the body moves at a constant speed the acceleration has to be there now let us look at a case of a passenger sitting on the rear seat of the car of a car where the car is turning left and so we assume that lets say this is moving we assume it is moving in a circular arc

so the car was initially moving straight and then it starts to turn left so we assume it is an arc of a circle through which the car is turning so now if there is a passenger which is sitting in a car so we are trying to analyze the situation of the passenger since the car is moving in a circle there is a force it's moving with a velocity v so there is a force mv^2 upon r acting on the car now suppose if there are n there will be four tires on the car and if the friction on the tires which provides the centripetal acceleration that means the friction force has to be equal to mv^2 upon r and let us say if there are four tires then the total friction force on the four tires will sum up to be give this value mv^2 upon r let us now analyze the passenger sitting on the rear seat of the car

so we draw the free body diagram of the passenger the car is moving like this now what we find is that there is a normal reaction which is coming out of the plane of the paper the passenger is sitting down a normal reaction this is from the seat of the car the weight of the passenger is in the direction perpendicular to the paper and normal reaction and weight they balance each other now on this particle what we also find is that there should be a friction force between the seat and the passenger this friction force is perpendicular to n and in this case the acceleration what we find is in the toward the center and this has to be provided by the friction force alone there's a friction force which is acting on the passenger and that is what causes this acceleration

so in the free body diagram of the passenger there will be n and w perpendicular to the plane of the paper and there is a friction force these are the three external forces acting on the passenger and this friction force has to be equal to mv^2 upon r where m is the mass of the passenger so this acceleration which the passenger is going has to be provided by the friction force and notice here the passenger is at rest with respect to the car but we have to write the acceleration of the passenger in terms of an inertial frame of reference and we are assuming any frame connected to the surface of the earth is inertial

so with respect to a person on the ground the passenger is travelling in a circle and so his acceleration is equal to v^2 upon r toward the center of the circle now if this friction if the velocity of the car is high then what will happen is mv^2 upon r increases and this may exceed $\mu_s n$ which means then if this exceeds $\mu_s n$ then friction force f will not be able to stop the relative motion of the body if mv^2 upon r is greater than $\mu_s n$ in this case n is equal to weight then the passenger will start to slip and friction force is acting in this direction so therefore this will be the direction of the relative slip so the unbalanced external force acting on the passenger is the force of

friction between the seat and the passenger and this force f is equal to mass times acceleration in the central direction or the radial direction and this we will write it as $m v^2$ upon r

so friction provides this acceleration to the passenger and the direction of friction is towards the center of the circle now what we can see is if the velocity is such that $m v^2$ upon r is less than $\mu_s n$ then we have a situation we have a case of no slip because friction is less than $\mu_s n$ and in this case we also realize that n is equal to mg

so the condition for no slip becomes $m v^2$ upon r is less than μ_s times mg

so this gives us v^2 upon rg is less than the coefficient of static friction between the passenger and the seat for this condition we have no slip the passenger keeps on sitting now once again realize the full picture this is the car which is turning we are looking at the rear seat the car is at this position and this is the direction of acceleration of the passenger as seen from the ground frame now suppose if $m v^2$ upon r is greater than μ_s times mg which of course we also know was the normal reaction then what will happen then the passenger will start to slip and what do we mean by slip

so acceleration of the passenger of the car is like this and what we will find is the passenger

so this is acceleration of the car which is equal to $m v^2$ upon r which is equal to v^2 upon r and we have the acceleration of the passenger with respect to the car

so the passenger starts to slip and let a_p be the acceleration of the passenger with respect to the car now what we find is we have a friction force but this is not able to generate the acceleration equal to $m v^2$ upon r

so therefore the passenger starts to slip

so if i this is the free body diagram that still stays the same in addition to this we have the normal reaction and the weight which cancel each other but if we look at the kinematics then in kinematics what we find is that we have an acceleration equal to v^2 upon r and this is the acceleration of the passenger with respect to the car

