so good morning

So yesterday i have discussed the concept of random variable the concept of discrete random variable and its probability distribution ah we have discussed the concept of mean or the expected value the variance and the standard deviation one particular discrete distribution that we studied is called the binomial distribution and we saw the origin of this distribution how it arises and also its mean and variance today i will solve certain problems on the discrete distributions and including some problems on the binomial distribution

So let us start with some of the problems let f be a set with two m plus one elements

So there is a set with two m plus one elements let a script e be the class of all subsets of f with odd number of elements

So we consider those subsets of f which have odd number of elements that means they may have one element they may have three elements they may have five elements etcetera and the set of all such subsets it is called the class it is denoted by script e a set is randomly selected from e and let x be the number of elements in the selected set

So you can understand that the selected set can have 1 3 5 up to 2 m plus 1 elements

So we want to find the distribution of x and expectation of x let me consider

So x can take values 1 3 and

So on up to 2 m plus 1. what is the probability that x is taking value say 2 i plus 1 now there are total 2 m plus 1 elements in f

So the number of sets which will have 2 i plus 1 elements will be 2 m plus 1 choose 2 i plus 1 total number of subsets of f will be 2 to the power 2 m plus 1 now those sets which have odd number of elements will be 2 to the power 2 m

So total number of subsets which have odd number of elements that is 2 to the power 2 m

So that is coming in the denominator and in the numerator i am having those sets which have 2 i plus 1 element

So such sets are 2 m plus 1 choose 2 i plus 1 for i is equal to 0 1 to m

So this is actually the probability distribution of x here why the sum must be 1 you can see if we add two m plus one c one plus two m plus one c three plus two m plus one c five and

So on up to two m plus one c two m plus one then the sum is equal to 2 to the power 2 m

So this is a proper probability distribution suppose i want to calculate expectation of this

So if we want to calculate the expectation of it that is equal to sigma 2 i plus 1 into probability of x is equal to 2 i plus 1 i is equal to 0 to m

So that is equal to sigma i is equal to 0 to m 2 i plus 1 2 m plus 1 c 2 i plus 1 divided by 2 to the power 2 n now this term combination term we will expand

So this becomes sigma 2 i plus 1 2 m plus 1 factorial divided by 2 i plus 1 factorial and 2 m minus 2 i factorial and then this 2 to the power 2 m will be there i is equal to 0 to m now this term we adjust when in the denominator we are having 2 i plus 1 factorial

So 1 of the 2 i plus 1 will cancel out

So we get 2 m

So this i write as 2 m factorial and 2 m plus 1 i take outside

So 2 m plus 1 and this divided by 2 to the power 2 m this term does not involve i

So we can take it out of the summation sign divided by two i factorial two m minus two i factor here

So that is equal to 2 m plus 1 divided by 2 to the power 2 m summation i is equal to 0 to m 2 m choose 2 i

So this is actually the terms of the type 2 m c 0 plus 2 m c 2 plus 2 m c 4 and

So on up to 2 m c 2 m the total sum of this is 2 to the power 2 m minus 1

So that is equal to 2 m plus 1 divided by 2 to the power 2 m into 2 to the power 2 m minus 1

So this becomes 2 m plus 1 by 2 that you can say m plus 1 by 2.

So the mean of this distribution probability x equal to two i plus one is equal to this the mean of this is m plus one by two ah let us take another discrete distribution example a package of four ics contains one defective the ics are tested one by one without replacement until the defective is found out let x be the number of testings required find distribution of x and expectation of x

So the procedure is as follows you will test one ic if it is defective then we know that it is defective

So the experiment is stops because we know that there is one defective among the four ics

