

okay friends today lecture six one linear programming problem

So let us discuss some problem a catering agency has two kitchen to prepare food at two places a and b from these places mid-day meal is to be supplied to three different schools situated at p q r the monthly requirements of the school are respectively 40 50 40 and 50 food packets a packet contains lunch for 100 students preparing capacity of kitchens a and b are 60 and 70 packets per month respectively the transportation cost per packet from the kitchen to school is given below transportation cost per packet in rupees So a two p 5 rupees from b to p 4 rupees from a to q 4 rupees and from b to q 2 rupees and from a to r three rupees b two are five rupees now problem is how many packets from each kitchen should be transported to schools

So that the cost of transportation is minimum also find the minimum cost

So this is problem this problem is called transportation problem

So we have to minimize the cost of transportation by using linear programming let the number of packet sent from a to p equal to x and the number of packets sent from a to q equal to y

So a catering agency has two kitchen to prepare food at two places a and b

So we have kitchen at say place a and second place b and after preparing foods midday meal we have to supply it to three different schools situated at p q r

So this is school p q and r

So number of packets sent from a to p x and number of packet send from a to q is why and the preparing capacity of kitchen a and b are sixty and seventy the preparing capacity of a is sixty and preparing capacity of b seventy

So we have we already sent x packet from a to p and y packet from a to q

So we have to send a remaining packet that is sixty minus x minus y to r it means all the 60 packets of a distributed to school p q and r now from b is called p q r the requirement monthly requirements of school are respectively forty forty fifty

So requirement of p is forty requirement of q is 40 and requirement of r is 50.

So school p already get x packet

So remaining 40 minus x packet will get from kitchen b similarly a school queue get y packet from a

So remaining packet 40 minus y will get from kitchen b now rest packet of b will be sent to kitchen iron and that packet will be 70 minus 40 minus y minus 40 minus x that is x plus y minus 10

So x plus y minus 10 packet will be sent from b to school r now the transportation cost is also given from p to a from a to p h five from a to q is 4 and from a to r is 3 from b to p is four and from b to q is two and from b to r is five

So total transportation cost

So total transportation cost means z we have to minimize the

So total transportation cost will be five x plus four y plus three into sixty minus x minus y plus four into forty minus x two into forty minus y and five into x plus y minus ten

So five x plus 4 y plus 3 into 60 minus x minus y plus four into forty minus x plus two into forty minus y plus five into x plus y minus ten

So after simplification five x minus three x minus four x

So five x minus three x minus four x plus five x

So x means three x now four y minus three y minus two i plus five y

So nine y minus five y

So plus four y now plus one eighty plus one sixty plus eighty minus fifty

So equal to plus 370.

So total transportation cost is three x plus four y plus three seventy we have to minimize the subject to constants subject to the constants x plus y less than equal to sixty and x plus y less than equal to ten and x plus y greater than equal to ten because x plus y minus ten is greater than equal to zero

So we can send

So from b to r we send x plus y minus ten greater than equal to zero and x less than equal to 40 we sent x packet from a to p and p have maximum capacity for t

So x less than equal to 40 and y also less than equal to forty and obviously number of packet will not be negative

So finally

So formulation of of lpp as minimize z equal to three x plus four y plus three seventy subject to the constants x plus y less than equal to 60 x plus y greater than equal to 10

$x$  less than equal to forty  $y$  less than equal to forty  $x$  greater than equal to zero  $y$  greater than equal to zero

So we have linear constants  $x$  plus  $y$  less than equal to sixty say first  $x$  plus  $y$  greater than equal to ten say second  $x$  less than equal to forty third  $y$  less than equal to forty fourth

So associated equation for first second third and fourth are  $x$  plus  $y$  equal to 60 this implies  $x$  by sixty plus  $y$  by sixty equal to one  $x$  plus  $y$  equal to ten this implies  $x$  by 10 plus  $y$  by 10  $x$  equal to 40 and  $y$  equal to 40. now draw the graph of these equations 10 20 30 40 50 60 70 10 20 30 40 50 60 70.

