

good morning ah

So now after introducing the basic terminology for probability i will give a basic definition of probability ah

So as i mentioned to you earlier the subject of probability theory ah was originated in 16 17th century europe and that through ah gambling games

So i mentioned to you that one of the originators of the subject garden ah cardano he was actually a compulsive gambler and in fact in his autobiography he has written that i used to i am ashamed to say that i used to gamble every day

So now through that gambling games they started considering probabilities of various type of events

So for example if you ah toss two dice then what is the probability that you will get it well what is the expectation expected number of throws that are needed to throw a 12 like that

So they started considering various type of possibilities and then they came into contact with various other mathematicians of that time ah namely format pascal james bernoulli ah heightens in fact there is a historical context in which some of the problem was posed even to isaac newton and it seems he actually answered that problem correctly

So that time the nature of the random experiments was that you have a finite number of outcomes because in all this coin tossing die throwing card drying games etcetera in all of them you have a finite number of outcomes and also you can have an assumption that they are all equally likely assuming fairness

So the first definition of the probability which is called mathematical definition or the classical definition of probability is based on this concept only

So classical or you can say mathematical definition of now ok the form of this definition which i am writing here is actually attributed to laplace the french mathematician and this was published in his book in 1813 theories deaths probabilities in this book this definition was published although the form of the definition was being used ah for almost 100 150 years before laplace

So the form that is given here and it is there in your class 11 and 12 textbook also

So suppose a random experiment has n possible outcomes

So i am just mentioning this number n here and you have to be careful about this part

So when i say n that means it is a number

So if you say tossing of a coin then you have two outcomes

So n is equal to two if you say tossing of two coins then it becomes four if you say tossing of two dice then it becomes thirty six etcetera that means you can count the number of outcomes here

So a random experiment has n possible outcomes and even more importantly which are equally likely

So this is again saying that we are assuming fairness

So as i mentioned to you that the origin of this definition is in the gambling games

So where inherently it is assumed that the coin is fair or the die is fair or when you draw a card from a deck of cards then all the cards are equally likely to be drawn etcetera

So that is why this type of restriction was placed in this original definition

So these outcomes which are equally likely and then we just to make it little bit more precise we say mutually exclusive that means the counting of the outcomes is you can say exact and precise that means there is no possibility that one outcome can be confused with another outcome etcetera

So mutually exclusive and then nothing is left out

So exhaustive that means total number of outcomes we are able to look at putting such restrictions that they are equally likely and there is no chance of overlapping and all of them are considered now in such a situation let e be an event

So that m of these outcomes are favorable to happening of the event e then we define the probability of event e as

So we use this notation P of e that is equal to m by n

So that means if there are total number of n outcomes which are equally likely mutually exclusive and exhaustive out of that m are favorable to the occurrence of event e then the probability of event e is defined to be m by n and in fact this is the definition which you actually use for solving various problems given in your ah class text books ah where there are lot of problems related to drawing of balls from some on

So there will be some problem like there are four black balls three red balls and two

white balls and suppose four balls are drawn then what is the probability that ah this four balls contain two black one red and one white like that this kind of problems you will be solving when you are solving these problems actually you are using this definition because you are using the randomness to assume equally likely outcome for all of them now

So i will look at ah solution of some of the problems but a little later but before that let me comprehensively discuss these definitions when i am saying that there are n possible outcomes i am actually assuming that i am able to count all of the outcomes and then putting all these restrictions also now look at this i ask a very simple question what is the probability that it will be rainy day tomorrow now this question cannot be answered from here because what are the possibilities of the weather tomorrow So you can say that it can be runny rainy it can be sunny or it can be cloudy So three possible outcomes each with ah equally likely

So this is not a reasonable out reasonable assumption that having sunny rainy and cloudy to be equally likely because in a whole year how many days are actually rainy and how many days are sunny and how many days are cloudy if you look at this over last 50 or 100 years then you wont find that they are equally and therefore this type of condition is not correct