if the first one is not defective then you will take another one and we will test that
So if that is defective we will stop otherwise we go to the third one now in the third one if it is defective then we know that that this is the defective even if this is not defective then we know that the remaining one will be defective
So x can take values 1 2 and 3 we don't have to test the fourth one
So the possible values of x are one two and three let us look at the probability distribution of this
So what is the probability of x is equal to one now there are four ics and one is defective and we are saying that the first one itself if we choose it is defective then the probability of that will be one by four how the probability of x will be two that means the first one is not defective that means we choose from the good ones that is three by four now there are three remaining out of which one is effective
So in the second one we are choosing the defective
So the probability of that will be one by three
So three by four into one by three which is again one by four now in fact you do not have to calculate the probability x equal to three because that is the only remaining value you have 1 by 4 plus 1 by 4 that is half
So this probability will be half however i will show you how the logical argument can also be given see you can have three by four then two by three now one is left
So that one can be defective or it cannot need not be defective
So you will have two cases now if you add these two you are getting one by four ah sorry one by two this is equal to one by 2
So this is probability x is equal to 3
So this is the probability distribution of x here now i want to calculate expectation of x
So expectation of x becomes 1 into 1 by 4 plus 2 into 1 by 4 plus 3 into 1 by 2
So this value is nothing but 9 by 4. let x be a discrete random variable with distribution probability x is equal to k is given by c divided by 2 to the power k for k equal to 0 1 2 up to n minus 1 for n greater than or equal to 1
So first of all what is the value of c what is the expectation of x now if this is the probability distribution then sum of all the probabilities must be equal to one that is summation probability x equal to k k is from zero to n minus one that must be equal to one
So if we sum this 1 by 2 to the power k k equal to 0 to n minus 1 then this is nothing but a sum of a finite geometric progression
So this value we can easily calculate the first term is 1 then you have half and
So on plus 1 by 2 to the power n minus 1. by the formula of the sum of a geometric progression we know 1 minus r to the power n divided by 1 minus r into a a is 1 here
So it is equal to c into 2 to the power n minus 1 divided by 2 to the power n minus 1 now we are saying this is equal to 1 which will give the value of c as 2 to the power n minus 1 divided by 2 to the power n minus 1.
So in the distribution of x the value of c is given by this now in order to calculate the expectation of x that is equal to sigma k into probability x is equal to k k equal to 0 to n minus 1
So probability x equal to k that is c times 1 by 2 to the power k k equal to 0 to n minus 1 now this is arthritic geometric progression or geometric arithmetic series
So we can write it as c the first term is 0 the second term is half then the third term will be 2 by 2 square then you will have 3 by 2 cube and
So on plus n minus 1 divided by 2 to the power n minus 1 now suppose i call this series as s
So s is equal to half plus 2 by 2 square plus 3 by 2 q n minus 1 divided by 2 to the power n minus 1 then s by 2 is equal to 1 by 2 square plus 2 by 2 cube and
So on plus n minus 2 divided by 2 to the power n minus 1 plus n minus 1 divided by 2 to the power n
So if we subtract from 1 to 2 that is 1 minus 2 if i do then i will get s minus s by two is s by two is equal to half then two by two square minus one by two square that becomes one by two square three by two cube minus two by three cube that is becoming one by two cube and
So on plus 1 by 2 to the power n minus 1 minus n minus 1 divided by 2 to the power n if you see this term it is again a geometric progression and we know the sum of it
So that is equal to half 1 minus 1 by 2 to the power n minus 1 divided by 1 minus half

minus n minus 1 by 2 to the power n
So we can easily simplify this and we get the value of s is equal to 2 to the power n minus 2 n divided by 2 to the power n minus 1. and once again expectation x is nothing but c times this term
So that is simplified it is becoming
So expectation of x is given by this term in this problem i have demonstrated here that there is a geometric series as well as you are evaluating the constant such that sum of all the terms is equal to 1. let x be a discrete random variable with probability distribution
So it takes values x i minus 3 minus 2 minus 1 0 1 2 three four and the corresponding probabilities are two k square for minus two the probability is k for minus one the probability is two k for 0 the probability is 3 k for 1 the probability is 2 k for 2 the probability is k for 3 the probability is 7 k square and for 4 the probability is k square where k is such that it is a proper probability distribution you have to find k expectation x and variance of x
So the sum of all the probabilities must be equal to 1
So if you sum this you are getting 2 k square plus 7 plus 1 that is 10 k square plus k plus 2 k plus 3 k plus 2 k plus k that is 9 k is equal to 1 which you can write as 10 k square plus 9 k minus 1 is equal to 0. which can be expressed as 10 k minus 1 into k plus 1 is equal to 0. now this gives you 2 values k is equal to 1 by 10 and minus 1. but k is equal to minus 1 is not possible because that will give you some probabilities equal to negative and also the probability is greater than 1 for example this will become 2 this will become minus 1
So these cannot be values of the probabilities
So k is equal to minus 1 is not possible
So the correct value is k equal to 1 by 10.
So if you have k is equal to 1 by 10 then you substitute the values here you will get the proper distribution of x here
So what are the probabilities then the probability distribution of x is probability x is equal to minus 3 that is equal to 2 into 1 by 10 square
So it becomes 1 by 50 the probability x equal to minus 2 that is k that is 1 by 10 probability that x equal to minus 1 that is 2 k that is 2 by 10 that is 1 by 5.
probability that x equal to 0 that is 3 k that is 3 by 10 probability x equal to 1 that is equal to 2 into 1 by 10 that is 1 by 5 probability x equal to 2 is k that is 1 by 10 and the probability x equal to 3 that is 7 k square
So that becomes 7 by 100 and probability x equal to 4 that is equal to 1 by 100 that is k square
So we have obtained the probability distribution of x it is taking values from 3 minus 3 minus 2 minus 1 0 1 2 3 and 4.
So expectation of x is nothing but minus 3 into 1 by 50 minus 2 into 1 by 10 minus 1 into 1 by 5 plus 0 into 3 by 10 plus 1 into 1 by 5 plus 2 into 1 by 10 plus 3 into 7 by 100 plus 4 into 1 by 100
So we can easily evaluate this it is equal to 19 by 100 in order to calculate the variance we can apply the simplified formula variance of x is equal to expectation of x square minus expectation of x whole square
So if we apply that we get expectation of x square is equal to minus 3 square into 1 by 50 plus minus 2 square into 1 by 10 plus minus 1 square into 1 by 5 plus 0 square into 3 by 10 plus 1 square into 1 by 5 plus two square into one by ten plus three square into seven by hundred plus four square into one by hundred
So if we evaluate it this turns out to be forty seven by 20.
So variance of x is expectation of x square minus expectation of x whole square
So if we simplify this it turns out to be approximately 2.3139 in this problem let me repeat we are given the probabilities of different values of x in terms of certain unknown constant k by applying the condition that sum of all the probabilities is equal to 1 we are able to evaluate the value of k here you can see that what is the feasible value we have to check because we are getting two values but one of them is not appropriate because that leads to either negative probabilities or probabilities which are greater than one therefore we choose the value which is giving you the proper probabilities after determining the distribution then expectation and variance can be calculated by using the given formulae not only that we can calculate certain probabilities also for example in this if i want to calculate what is the probability