So for first equation  $x$  by sixty plus  $y$  by sixty

So  $x$  intercept sixty and  $y$  intercept sixty  $x$  plus  $y$  equal to sixty second equation  $x$  by ten plus  $y$  by ten equal to one  $x$  equal to 40 is a line parallel to  $y$  axis and  $y$  equal to 40 is a line parallel to  $x$  axis

So by origin test four one origin means zero plus zero equal to zero less than equal to sixty is true

So four one origin belongs to solution reason origin test zero plus zero equal to 0 greater than equal to 10 false for second origin does not belongs to solution region

So feasible reason

So  $x$  plus  $y$  equal to 10 origin does not include and for  $x$  plus  $y$  equal to 60 origin include and  $y$  less than equal to 40 it means below the line and  $x$  less than equal to 40 means left side of the line

So feasible reason and this is  $x$  greater than equal to zero and this is  $y$  greater than equal to zero

So feasible reason will be this region and corner points of this bounded feasible region is this

So its fair graph is like this

So we have corner points six corner points a forty twenty b twenty forty c zero forty d zero ten e ten zero and f four t zero since feasible reason is bounded and convex feasible reason a b c d e f is bonded and convex

So minimum value of  $z$  equal to three  $x$  plus four  $y$  plus 370 exist at corner points and corner points are a forty twenty b twenty forty c zero forty d zero ten e ten zero and f for t zero

So value of  $z$  equal to three  $x$  plus four  $y$  plus three seventy at corner points  $z$  a equal to three into forty plus four into twenty plus three seventy equal to five seventy  $z$  at b three into twenty plus four into forty plus 370 equal to 590  $z$  three into zero plus four into forty plus three seventy five thirty  $z$  r d three into zero plus four into ten plus three seventy equal to four one zero  $z$  at e equal to three into ten plus four into zero plus three seventy equal to four hundred and  $z$  at f three into forty plus four into zero plus three seventy eight equal to 490

So  $z$  at e is minimum since feasible region is bonded and convex

So  $z$  at e equal to 400 will be the minimum transportation cost when 10 0 and 50 packets are supplied from a and thirty forty zero packets are supplied from b to school at p q r respectively

So we can minimize the transportation cost by using the concept of linear programming now another problem this problem is related to postal services the post master of a local post office wishes to hire extra helpers during the deepavali season because of a large increase in volume of mail handling and delivery because of the limited office space and budgetary conditions the number of temporary helpers must not exceed 10. according to past experience a man can handle 300 letters and 80 packages per day on the average and a woman can handle 400 letters and 50 packages per day the master believes that the daily volume of external and packages will be no less than 3400 and 680 respectively a men receive rupees 225 a day and a woman receive rupees 200 a day how many men and women helps should be hired to keep the payroll at a minimum formulate an lpp and solve it graphically let the number of men heard per day equal to  $x$  and the number of women hide per day equal to  $y$

So according to question we have to how many men and women helpers should be hired to keep the payroll at a minimum

So we have to minimize the cost minimize  $z$  equal to 225  $x$  plus two hundred  $y$  subject to the constants the number of temporary imply helpers must not exceed ten

So  $x$  plus  $y$  less than equal to ten now a man can handle three hundred letters and eighty packages per day and women can handle 400 letters and 50 packet per day

So  $300x$  plus  $400y$  and total the daily volume of extra mill and package will be no less than three thousand four hundred and six eighty

So three hundred  $x$  plus four hundred  $y$  greater than equal to three thousand four hundred and number of packets

So men can handle 80 packets per day and women can handle 50 packets

So  $ix$  plus  $50y$  the daily volume of external and package will be no less than three thousand four hundred and six eighty respectively

So this is greater than equal to six eighty

So this can be written as three  $x$  plus four  $y$  greater than equal to thirty four and  $ix$  plus five  $y$  greater than equal to sixty eight and number of men cannot be negative number of women cannot be negative