So when i say

So let me write here disadvantages or you can say drawbacks of this definition

So one is the events or you can say outcomes need not be equally likely let us look at ah other use of this equally likely thing even if i say only a coin ok or i consider a die then if i am considering a die and i am assuming that all one two three four five six have the same probability of occurring that means i am allocating probability one by six to each of them now is it reasonable suppose i am considering suppose i am actually talking about real gambling and in a real gambling there are players somebody has supplied the die now if that person who has given the die he is dishonest then maybe he is in league with one of the parties or one of the players and he may actually give a biased die such that it will fair one it will be favoring one of the players for example may be it is heavily favoring 5 and 6 and the person knows it the other player and therefore he will call more times five and six he will bet on that and he will be winner if you remember ah some sort of mythological story in mahabharata

So there was a gambling game between kawarewas and pandavas and then yudhisthira was losing because shakuni was playing with duryodhana and it is said that he used a biased set of ah not dies that time whatever thing they had ah they used that thing they called it passa those days

So therefore the game favored one of the parties now in such a case you cannot apply the definition of classical definition because equal equally likely condition is violated So that is the first thing that the outcomes need not be equally likely another point which is ah also logically incorrect that is we are actually defining probability that means i am saying that through this the way we are actually finding out the probability when i say equally likely i am already putting the definition of probability in the definition itself which is circular logic

So from a logical point of view also this definition is failing on this account let us consider another typical problem i talked about life of an electronic equipment ok suppose we are considering life of an electronic equipment ok

So usually maybe we know for example a good quality laptop

So if i am counting the you know life in months then we will put say life of the laptop is between zero to say sixty ok

So that means up to five years you can say but if you look at how many possibilities are there what is n here now here n cannot be defined because this is actually in finite i am taking an interval

So all the values between 0 to 60 are there

So it is not necessarily only integer months

So the next assumption that n we will be able to tell something is also erroneous that means every in every situation this capital n need not be possible ok

So here n is not finite now to overcome these things the statisticians and people in other sciences for example in physics or economics etcetera they started thinking that we can have a definition which is based on observing of the outcomes that means over a long period of time if you observe and then you look at what is the proportion of the number of events that is occurring then we consider that as the indicator of the probability

So that is called empirical definition that means the definition is based on the experience ok

So this second definition that is called relative frequency or empirical definition of probability it is attributed to actually phone mices the definition is as follows suppose a random experiment is conducted a large number of times independently under identical conditions suppose in n trials of the experiment an event e occurs a_n times

So i am putting this as a subscript notation here just to say that if i am conducting 10 trials then how many times event e is occurring i call it a 10 . suppose i conduct the trial 20 times out of 20 times how many times e occurred suppose i call that number to be 8 1 t that numbers can be anything it can be three seven eleven or anything

So those numbers are recorded as a_n that means in n trials how many times it occurs i call it a a_n ok then if the limit a_n/n as n tends to infinity exists we define probability of e to be limit of a_n/n as n tends to infinity

So physically speaking what is it denoting out of total number of trials how many times your particular event is occurring if that ratio is having a limit that means over a long term then that is defined as the probability of that event

So for example we say ah when a new child is to be born

So we allocate equal probability for a boy or a for a girl child

So why do we do that because over thousands of years it has been observed that out of the total number of burs nearly 50 percent of buyers and 50 percent are girls

So if we are considering the ratio of the number of say boys to the total number of birds the ratio is almost half

So that is why we allocate the probability half similarly if i say that the probability that the next year the monsoon will be normal normal means they are defining in certain that this much amount of rainfall is there the probability is point nine then we say that we say this kind of statement because over the past hundred or two hundred years it has been observed that almost if you observe for ah twenty years then out of twenty years two times the monsoon is not normal if we observe say forty times then we observe that around four times the monsoon is not normal if we observe hundred years then we observe that around nine to ten times the monsoon has not been normal over the past hundred years

So that lends the credibility to the statement that saying there is a 90 percent chance that the monsoon will be normal in the next year because it is based on the experience

So we are looking at the total number of experiments total number of trials and out of that how many times the particular event in which we are interested has occurred to show you that how ah this kind of thing is practically applicable let me take one ah

theoretical example ok theoretical means basically hypothetical experiment ok suppose a an hypothetical sequence of tosses of a coin results in the following outcomes ok

So i am just putting it hypothetical

So suppose head head head tail head head head tail head head head tail etcetera

So i mentioned it is hypothetical in practice it will be little bit varying but here i am putting in a very nice structure let us see what is the probability of head

So if we want to calculate the probability of head i have to consider the ratio a_n/n let us see what is the ratio here i will write in this particular sequence in the first trial i observed a head