So let us ask some probability problems here in this distribution find probability mod x say greater than or equal to 2.

So now we are saying mod x is greater than or equal to 2 that is equivalent to saying x is greater than or equal to 2 r x is less than or equal to minus 2

So that is equal to probability x is equal to 2 x is equal to 3 plus x is equal to 4 similarly if i say x is less than or equal to minus two then it is equal to probability x equal to minus three and x equal to minus two x is equal to minus 2 here now all these probabilities are available here

So we have to only sum all of this here given any probability distribution all the probabilities that is the probability that random variable will take this value random variable will take value in this range the mean the variance or the standard deviation all of them can be determined let us take one more problem let x be a discrete random variable with possible values minus 2 minus 1 1 and 2. it is given that probability of x is equal to minus 2 is 1 by 3 and probability x is equal to 2 is 13 by 60 but probabilities of minus 1 and plus 1 are not given further it is known that expectation of x is equal to minus 17 by 60 determine probability of x equal to minus one and probability x is equal to one

So from the given information we have to calculate the probabilities of x is equal to minus 1 and x is equal to 1.

So we apply the condition that sum of all the probabilities is equal to 1

So probability x is equal to minus 2 plus probability x equal to 2 plus probability x equal to minus 1 plus probability x equal to 1 is equal to 1

So if we apply this condition 1 by 3 plus 13 by 60 now these values are not given to us

So we make some assumption let us assume probability x equal to minus 1 is q and probability x equal to 1 is p

So this is q plus p is equal to 1

So this will give you p plus q is equal to

So 13 by 60 plus 1 by 3 you add and subtract from 1

So we get here it is equal to 9 by 20 let me call this equation 1.

So we get one condition on the value of probability x equal to 1 and the value of probability x equal to minus 1. it is in the form of this equation p plus q is equal to nine by twenty now second condition we can determine because expectation is given now expectation formula if we apply then it is minus two into 1 by 3 x can take value plus 2 probability is 13 by 60 plus q into minus 1 plus p into 1 that is equal to minus 17 by 60.

So once again we can easily simplify this

So minus 2 by 3 this is 13 by 30 you subtract this number and take it to the other side

So you get p minus q is equal to minus 1 by 20.