so the net acceleration of the passenger with respect to ground this will become equal to v^2 upon r minus a_p in the direction toward the center of the circle and now when we apply newton's law what we will get is friction force is equal to m times v^2 upon r minus a_p and now this friction force will be equal to μ_k times n which will be equal to μ_k times mg and we can if other things are known the acceleration of the passenger with respect to the car can be worked out the minus sign tells us that the passenger will be thrown outwards and

so this is how we work out a problem like this next let us now take up some simple examples and from there we will move to more complex cases we have seen how to apply newton's laws and now in the remaining part of today's class and in the next one or two classes we will solve some problems where we will use newton's second law in a direct fashion with f is equal to $m a$ and as we have discussed ah the more complications in these cases will come because there may be a one or two bodies connected together they may be connected with a string they may be touching each other and then we will have to analyze each of these bodies separately and we have to analyze each of the kinematics of each of them to find the acceleration and the relation between the accelerations of the bodies

so as a very simple first case what we consider is there is a pulley on which

we have a string connecting two masses m_1 and m_2 and the string is on we can say is mounted on a pulley and it is given that the string is light inextensible and the contact with the pulley frictionless contact with pulley and what we have is there is a mass m_2 there is a mass m_1 ah it is given to us that m_1 is equal to five kgs m_2 is equal to four kgs and we want to find the tension in the string and the magnitude of acceleration a of the blocks of mass m_1 or m_2

so this is what we have to find now let's start analyzing this problem what we find is mass m_1 is 5 kgs m_2 is 4 kgs and this is the string which is i am showing it as a double this thing this is going on a pulley now the contact with the pulley is frictionless

so what we find since the mass 1 is heavier than mass 2 then we know that this mass 1 will start to move down mass 2 will start to move up if we leave the system as it is

so then let us start analyzing the problem and solve this the other thing what we realize is that that because the string is inextensible and in frictionless contact with the pulley the tension in the string throughout will be constant the magnitude of tension is constant its direction changes as it as we go along the string at least for the non straight parts and the direction will change

so let's draw the free body diagram of the mass 2 but before that we realize another thing because the string is inextensible the magnitude of acceleration of one and two will be equal

so these are the two principles we have in mind the tension is constant and accelerations of one and two are equal in magnitude of course the reason why we say is the directions we know one is moving up the other is moving down

so we draw the free body diagram of mass 2. now on mass 2 we have its weight which is equal to m_2g acting down and the string pulls this mass with the force t we call it t

so this is the free body diagram of mass two now let's draw because this will move only vertically

so we have only one direction it's a one dimensional motion let's say y is upwards here

so what we have is when we this is the free body diagram and our when we apply newton's law what we get is $t - m_2g$ is equal to m_2a because so here we have implicitly we have written here mass 2 is moving upwards because y is in the upward direction

so we look at the direction when we write the left hand side we only look at the free body diagram all positive forces with a plus sign all forces pointing downwards because y is upwards with a negative sign

so we get $t - m_2g$ is equal to m_2a this is one equation now we realize there are two unknowns t and a here

so to solve for the second unknown we go to mass one

so then we draw the free body diagram of mass one this is m_1 now once again the string pulls this mass with a tension t which we have said is the same as what was there in the other part and its weight which is m_1g now for analyzing mass 1 i can choose by going downwards i need not take y going upwards because here i know a priori i know that mass is moving down

so acceleration is downwards

so i choose y going downwards in some problems if you come across problems where you don't know the direction of the motion then you choose your x and y as a particular direction assume the acceleration to be positive if it is in the other direction you will get your answer with a minus sign

so now here when we write this what i get is $m_1 g - T$ is equal to $m_1 a$ let us look at the other equation the other equation was $T - m_2 g$ is equal to $m_2 a$ this was equation one this is equation two and now i have two equations and two unknowns

so i can solve them let us add these two equations and what we get is $m_1 - m_2$ times g is equal to $m_1 + m_2$ a