So the ratio is one by one that is a_n/n i am writing down when in the second trial was conducted again we got the head

So the ratio is two by two the third time again head came

So the ratio becomes three by three in the next one a tail came

So that means out of total four trials three heads have occurred

So the ratio becomes three by four you observe it one by one two by two three by three three by four let us go to the next one again a head has been observed

So out of five trials you have four heads out of six trials you have five heads out of seven trials you have six heads next again a tail is observed

So out of eight trials you have six heads let me continue for a little more ah period of time next is seven by nine you have 8 by 10 you have 9 by 11 and then you have 9 by 12

and

So on now i want to find out the limit of this a_n/n whether the limit exists because if the limit exists then that will be the probability of head here ok

So to this end i will consider little bit of mathematical formulation here

So you observe critically how the sequence is looking if you look at the fourth term here that is three by four if you look at the eighth term the eighth term is ah six by eight

which is actually three by four then again if you look at the twelfth term that is again nine by twelve which is three by four

So i can put it in a little bit more mathematical form like

So this i put like this

So we can express a n by n can i write it as $\frac{3k}{4k}$ see initially it is three by four then it is three into two by four into two then it is three by into three divided by four into three that means if n is of the form $4k$ then out of $4k$ trials $3k$ trials are head ok

So this one i am able to represent now let us see one before that that means $4k - 1$ now if you look at here $4k - 1$ the number of heads is actually $3k$ see in here it is 8 here it is 7 but number is 6 only that means in here also you see out of twelve trials you had nine out of eleven also you add nine

So that means this ratio we can write it is $\frac{3k}{4k - 1}$ if n is of the form $4k - 1$ let us look at the next one suppose i look at this one So here it is $4k - 2$ and here it has become one less that is $\frac{3k - 1}{4k - 2}$ you can observe here also and you can observe here also that means it is of the form $\frac{3k - 1}{4k - 2}$ if n is of the form $4k - 2$ and the next one if you see it is actually $\frac{3k - 2}{4k - 3}$ if n is of the form $4k - 3$ for k equal to one two and

So on

So you can see here this complete sequence a n by n in a mathematical form i am able to write as a union of four subsequences and now my aim is to take limit as n tends to infinity now if n tends to infinity actually k tends to infinity and let us consider the limit in each of them here actually this term is there is no question of limit k k cancels out

So actually it is three by four this one if you consider what is the limit three if you divide by k $4k - 1$ by k

So if i take limit as k tends to infinity this limit is also three by four if i consider this term this term is also $\frac{3k - 1}{4k - 2}$ by k

So if i take limit this goes to zero this goes to zero

So the limit is three by four similarly if i take limit here it is $\frac{3k - 2}{4k - 3}$ by k

So if i take limit here this becomes three by four consequently what we are saying here is that the all subsequences converge to three by four therefore the limit of the sequence a n by n is three by four

So limit a n by n is equal to three by four that is the probability of head

So in this hypothetical experiment in which i had three heads followed by a tail repeatedly we expect that the head is three times more likely to occur than the tail and i have shown you here by actually applying the relative frequency definition that you will get actually the same answer

So the purpose of this exercise was to show you that this relative frequency definition is the actual practical definition of the probability

So when we make loose statements we say that this year the average per hectare production of wheat will be more than the last year then actually i am observing over the years and over the years we have observed that in this particular type of climate or in this particular type of situation where the irrigation facilities are good or seeds quality has been good then the average production per hectare is more means higher

So this loose statement is actually you can say experience based definition or empirical definition or the relative frequency definition of the probability and in the real life situations except in the textbook kind of situations where we talk about ah coin tossing die throwing or on drying i mean drying of balls etcetera problems in the usual practice we actually apply relative frequency definition ah that is not to say ah to tell you that how you will do a problem in the classroom or in the exam when the problem is given in the exam then we are actually describing the experiment

So therefore you can actually apply the conditions of ah equally likely etcetera

So therefore you are solving the problem based on that or in some other problems ah some of the basic probabilities are already given there that means you are not asked to calculate those but based on those you are asked to calculate say probability of a union b or the probability of a union b union c etcetera if the basic probabilities are given to you ok ah now the another question is that whether this definition can be adopted as a universal definition of probability the answer is again no because in order to have a

mathematically universal definition that means the framework should be useful everywhere or that means whatever problem is coming you should be able to solve within that framework again I will show you that there are problems with this definition also