So now we have two relations that is two equations in p and q

So we can easily solve them p is equal to one by five and q is equal to one by four

So probability x is equal to one in this problem is one by five and probability x equal to minus one is equal to one by 4

So from the given condition we are able to derive the values

So ultimately what we are checking in all these problems that it should be a proper probability distribution that means the probabilities lie between 0 and 1 and sum of the probabilities is equal to 1 and in case we are required to calculate expectation are the variance then we are applying the relevant formula for that a similar problem is next problem suppose x is a discrete random variable with distribution given by probability x equal to minus one is equal to one minus two alpha by three probability x is equal to one is equal to one plus two alpha by three probability x is equal to zero is equal to one by three where alpha is a real number find range of alpha that means for what values of alpha this is a proper probability distribution also determine values of alpha for which variance of x is maximum or minimum

So first of all let us check whether it is a proper probability distribution

So some of the probabilities must be equal to 1 let us see 1 minus 2 alpha by 3 plus 1 plus 2 alpha by 3 plus 1 by 3.

So this 2 alpha by 3 minus 2 alpha by 3 cancels out we get 1 by 3 plus 1 by 3 plus 1 by 3 is equal to 1.

So one condition is satisfied now second condition is that the probabilities must lie between 0 and 1.

So if we apply that condition you should have 0 less than or equal to 1 minus 2 alpha by

3 less than or equal to 1. now this can be easily simplified you get 0 less than or equal to 1 minus 2 alpha less than or equal to 3 which is equivalent to saying now if you apply this condition alpha becomes less than or equal to half and if you apply on this side you get alpha greater than or equal to minus one let me call it condition number one similarly if i apply the condition zero less than or equal to 1 plus 2 alpha by 3 less than or equal to 1 that is this probability is between 0 and 1 then this will lead to 0 less than or equal to 1 plus 2 alpha less than or equal to 3 then this is equivalent to now if you look at this alpha is less than or equal to 1 and if you apply on this side you get alpha greater than or equal to minus 1 by 2.

So let us look at these two conditions here alpha is between minus one to half and in the second one we get alpha is from minus half to one

So if you take the intersection of the two regions i will get alpha is from minus half to plus half

So taking intersection of two regions in one and two we get the range of alpha as minus half less than or equal to alpha less than or equal to half

So in order that the probabilities 1 minus 2 alpha by 3 1 plus 2 alpha by 3 and 1 by 3 define a probability distribution the relevant range for alpha will be minus half to half now the second part of the problem is that we want to determine the values of alpha for which variance of x is maximum or minimum

So we calculate the variance

So first of all what is expectation of x

So expectation of x is minus 1 into 1 minus 2 alpha by 3 plus 1 into 1 plus 2 alpha by 3 plus 0 into 1 by 3 here you can see it becomes simply minus one by three plus one by three cancels out here we get two alpha by three and here also you get two alpha by three

So it becomes four alpha by three similarly if i calculate expectation of x square then i get minus 1 square into 1 minus 2 alpha by 3 plus 1 square into 1 plus 2 alpha by 3 plus 0 square into 1 by 3

So that is equal to 1 minus two alpha by three plus one plus two alpha by three

So it becomes simply two by three

So variance of x is expectation of x a square minus expectation of x whole square that is equal to 2 by 3 minus 16 alpha square by 9 easily you can see you are getting alpha square term in the negative

So the

So that means this term is always positive

So this term will be minimum if alpha is minimum that is alpha is equal to 0 that will give me the maximum value of variance

So variance x is maximum then alpha is equal to 0. now to look at the minimum you have to have the maximum value of mod alpha now in this range the maximum value of mod alpha will be alpha is equal to plus half because alpha is equal to minus half and plus half both lead to mod alpha is equal to half

So that will give me the minimum value of variance x of course you can give a different argument also by using direct analysis let us call it say g alpha this function let me call g alpha that is equal to 2 by 3 minus 16 alpha square by 9.

So if i look at g prime alpha that is equal to minus 32 alpha by nine

So that is positive if minus half less than or equal to alpha less than zero and it is less than zero if 0 less than alpha greater than half

So the shape of this function g alpha if we plot at minus half to 0 it is positive that means it is increasing and then from 0 to half sorry i wrote this as 0 to half it is decreasing because it is negative

So the maximum value is here and we are looking at the range from minus half to plus half

So the minimum value that is obtained that is at minus half and plus half the function is like this and in fact the value at minus half and plus half is equal also

So the minimum value of g alpha is attained at alpha is equal to minus half and alpha is equal to plus half ah let us deal with one problem related to bernoullian trials

So independent questions are posed in a quiz to a candidate if the candidate fails to answer he or she has to leave the quiz let the probability of answering a question be say b it is known that the candidate answers an even number of questions and then fails is 0.9 what is p

So let us look at the question independent questions are posed in a quiz to a candidate