So in this way we can formulate the given problem as lpp

So finally the formulation of the problem like this now the linear constants linear constants are three  $x$  plus four  $y$  is greater than equal to thirty four  $i$  takes

So  $x/34$  by 3 plus  $y/34$  by four equal to one and eight  $x$  plus five  $y$  is greater than equal to is greater than equal to sixty eight this implies  $x$  by sixty eight by eight plus  $y$  by sixty eight by five is greater than equal to one

So associated equations 4 1 and 2  $x/34$  by 3 plus  $y/17$  by 2 equal to 1 and  $x/17$  by 2 plus  $y/68$  by five equal to one

So when we draw the graph of these two lines we will get and one constant that is  $x$  plus  $y$  is less than equal to ten

So this is first this is second and this is third

So  $x$  by ten plus  $y$  by ten equal to one

So when you draw the graph of these three equation we will get the graph of these three equation of line which intersect at 1 points  $p(6, 4)$  means all these  $c$  lines are concurrent lines

So the feasible reason here the feasible reason for all these three constants in equation is a point only because all the three lines are concurrent since all the three lines are concurrent at  $p(6, 4)$

So feasible reason will be point  $p(6, 4)$

So the value of  $z$  equal to two twenty five into six plus two hundred into four equal to two one five zero

So payroll is minimum rupees two one five zero per day when six men and four women are implied now let us take another problem this problem is related to construction activities the standard weight of a special purpose brick is five kg and it must contains two basic ingredients  $b_1$  and  $b_2$   $b_1$  cost rupees five per kg and  $b_2$  cost rupees eight per kg strength consideration dictate that the bricks should contain not more than four kg of  $b_1$  and minimum 2 kg of  $b_2$  since the demand for the product is likely to be related to the price of the brick find the minimum cost of brick satisfying the above condition formulate this situation as an lpp and solve it graphically let the weight of ingredients  $b_1$  equal to  $x$  kg and the weight of ingredient  $b_2$  equal to  $y$  kg

So in gradient we have two ingredients  $b_1$  and  $b_2$  and weight in kg is given as  $x$  plus  $y$  let  $x$  and  $y$  we have already taken and cost per kg in rupees five and eight

So we have to minimize the cost find the minimum cost of brick satisfying the above condition

So  $x$  plus  $y$  weight the standard weight of special purpose brick is five kg

So  $x$  plus  $y$  equal to five and the cost function  $z$  equal to five  $x$  plus eight  $y$  and condition on  $x$  the strength consideration dictate that the bricks would contain not more than four kg of  $b_1$

So condition on  $x$  is  $x$  less than equal to four and minimum two kg of  $b_2$  and  $y$  is greater than equal to two and obviously  $x$  greater than equal to zero  $y$  greater than equal to zero

So formulation  $z$  equal to five  $x$  plus eight  $y$  this has to be minimized subject to  $x$  plus  $y$  equal to five  $x$  less than equal to four  $y$  greater than equal to two  $x$  greater than equal to zero  $y$  greater than equal to zero

So when you plot the graph of these three constants will get the graph like this

So  $x$  plus  $y$  equal to five  $x$  less than equal to four means left of  $x$   $y$  greater than equal to two means above  $y$

So when we considering all these three condition will get the feasible solution on line  $a$   $b$  only

So feasible region for this linear programming problem is a line it means all the points on this line will give the solution but we have to find exact minimum value  
So the corner points of this feasible region will be a corner points a zero five and b three two a zero five and b three two  
So z at a is equal to five into zero plus eight into five equal to forty and z at b equal to five into three plus eight into two equal to thirty one  
So z minimum equal to thirty one at b three two  
So weight of b one equal to three kg and weight of b two equal to two kg  
So in this way we can use the concept of linear programming problem in construction activities also  
So ok friends now we discuss various types of problem in linear programming problem ok  
thank you you

Prutor@nitk