So one or two you can easily appreciate

So the first thing is that you should have enough empirical data that means previous experience from where you can actually calculate the probability in case that is not available then the relative frequency definition cannot be applied

So we should have sufficient number of trials and their outcomes

So if there is a sudden problem posed to you ah for which you do not have any method of knowing that what were the trials and what were the outcomes then you will not be able to apply this definition

So for example you are sitting in a room ah where there are large number of chairs you sit down on a chair and the question is ask what is the probability that when you are sitting on the chair the chair will break

So naturally this type of question we may feel like laughing on it but its a valid question but the answer cannot be provided because you do not have the data that means earlier when

So many students sat on the chairs how many chairs broke

So this question cannot be answered in this particular question

So many times people pose questions and people may feel like laughing on those questions but they are perfectly valid statistical questions the thing is that we since we do not have sufficient data to apply your theory therefore you cannot answer those questions it does not mean that the theory of probability is invalid or it is incomplete or no such thing is there actually the theory is proper but you cannot answer all the questions unless the sufficient amount of evidence or you can say data is available to you ah this question is also posed many times for example ah opinion polls are there exit polls are there when you are conducting ah general elections or other kind of elections are there and then the questions are asked what is the probability a particular political party is going to win now you will might have observed that there will be answers ah which will be varying because there will be several agencies which will be giving the same which will be answering the same question but their answers may be little bit different it will be different in the terms of percentage of votes it will be different in the number of seats one the thing is that in each of these cases your sample space itself changes for example if you are looking at the number of seats then the sample space is different if you are looking at the percentage of force then your sample space is different

So depending upon what methodology those agencies which have conducted the survey based on that the answers will vary and that is why you will have quite you can say substantially different answers it does not mean the theory of probability is not applicable here it is applicable but the practical application requires large number of conditions sufficient data and whether it has been applied correctly or not

So if it is not done in that way then there will be problems

So we should have sufficient number of trials and their outcomes recorded ah for example industries usually this manufacturing industries

So they will routinely apply this empirical definition for example they say what is the number of defectives

So out of every 100 product ah 100 units of the product they will take a sample of 10 out of that they will check how many are ok or not

So suppose out of 10 all are ok

So suppose like this they conduct the trial ten times in a one hour period

So out of um in one hour period for example they might have produced say ten thousand things and out of every hundred they have taken ten and they are recording the number of good ones or number of bad ones

So out of ten thousand suppose you have taken hundred samples out of hundred sample that means total one thousand units you have taken now out of 1000 units suppose only 3 are bad then you are able to tell that out of 1003 are bad that means the probability of bad is zero three you will not say it is out of ten thousand because you have checked only thousand now to generalize it to ten thousand you are giving the probability point zero three

So now in a large amount of production the company knows that roughly point zero three percent of the items may be defective similarly if they are looking at the life of the ah item

So that is another quality parameter for example they may like to give the warranty period then they need to know that what is the average life and what is the life beyond which ninety percent of the objects are working what is the life beyond which ten percent of the objects are working etcetera the different time points at the ah for the manufactured item

So if they find that ninety percent of the items they work beyond say three years for example its a electric fan then they are very safe to give two year warranty or one year warranty because then they will know that almost all the fans will be actually working beyond one hour one year because the average life is three years

So most of the things will be actually working beyond that

So they are very safe to give a warranty period of one year for that particular product

So this is all real applications of the empirical definition of probability the same thing is applied when a company is say an insurance company is launching a product

So they will say this particular policy is for service class implies that means the people who are in a service class and then they will say that the maturity amount will be given at the age of 60 years something like that they will give a statement now some premium is fixed

So that premium is calculated on the basis of expected life expectancy of that particular class of people because if they find that say 95 percent of the people they will survive beyond age of 60 that means they do not have to give the benefits at death are accidental death etcetera because then the people are likely to survive beyond 60 that means their amount they are getting the full premium and then they are paying from that only

So that that is how the insurance companies survive in the market if they put an unrealistic small premium and they try to give lot of maturity benefits then the companies will run in the losses because then if more people are say claiming the benefits before the maturity of the policy then they will be in the loss

