

We will continue our discussion on error analysis and then we will end today with the discussion on dimensional analysis in the last lecture which we saw all the rules about significant numbers and how we can calculate for them. Let's start with the number one problem where it is given that the mass of a box is given as two points three kg and the box contains two marbles of mass 2.15 g and 12.39 g.

And the question is how accurate is the total mass of the box up to a significant number of such problems? Each of these and we use the rules we've seen and if I repeat them, if we add or subtract quantities, we go to the number of decimal places that have the least, ah, the least accurate measure, or the number that has the least number of decimal places, and the final answer is a lot of decimals. We have to keep in terms of the space where if we use a multiplication or division we count the significant numbers so now when we see this problem we have to realize that the given units are different

so all we have to do is put everything in the same unit. Convert

so let us do this

so that we have this mass which we convert everything into kg

so the total mass of the box is marble

so let's see the real problem The mass of the box was 2.3 kg and the marbles weighed 2.15 grams and 12.39 grams

so when we add it we We will get this as 2.3 kg and we will convert 2.15 gm to kg

so that it is 0.00215 kg and the third one which has twelve points three nine grams plus zero. The unit zero will become one two two three nine kg and when we add them all we get our answer as 2.31454 kg and now we have to work on the significant numbers here which we can see the first quantity we added 2.3 kg is correct one decimal. Given the space where other quantities are going. This is going to a decimal place. Both are going to five decimals. So we have to complete the answer in the first decimal place. So it is 2.314. Showing because the box itself was 2.3 kg you added two marbles but the answer did not change and the reason is that the mass of the box that was given to us was 100 grams correct up to 0.1 kg and these two marbles are much less than 100 grams and even their total. The mass when you add them is less than 100 grams

so now it does not affect We make a small deviation in the question, suppose the mass of the box is two points three Zero is given as zero kg but how will this answer change and here you have to understand now when you say the mass of the box is two points three zero zero kg means it has been accurately measured up to 0.001 kg

so we have to go to 3 decimal places here

so final In the answer we see we have two points three one four five four four to go up to three decimals

so now we have to go two points three one four now the next digit 5 is a 4

so then when we round it it is 2 points 3 1 will be equal to 5 kg

so if the actual data given to us becomes 2.300 instead of 2.3 then how will the answer change and we should take such care when we do something in wrong analysis and because of unit change we have to make sure that our units match

So let us say that two lengths l one and l two have been measured and they have been given that l 1 is equal to 9.99 m and l 2 is equal to 9.99 mm and we need to find the sum to correct the significant numbers. We need to find the result

so once a profit we have to be careful we convert both to the same unit

so l 1 is equal to nine points not nine meters and l 1 is equal to two zero points is not equal to nine meters we convert it to thousands

so we What we will do is get and now when we add two we get 1 one plus 1 two is equal to nine points is not equal to nine meters

so now in those two we have to go to the least amount of decimal place we can see at least one decimal place per meter where 1 up to four decibels of space in two meters

so we have to write our answer at the end .

So the round up answer would be equal to ten points zero zero meters. Now you could convert it to millimeters and do the same thing. The same answer could be found. Let's take an example which involves multiplication. Let's say, given that each side of a cube is equal to 5.402 centimeters and now we need to find the surface area of the cube is inappropriate.

Equals six squares which means now a multiplication is involved because a multiplication is involved. The final answer must have a significant number equal to the original number. I calculate six squares and I can use the correct calculation calculator I get my answer as one point seven five point zero eight eight nine two four centimeters square and this is the point we are saying my answer does not need to be expressed in decimal places because we have real data up to four important numbers are correct

so now if our number is non All we have to do is we have one seven five points zero eight nine six two four centimeters square and we have to write it right up to four significant numbers

so it will be equal to one seventy points now zero is the fourth most important number I see after eight

so I We round it up to one point seven one seventy five point one ah centimeter square

so thus calculating for an important sum when we do the problems related to multiplication now let us see a problem that involves or uses all the formulas we have learned and our problem is Determining the acceleration due to gravity g One uses the formula for the duration of a clock. Which is equal to two pi times seven times the square root r minus r by five g This period is probably some pendulum or some hanging body and this formula has been given to us