so therefore acceleration turns out to be equal to $\frac{m_1 - m_2}{m_1 + m_2} g$ and we can also then work out the value of T and the value of T turns out to be equal to $\frac{2 m_1 m_2}{m_1 + m_2} g$

so again now small checks very obvious things but you should just check these things the dimensions of acceleration are the same as the direction dimensions of acceleration due to gravity and

so therefore the coefficient which comes in front should be dimensionless and here we see this mass divided by mass

so therefore this is dimensionless tension is a force

so its dimension are mass times acceleration we have a g here

so the coefficient in front should have dimensions of mass and we do see that $\frac{m_1 - m_2}{m_1 + m_2}$ these are very small checks but these you should keep in

mind and they with this you can then work out the problems ok now let us

take this was a very straight forward case but let us now take a second case

of a passenger of mass m who stands on an elevator on a weighing scale that means let us say this is the compartment of a lift on which there is a weighing scale and a person is standing on the left let the reading taken by the lift be w' i am taking it as w' because when we normally we expect the weighing scale should tell you the weight which we call as w but what we will see is depending on the acceleration of the elevator w' may or may not be equal to mg where m is equal to mass of the passenger

so here let us draw a free body diagram of the person

so we will analyze these cases for different values of acceleration of the element

so first let us take the case when acceleration a is the acceleration of the elevator as seen from a ground frame and first we look at the case when a is equal to zero that means the elevator is at rest its something like i am standing on a weighing scale and then what we find is if we draw free body diagram of the person we have his weight acting down and n this is the normal reaction from the weighing scale and this free body diagram stays the same whether the elevator is moving or not moving and then when we look at the weighing scale then n is the force being applied by the person and this n will be equal to w' the reading which will be shown by the scale

so now when acceleration is equal to 0

so the free body diagram in all cases will be n and mg and i am replacing n by w' because n is equal to the reading which will be shown now when

acceleration is equal to 0 we get w' is equal to mg

so the weight which the reading scale shows is by weight which is correct now if acceleration is upwards is

so that means is positive upwards then what will happen is $w' - mg$ will be equal to m times a the free body diagram stays the same but now the person is not at rest person is at rest with respect to the elevator but since the elevator is moving up

so this acceleration is acceleration of the elevator

so what we get is w' is equal to m times a plus d

so if the elevator is accelerating upwards it looks as if the person has got

some extra weight and when we take the other case when the elevator
so once again we have w' we have mg and if the elevator moves down with
acceleration a
so if this is the acceleration then what we have is mg minus w' is
equal to m times a and w' will be equal to mg minus $m a$ where a let me
just use the subscript d to show that it is downwards
so now when the elevator is moving down the same weighing scale shows a reading
of $m g$ minus m times a_d
so it shows as if the person has lost weight
so this one has to ah keep in mind therefore the same scale shows as if the
person's weight has changed
so now let us consider the case if the elevator's cable breaks down
so that the elevator is in a free fall if the elevator is in a free fall then
the downward component of acceleration a_d this will be equal to g
so let us look at the case of free fall and once again as we have emphasized
the free body diagram stays the same this is w' this is mg and the
downward acceleration is g
so then what we will have is mg minus w' will be equal to m times g
and therefore what we will get from here is w' is equal to zero
so the weighing scale will give a reading as if the person is weightless
there is no weight for the person if he is falling down freely and on an
element on a weighing scale now one thing we realize while we have talked of
the change in weights these change in weights come with the acceleration in
downward or upward direction if the elevator is moving up or down with a
constant speed then the acceleration will be 0 and the weight read by the
scale will still be mg it is only when the elevator is accelerating up or down
that the scale changes its reading
so in today's class what we have seen ah first we look at the basic
principles as to how to apply Newton's second law to various problems and then
we have looked at some very simple examples in the next class we will take up
some more examples of application of Newton's laws to problems and maybe some
more complex examples thank you you