So to ah in order to apply this relative frequency definition that first thing is that we should have sufficient amount of data sufficient amount of trials to be performed and then their outcomes to be recorded and second thing is that it should be done in a reliable fashion that means we should not have something which is not reliable that means when the data is recorded and it is reported falsely or it is not collected properly that means when the experiments are being observed then the data is not recorded correctly then also you will results will be wrong ah

So that has mostly attributed to ok say that oh probability theory is not correctly applied or ah probability theory does not give the solutions to these problems that is not the thing the thing is that people are not applying it properly second thing is that some experiments are destructive in nature

So in that case there is a loss of the property for example you are considering how many match sticks in a match box of 50 sticks are ok

So if the actual experiment is conducted then out of every 50 will you try to light all 50 if you light all the 50 then the entire boxes destroyed

So there are experiments which are actually destructive second thing is that there can be experiments which are very costly for example launch of the satellites ok

So you cant keep on launching and seeing how what is the probability of the success these are actually done in the due course of time you record the previous data and on the basis of that you calculate the probabilities and there can be something counter intuitive also for example if i consider ah suppose a n is equal to square root of n okay that means out of every n trials square root of n times the ah event is favorable to e

So then if i consider a n by n then that is root n by n and if i take the limit that goes to zero now inherently we understand that if the probability is 0 it means that the probability event does not occur it is impossible even but actually event is not impossible only thing is what is happening is that as compared to n the number of trials is much less where the event is observed let us look at this out of 4 trials 2 times you have success if you have 9 then you have three times if you have sixteen then you have four times

So the event is actually occurring but the occurrence is becoming rare and rare as the number of trials increases therefore the probability zero does not mean impossible it means the event is rare event

So this is little bit counterintuitive if you are talking in very strict mathematical terms we say event is impossible then we allocate the probability zero but here we may have the probability zero event is not impossible

So here it means probability of rare event ok is 0 but it does not mean that the event does not occur ah the reverse of this is also true we can have say a n is equal to say n minus n to the power one by three ok

So here a n by n is equal to n minus n to the power one by three divided by n that is one minus one by n to the power two by three that converges to one

So again you can see here the event is not sure even because there are sometimes the event does not occur but again it means that the times when it does not occur is rare that means almost you can say with almost certainty the event occurs it is not with certainty with almost certainty

So you can say probability of events which may not occur sometimes is also one

So this is again little bit of counter intuitive but nevertheless it is extending the definition of probability to little bit more now i have given you two definitions of probability which were developed long back and then what happened that when we when the other mathematicians got to know about that there are problems in the definitions that means on a theoretical framework these definitions are not universal then they thought that probably the subject is not very strong in foundations ah around the same time david hilbert in mathematics he brought out a framework for formalizing entire mathematics and therefore there was a need to formalize the definition of probability also

So then in 1933 the russian mathematician in kalmogorov he was successful in providing an axiomatic foundation for the probability

So let me give this definition here the axiomatic definition was that this is a framework only it does not tell you how to calculate the probability but if there is a probability the validity of that probability and also there will be certain rules which can be given for calculation of probabilities

So suppose we consider s is a sample space and let us consider

So events are actually subsets of this

So let us consider a class that means a set of subsets of s ok ah let us give some name to it let us denote it by say some notation i will use c i am using some little bit different notation because see this s a b c e f etcetera we are using for events ok now i am considering set of events

So i am giving little bit different notation

So i am using this script notation script c sometimes it is written as script b etcetera

So any notation you can use let me put here c now let this class satisfy the following two conditions one if e belongs to c then it implies that e complement belongs to c what does it mean

So this means that see i can use this notation i can use this notation i can use this notation

So these are all similar it means the class of sets

So there are various ways of writing down this it means that if e is an event to be considered then its complement is also a valid event secondly if i am considering say e1 e2 and

So on these are all valid events then this implies that union of e i is also a valid event now you may wonder that why i am considering like this the reason is that earlier i discussed when i talk about sets events as sets then i should be able to talk about their unions intersections complementations differences because they are all denoting various kind of events

So then for when i am defining a probability structure then these should be all valid

So this definition as i mentioned this is for formalizing the theory of probability therefore it considered such a class of sets such that whenever an event is considered then all such things should also be even that is the unions intersections complementations differences that will be event

So this is structure is actually satisfying this in mathematics this is called a sigma field but at your level i need not talk about that formal definition of a sigma field but these are the basic conditions which are satisfied here