so now what is given is the lue of v r is measured as 60 plus minus 1 millimeter. The value of small r is 10 plus minus 1 millimeter and the test is repeated 5 times with the clock to find out the duration and the duration is measured by the time which is measured in 5 tests 0.52 seconds point five six seconds point five seven seconds point five four seconds And point five is nine seconds and the minimum count of the clock is given as zero point zero one second

so all this data is given that we need to find the small rt and find the percentage error in g gauge basically find the percentage error in our g We need to find out but we need r and t

so we also look at all these quantities

so what we do is we first see the value of t is given to us as a test again and again

so we calculate the average value of t for the period. And we add 5 given data which divides by 5

so we get the answer as t means 0.556 seconds but we understand o we get the correct information up to 0.01 seconds

so we will round up these six second decimal places

so five a This will be followed by six

so it means zero point will turn into five or six seconds. This is an important step. Finally you have to take the average value as one which has a significant number equal to the original data. Now let's find the percentage error in t .

We calculate the individual error in each of the measurements

so that our first measurement is 0.52 0.52 minus 0.56 it is subtraction 0.04 we take the absolute value it is 0.04 for the second measurement we will take point five six minus point five six so delta t will be zero third measurement point five seven minus point five six

so that it will give us zero point one fourth measure will give us zero point two and similarly if we do the fifth measure we will give our point zero three so we can let's calculate the error. The average error is the sum of all these divisible by five. We get it by dividing the unit by five is equal to 0.02. If so we should have rounded it up. We don't have to do this here until the second decimal place

so now that we know the average error of t, the percentage error of t will be 100 by delta t by t which is 0.02 divided by the value of t which means t. So 0.56 divided by 100 and when we calculate this percentage it comes out to 3.57 percent if we want to find the percentage error in small r and capital r so we go back to the data small r is given as our 10 plus minus 1 millimeter so the percentage error in this will be in small r where 1 divided by 10 is equal to 100 and this is equal to 10

so the error in this small r will now be written first to find the error in g or the percentage error. The formula of g which means we were given the formula with the riod in terms of time we get the square and when we write the formula in terms of g you see that g comes out eight feet square over five times r minus r by t then we analyze our error let's use. Write delta g on g. The change of error will now be equal to twenty eight pi square and five constant so there is no error which means we have to calculate r for subtraction r and t for square error

so it will be equal to r minus r adding r minus r Now these are t squares so by 2 times delta t now if you can understand the rules that we see whether multiplication or division we always add relative errors so this is now let's find the error of r minus r .

Since one millimeter delta was small r was one millimeter so the total error in r minus r will be two millimeters an dr minus r sixty minus ten was given so it is fifty millimeters

so we will put it in the delta of r minus r by r minus r two By fifty and then we have twice the delta tee that we have already calculated so it is equal to the point zero six times by point five six. We can multiply by hundreds

so we multiply both sides by hundreds. In delta g by g 100 we get this because the first number will be 50 by 200. The second number that we have calculated is 3.57

so we have multiplied by 2 and multiplied by 100

so we get 4 plus 7.14

so the percentage error is eleven points one four percent

so in this example we have seen how we calculate the error when you add a How do we take care of the error if there is a product or a division? First we calculate the individual error in the quantity of the formula where the sum comes

so two errors are added to the sum and then we find the relative error in that quantity and the second quantity which is a division of it

so we are relative to it. Add the errors and thus we calculate all the errors. Now we look at the dimensional analysis which shows that all the quantities that we measure can be expressed in seven basic dimensions and it is seen that they do not require seven basic dimensions for all and These are the seven