So if the candidate fails to answer then the candidate has to leave the quiz

So that means as long as the candidate is able to answer he continues in the quiz

competition now the probability of answering every question we are taking to be p
So it becomes a bernoullian trial that means a question is asked if the candidate answers correctly the probability is p the candidate does not answer correctly the probability is 1 minus p and since i have made assumption that independence
So it becomes actually independent bernoullian trials
So now let us say x is the number of questions asked to the candidate then x can take values 1 2 and
So on
So if he is totally asked k question that means the last question he did not answer
So the probability is 1 minus p or we also can call it q and before that he is able to correctly answer k minus 1 questions
So it is p to the power k minus 1 into 1 minus p where k is equal to 1 2 and
So on now it is given that the candidate answers and even number of questions correctly with probability 0.9 that means probability that x is equal to 2 k plus 1 ok what does it mean k equal to 0 means x is equal to 1.
So that means he does not answer any question correctly then if we look at x is equal to 3 that means 2 questions he answers third question is not able to answer
So like that
So this is equal to 0.9 now this is equal to p to the power 2 k into q k equal to 0 to infinity that is equal to now this is nothing but an infinite geometric series the sum is q divided by 1 minus p square that is given to be 0.9
So we can easily solve this equation because this is nothing but 1 minus p divided by 1 minus p into 1 plus p that is equal to 0.9
So this cancels out and you are getting 1 plus p is equal to 10 by 9 that means p is equal to 1 by 9 that means the candidate can answer each question correctly with probability 1 by 9.
So this is an example where we have used ah bernoullian trials here let me give another example in which binomial distribution is used a missile can successfully hit a target with probability 0.75 if three successful hits can destroy the target completely how many missiles must be fired simultaneously
So that the probability of completely destroying the target is not less than 0.95
So let us assume that n missiles are fired and x is the number of missiles hitting the target now this you can consider independent firings
So each missile can hit the target or it may not hit the target
So it becomes a bernoullian trial which are performed independently and the identical probability of success is 0.75
So if x is the number of success then x will have binomial distribution with n is equal to with n and p is equal to 0.75 this is p p is equal to 0.75 now we want probability that x is greater than or equal to 3 to be greater than or equal to 0.95 because if more than or equal to 3 hits are correct then the target is destroyed
So we want probability of x to be greater than or equal to 3
So this we can easily calculate this we can write as 1 minus probability x less than 3 greater than or equal to 0.95
So this is equivalent to probability x less than 3 less than or equal to point 0.05
So that is probability of x is equal to 0 plus probability x is equal to 1 plus probability x is equal to 2 less than or equal to 0.05
So from the binomial distribution the probability of x is ncx p to the power x 1 minus p to the power n minus x
So probability x equal to 0 becomes 1 minus p to the power n
So that is 1 minus 3 by 4 to the power n plus n c 1 1 minus 3 by 4 to the power n minus 1 into 1 by 4 plus n c 2 1 minus 3 by 4 to the power n minus 2 1 by 4 square less than or equal to 0.05
So this becomes a little bit of algebra here this first term is 1 by 4 to the power n plus n 1 by 4 to the power n minus 1 into 1 by 4 ah 3 by 4 this is 3 by 4 plus n into n minus 1 by 2 1 by 4 to the power n minus 2 3 by 4 square less than or equal to 0.05
So we can simplify this 10 into 9 n square minus 3 n plus 2 less than or equal to 4 to the power n now we have to check for what values of n this becomes true for example if i take n is equal to 1 then the right hand side is 4 and the left hand side becomes 9 minus 3 that is 6 plus 2 is 8 8 tangent is 80.
So this condition is not satisfied if i take n is equal to 2 3 4 5 the condition is not satisfied the condition is satisfied first for n is equal to 6 this condition is

satisfied for n greater than or equal to 6.
So the minimum value is minimum number of firings must be six
So the physical interpretation is that if each missile can successfully hit with probability three by four and we need at least three successful hits then we should fire at least six missiles to have more than 95 percent chance of hitting the target or destroying the target completely uh let me just give one problem in a quick fashion an item is defective with probability point zero one
So this is an industrial item in the industrial item ah there is an assembly line where the items are being produced
So on the average out of every 100 items one item is defective now a customer has to buy a pack of 10.
So what is the probability that in a pack of ten none are only one defective is there okay
So if x is the number of defectives out of 10 then x follows binomial 10.01
So probability x less than or equal to 1 will become probability x equal to 0 plus probability x is equal to 1 that is 0.99 to the power 10 plus 10 into 0.99 to the power 9 into 0.01
So that is equal to 0.9957 approximately i will spend one more class on various tutorial problems on various probabilities okay you