So now let us consider ah

So s is a sample space and then there is a class of subsets of this then the probability is defined as a function

So we call it notation p this is defined on c to zero to one set probability is a number between zero to one satisfying the following three axioms first axiom is that the probability of every event is always non negative second is the probability of the full sample space is equal to one and the third axiom is that let e one e two and

So on be pairwise disjoint i defined to you earlier what is the meaning of pair wise disjoint that means if i take any two of them then they are disjoint then probability of union e_i is equal to one to infinity that is equal to \sum probability of e_i that means if the events are disjoint then the probability of occurrence of at least one of them is nothing but the sum of the probabilities this notations may you if you are not very familiar let me write it like this actually it will mean something like this suppose i am considering two then probability of a union b will become probability of a plus probability of b if i am considering probability of a union b union c then that will become probability of a plus probability of b plus probability of c

So here a b c etcetera they are disjoint disjoint events ok now you may think that why this definition was given

So the first thing is that it is non negative second is the probability of the full space will be one that means whenever you are calculating probability of an event it is a proportion between zero and one and the third is that the probability is an additive function that means if i am considering ah some event then another event then another event if i know individual probabilities and i know that they are disjoint then the probability of union will be some of the probabilities now from here

So this is called actually the axiomatic definition of probability this kalmogorovs thing this is called exomatic definition of probability now on the basis of this several other rules of the probability could be easily established

So for example we can establish some consequences of the axiomatic definition probability of an impossible event is always zero

So it is very easy to prove this one in axiom three let us take event e_1 to be s and e_2 e_3 and

So on is equal to phi then what is statement i will get here the left hand side is union of e_i in this union of e_i the first set is s and other sets are phi then the union will become s itself the right hand side is probability of e_1 that is probability of s plus probability of e_2 that is probability of phi plus probability of e_3 that is probability of phi and

So on

So you look at this statement carefully what i have written p_s is one

So i am writing one is equal to one plus p_ϕ plus p_ϕ and

So on now when is it possible this one gets cancelled out on both the sides

So what you are saying if i am summing p_{ϕ} p_{ϕ} p_{ϕ} and

So on that is zero that means p_{ϕ} must be zero then the second consequence is that if i am considering say e to be a subset of f then how it is suppose i use the vein diagram

So this is my set e and this is an event f then what is this part this part becomes e minus f sorry i wrote wrongly this is ah

So here actually it is f is a subset of e

So this is e that is outer set and this inside set is f

So i can write here this set e as f union e minus f

So this e is equal to f union e minus f this complete set e is written as union of two disjoint sets

So probability of e will become probability of f plus probability of e minus f because f and e minus f are disjoint

So now for every set what we have written is that the probability is non negative probability of e is greater than or equal to zero

So if we use this then this term is non-negative that means probability of f is always less than or equal to probability of e

So first of all we can get several statements from here we can write probability of e minus f is equal to probability of e minus probability of f

So this is the statement that we are getting that if f is a subset of e then probability of e minus f can be written as probability e minus probability of f and this is greater than or equal to zero this means that probability of e is always greater than or equal to probability of f now this is an important statement that we have made if f is a subset of e it what does it mean it means that event e is more likely to occur than the event f and then probability of e will be greater than or equal to probability of f

So this is satisfying the basic you can say monotonicity property of the probability that means if an event has more chances of occurrence it has more favorable outcomes then it should have a higher probability of occurrence that is probability is a monotone function probability is a monotone function we can also prove another ah useful property if i

consider say e union e complement then that is equal to the full space

So if i apply then that is equal to p_s that is equal to one that means probability of e complement is always one minus probability of e that means the probability of the complementary event is one minus the probability of the original event here that also we are able to prove using this now ah these are some of the elementary rules that follow immediately

So what we are able to prove probability is a set function that means for every event it is defining are allocating a number between zero and one it is monotonic function the probability of full sample space that is the sure event is one the probability of impossible event that is ϕ is zero

So all other probabilities lie between these two extremes probability is additive that means if i have disjoint events and the probability of the union is equal to some of the probabilities probability is monotone that means if an event is more likely to occur then its probability will be higher these are some you can say basic framework under which this axiomatic definition was given and based on this there will be certain other rules that can be derived

So in the next lecture i will give all these rules and then we will see how we can apply this to solve various practical problems thank you