basic dimensions that we have chosen for the length and mass. There will be a dimension when we come to electricity we get a dimension that we use. And the current and temperature for which we can use a symbol to separate it from time or sometimes people use the Greek symbol theta for temperature and then the fifth-sixth quantity will be the intensity and we can use ah this is the symbol using the illuminated intensity in si unit Cadillac
 so we keep Cadillac we can use anything else and finally the amount of mole in a substance that will use mole
 so we can use mole
 so these are the seven basic dimensions and any quantity we write is expressed in terms of product or division of these dimensions. Maybe now it is not sacred that we just take the mass of length and time as the basic dimension. Someone else can take the ball mass and time as the basic dimension and that too will be valid but then if we use force mass and time then take length as the basic dimension. And the rule is that within these basic dimensions we cannot create quantities that can be made from these dimensions. Let us give ourselves an example. If I look at area then area length is equal to square so either I can use area as basic dimension or length but I can't use both length and area as basic dimension. Because they can be derived from each other which means they are not independent
 so given these basic dimensions we can use some analysis and it can be useful so we have one thing that we have to be careful when we write all our equations. What we write in physics should have dimensional consistency and dimensional consistency means that quantities of the same dimension can be added or subtracted which means if two quantities are added or subtracted or I say a is equal to b then the quantity must be the same as b There are dimensions and this principle must be followed
 so for example we cannot add force and velocity because the dimension of the ball will be mass times acceleration,
 so if we express it it will be m times l by t square where velocity will be when you have problems with dimensional analysis One of the first things you need to do is to express them in dimensions for quantities other than the basic dimensions. Be able to easily do if you remember some formulas some of which we have not seen, but you may have seen them in previous classes. Equal we know the distance with time
 so its dimension will be equal to t by l and we express it as l to the power of times t minus 1 and we do this for all the other sums we have
 so say the same as we say
 so say equal Either you remember that the force is given from your initial ah to the previous class as mass bar acceleration
 so the dimension of the ball will be equal to the mass of the mass of acceleration which is equal to the square of l by t
 so we will express the total dimension of the ball as m times l times t With energy
 so any new quantity that you should be able to come across you should be able to write its dimensions in terms of basic dimensions now how we will use dimensional analysis as we said M that all the terms of an equation that are added or subtracted have the same dimension
 so its principle is called dimensional similarity All the terms of the same equation which have addition or subtraction have the same dimension This leads to things like b because it is seen as b minus zero
 so an equation must have the same dimensions of both a and b
 so what can one do now if I know if I have been given an equation as we saw in the previous example Where we saw there was a formula that was given to us in

terms of time which we saw in this problem t is 7 times the root of $2\pi r$ minus r by $5g$ should be equal to the right e dimension where we have a constant which is non-dimensional like 2π and 5 but then we have r minus r by g

so dimensionally consistent on both sides and this dimensional consistency of the equation is what anyone can use and it This gives us how dimensional analysis can be used to predict some formulas but one thing we need to make sure is that one thing we should realize or catch is when dimensional adjustment is required which means that two terms in the equation must have the same dimension a dimension b It is equal to ab if it is equal to but if the levels of a and b are equal then it may not be sure that equal to ab it is only a way to test the first step dimensional consistency which does not guarantee that the formula is correct but not dimensional consistency. Wrong

so dimensional consistency does not guarantee that the formula is the correct constant that is there may be wrong but if dimensional consistency is missing then the formula or equation is definitely wrong

so care must be taken and also what we understand is that there are certain quantities which are non-dimensional and Using Dimensional Analysis We Cannot Do Anything About Dimensional Quantities Example of Dimensional

Quantities Mothers may have similar physical ratios, so if you divide one quantity with another whose dimensions are the same, the result will be non-dimensional. Dimensional

so it would be a non-dimensional quantity, so how does it use the dimensional principle of dimensional analysis to determine quantities and let's look at it with that. With the help of some examples and we will see how we can predict a specific formula. Let's see if we think that the duration of vibration of a drop depends on the surface tension, its radius r and the density of the fluid ρ and we want to find an expression for the duration. Let us put this

so we want to find an expression for t

so here we will express these problems in a way that we will now understand one thing that it will work when we are talking about period is a function of surface tension radius and density So if we write the basic dimensions that are involved with all these quantities then we will see that time is the time involved. Surface tension is a quantity that is the force per unit length so it will include all mass lengths and time. Length

so completely we see here m and t involve three basic dimensions

so here duration is a function of the other three sums s and ρ and s Bottom way we can get our formula if four has four quantities and three basic dimensions s then the fourth sum can depend on at most three other quantities if it depends on four or five quantities then we cannot get a formula using dimensional analysis and we have there We need to look at the number of independent variables and this will give us that sometimes the number may be even less

so let us first solve this problem

so we want to find out

so the way we do this is that we assume that the period with s is from power α r Power is proportional to $\beta \rho$. The power γ is now unknown here $\alpha \beta \gamma$ and these values we will be able to get using dimensional analysis and how we will do

so we determine the dimension of each of these that the period dimension we will write will be equal to what we write as lmt for each quantity .

Will express in terms and if not involved in any quantity, we will keep it as θ energy. Now the dimension of s is this and this is how you work it pulls the

ball surface per unit length

so it will be equal to the ball equal to what we saw as mass multiplication acceleration and then we have a length

so it will be equal when we work we see the other two The energy goes from minus two to m times t . We express things in terms of negative energy and then we have the dimension of r is exactly the length

so it will be the power of l times m from the energy of zero t to the energy of zero if you want. You can just write it as l and when you write in the final expression you will carry all the elements and now the dimension of ρ will be the mass of each unit volume

so it divides the mass by volume

so ρ can be written as l^3 by m So the dimension will be m into l minus the power of minus three,

so now when we finally express it, what we get is the power of a person from zero m to zero is equal to the power of minus two to the power of minus two. Is r and m times l Dimension l minus three energy γ energy is now here the dimension of s is the dimension of r and this is the dimension of ρ now we calculate

so we now equate the equivalent powers of lmt separately it will give us three equations and

so I said if we have If more than three, we can only reduce the number of variables. We will not be able to give the final form. So here we do it. Equal $\beta - 3$ is equal to γ . This is what l gives us. Next we will go to m again. The power of m on the left is equal to 0 $\alpha + \gamma$ and then we have the third quantity which is t

so here we have 1 equal $\alpha - 2$ and nothing else

so First we solve this equation it gives us α equal $\frac{1}{2}$ and then we go to the second equation we equal the γ $\frac{1}{2}$ and then we go to β the third equation $\beta - 3$ is equal to γ

so β will be equal to t by three two

so now we have the original equation Let us go back to the fact that our original equation t is proportional to the power α r from power α r to the power β ρ

so now we can write the power subtraction of k equal to s with the power of half r to the power three by two ρ half. And if I write it in terms of common denominator, I get it as k times the square root of s over $r^3 \rho^{\frac{1}{2}}$

so that I can use it to find a formula Dimensional Analysis Let us see another problem. The potential energy varies with the x distance from the origin and the formula is given to us u which is equal to the potential energy a multiplied by x^2 square root $x^2 + b$ where a and b is a dimensional constant and the problem says find the dimensional formula of ab we have the dimensional formula What I mean by that is the dimension of ab in terms of the basic dimension

so here it means we have to find out the dimension of ab find the n dimension of a and the dimension of b

so let's see the original formula here we have If u is equal to $a \sqrt{x^2 + b}$ then how do we find the dimension of b let us first find the dimension of b . See b is added to the square of x

so if this formula is to be correct then the dimension of b must be equal to the dimension of x^2 , because we have $x^2 + b$ it means the dimension of b is equal to the dimension of the square

so it means the dimension of b is l^2 The square will be equal because x is a distance given to the distance x from the origin

so now let's look at the length we want to find the dimension of a

so we have given the formula for you to invert the formula
 so we now write the formula for a
 so that a will be $u \sqrt{x^2 + b}$ Multiply $x^2 + b$ is divided by the square root of x ,
 so now if I write, then the dimension of a is given for a,
 so a is given in this way,
 so the dimension of u is equal to the dimension of o, f is equal to x^2 or b one of these
 so let's say x^2 dimension divided by the square root of x Now what we need to find is the u level given to us as the potential energy of a particle of potential energy and now when you don't know anything of this amount Only It is possible to start by memorizing the dimensions but once you know what these quantities are, remember that your dimensions do not need to remember the potential energy. If you think we can say that we have the same level of momentum then it We can think and use the way we can get the mass of energy v as the dimension of the square
 so it will be equal to $m \cdot v$. So when we have to work with the dimensions of ah , then the dimensions of a will be equal to the dimensions of u,
 so the dimensions you have are $m \cdot l^2 \cdot x^2$ dimension x^2 dimension 2 times the power to subtract the power we have here. So just write this then x will be the dimension of the square l^2 and then we have divided by the square root of x
 so it will be half the subtraction of the power subtraction because we have taken it into numbers
 so that's how we work and when we When we write this the dimensions are now we write
 so the dimension of a bend is equal to $m \cdot l$ to the power we have 4 minus half
 so the power subtraction of 1 to 7 by 2 t is 2 and then we need to find the level of a multiplication b is minus 7 by its power The power of two t will be equal to two times b which is the square of l
 so the final answer we get is to subtract the power of $m \cdot l$ from eleven by two t is two
 so these kinds of problems that are there are pretty simple now I want to explain The analysis shows how this could be really useful in life and only towards the end of World War II America was conducting some experiments on atomic bombs and they had very classified data and those experiments were carried out in Los Alamos Lab Didn't do what was coming out because these were the nuclear explosions that were done there but what they did was they published pictures of that explosion with the period that the front part of the bomb exploded and G.T.L. He was a famous scientist in the United Kingdom at Cambridge. He was a mathematician and was able to estimate the power of the explosion using what Taylor did as he traveled from the front of the wave to the picture he made. Dimensional analysis was actually used to obtain a lot of classified data that was not provided otherwise and it was found that the Americans were later embarrassed to hate the scientists because they thought they only gave images of explosions but some predictions of bomb energy data from wave speeds using dimensional analysis teller. Was able to now have some limitations of dimensional analysis. Dimensional analysis alone is not able to give you everything, let us look at some limitations. A good limitation is that two physical quantities which are not related can have the same dimension and as far as quantitative analysis goes it will make both quantities the same and its a very Let us give a simple example. When we talk about the moment of a ball or a rotating force, it is a measure of the distance of the force

multiplied and when we look at it, it is a quantity and the second quantity that we see is that they have the same dimension of force or action. Like the distance where these two quantities are physically very different which gives us a bending effect the other energy gives us strength or accomplishes the task but if we use dimensional analysis it will consider these two quantities as the same but secondly you must understand that we have a Find the formula that we get from dimensional analysis to get a constant k correct that non-dimensional now dimensional analysis cannot help us to find the value of k so dimensional b The third limitation is that we cannot use dimensional analysis to predict behavior where the equations contain quantities other than the product strength because we have seen that using dimensional analysis we can only relate to things whose product So for example if we have a formula y is equal to $y \sin \omega t$ we cannot get $\sin \omega t$ from dimensional analysis but a subject dimensional analysis can tell us because sine has an argument which is an angle then ωt itself must be dimensionless But we cannot find a form like $\sin \omega T$ from a dimensional analysis or even if we have a formula that we will encounter in mechanics because s equals ut plus half a square now this formula cannot be found from dimensional analysis but what dimensional analysis again In this formula we can say that ut and s must have the same dimension, similarly halves in square and s have the same dimension. But we cannot predict this form ut plus half a square for those of you who are more interested in dimensional analysis. What we understand is that this dimensional analysis as we have seen here is a limited form of what is called Buckingham's pie theorem a Buckingham pie. There are theorems and what we have seen here is basically a finite form. this theorem which we have done here because as we have seen if there are three basic dimensions in a problem then we can only find relations up to four variables at most and sometimes even that may not be possible if there are only two basic dimensions appearing or if some come in some combinations the dimensions cancel out see for example there is a formula for a frequency of tuning fork the formula for frequency of tuning fork in some conditions is given as frequency is equal to d by l square times v now a formula like this cannot be predicted by dimensional analysis even though we have three quantities u on the right hand side and one quantity totally there are four quantities and here if you realize the reason why this doesn't work why dimensional analysis cannot be used to predict this is because there are only two basic variables l and t in this so that means at most three quantities we could have derived this formula so this formula cannot be derived from dimensional analysis so we should not go by just blindly counting the number of variables the number of basic variables here are only 2 l and t so therefore we could at most get three variables involved but there are four variables d l square and v so therefore such a but once again the dimensional consistency has to be ensured the dimension of f has to be the same as dimension of d by l square times v so that dimensional analysis can check so this is how we see while dimensional analysis is a powerful tool but it has its own limitations you