so good morning today i am going to start the topic probability theory now the terms such as chance randomness uncertainty probably have been in use since time immemorial people realized long back that things do not happen as planned

so i am giving some examples of uncertainty in events like what will be the weather tomorrow

so whether it will be rainy day whether it will be very cold or whether it will be moderately cold or whether it will be warm or whether it will be cloudy

so this is uncertainty a child born today what will be the height that he will achieve when he or she is adult

so so we know that in everyday life we cannot predict that what would be the actual height that an adult will achieve

so this is uncertainty we can consider for example what would be the total amount of food grains production this year we may have a fixed area or fixed amount of seeds sown for a particular crop but the ultimate food grain is dependent upon various things for example what is the ah irrigation how much is the fertility whether there are any natural calamities during the year etc

so the total amount will be variable we can never predict the complete age of a person

so a person may be very healthy but still he may die young due to various reasons a person may be a precarious but he may actually live much longer during a cyclone what is the height of a storm surge if we take a medicine suppose you get a common cold and you take medicine but how much time you will take to get cured is not certain what would be the blood pressure of a person

so we measure the blood pressure when a patient goes to a doctor

so what is the actual whether it is 120 by 80 whether it is 130 by ninety whether it is one fifteen by seventy five etcetera

so that is uncertain what is the when we conduct a exam

so how many students score how many marks

so this is uncertain

so for example i have written how many students score more than 75 percent marks in a test similarly whenever we consider any mechanical electrical or electronic equipment then what is the total life of that

so for example if we are considering a tube light what is the total life say in hours whether it will light for 100 hours whether it will light for 500 hours whether it will light for 1000 hours etcetera in fact i am giving one quotation by a roman playwright plotters that is everyone ought to keep a sharp eye on the main chance

so there are several such statements that randomness is an indispensable ingredient of the human life ah i will give some historical evidences

so the theory of probability originated in the middle of seventeenth century ah there are mainly studies by some mathematicians namely format who was sixteen hundred one to sixteen sixty five pascal between sixteen twenty three to sixteen sixty two heinz sixteen twenty nine to sixteen ninety five james bernali between sixteen fifty four to seventeen hundred five etcetera ah these are you can say some of the prominent mathematicians who started discussing among themselves various problems related to probability and through their discussion and solving some of the problems the subject probability started to grow in fact we can say that probably the physician and mathematician g cardone whose time is from 1501 to 1575 probably he was the first to develop the systematic theory of probability in fact he was a gambler he that means he used to play gambling games of dice coin cards and therefore his interest in the probability was actually originated to find out the probabilities of various possibilities when he is playing a game of cards or when he is playing a game of dice

so just to show you the photograph he is actually cardono his work was published in 1663 much after his death and it was a 15 page notebook consisting of 32 small chapters the book on games of chance and he solved a few problems which related to ah coin tossing die throwing etcetera and you can say that ah he is the first one who systematically started to formulate the ah subject you can say elementary concepts of probability ah thereafter the subject was picked up by various other mathematicians i named format pascal etcetera and through the their discussions the subject is started to crystallize now what i will do i will give you some elementary concepts of the probability and through that i will give you some definitions of probability through which the probabil the problems of probability can be solved

so let us look at the terminology first

so the first terminology is the term experiment

so what is an experiment in scientific terminology an experiment is observing something happen or conducting something which results in an outcome

so let us consider examples here ah i gave you few examples in the beginning let me repeat some of those examples and say that how it is an experiment

so i mentioned here weather tomorrow

so here we are not conducting the experiment rather we are observing but the outcome will be observed

so for example whether it is cloudy tomorrow or whether it is very cold tomorrow or whether it is sunny day tomorrow etcetera

so we are observing similarly i have given what is the height of a child

so we are only observing that means we are not conducting the experiment but something is happening and we look at the outcome

so an experiment is observing something happen second is like conducting something which results in an outcome

so this is like experiments which are done in the field of science

so for example you conduct experiments in physics you conduct experiments in chemistry you conduct experiments in biology in genetics there are large number of theoretical or practical experiments which are conducted in the laboratory conditions where the outcome will be something now there are two types of things one is deterministic experiment

so in the deterministic experiment if after conducting the experiment we know the outcome of the experiment we know the outcome of the experiment

so for example you know in your ah physics lab or chemistry lab etcetera you conduct some experiments and many times you know already what will be the outcome for example a simple experiment is certain ah mixing of certain chemicals and then looking at the reaction since already this type of experiment is known

so you know what will be the outcome the simplest thing is for example if you say two molecules of oxygen and a molecule of hydrogen then you know that this is a formula for the water similarly if i take a vessel and put water in that and we put it on a heater we observe that the temperature raises to say 100 degree celsius and the atmospheric pressure is say 760 then we know the outcome that the water will boil

so these are some of the scientific experiments in which the outcome is known these experiments are called deterministic experiments in probability theory we are not bothered about such experiments we are bothered about experiments which are non deterministic which we also call random experiments in non deterministic or random experiments we observe the experiment or we conduct the experiment but the outcome cannot be predicted in advance

so starting with the simplest one like if we consider a coin and we toss it then we do not know whether head will come up our tail will come up if we toss a

die then we do not know whether you will get one two three four five or six if we consider a well shuffled deck of cards and we draw a card at random then we do not know which of the fifty two cards will come out now these are some of the you can say textbook or classroom kind of experiments but you generalize it to the experiments which i mentioned just now

so for example observing of the weather

so no matter what scientific knowledge we have or how much scientific development we have but to say exactly at what time what temperature of the day will be there with certainty we cannot say

so therefore there is uncertainty here it is random experiment similarly about the height of a child the life of a person ah or life of an equipment

so no matter how scientifically precise we produce a manufactured item say for example a light bulb but can we say that what would be its actual life whether it will be 5 hours whether it will be 20 hours whether it will be 1000 hours we cannot say exact one cannot be you may give a range maybe you can say it will be between 5 hours to 50 hours or a statement like that

so you can give an approximate statement but you cannot give a fixed statement therefore these are all examples of non deterministic are random experiments

so formally i can give a definition when an experiment is conducted but the outcome cannot be predicted in advance then it is called a random experiment now one question arises if an experiment is random then what is the use of studying it

so for example i say that in a coin tossing experiment the next class will give me a head or tail that is not known to me then why should i actually spend my time to develop a subject and study this thing now the justification for this is that although in each trial i do not know whether a head will come or a tail will come but in the long term if i know or if we feel that the coin is unbiased or a fair coin then maybe out of thousand trials you will have a 500 heads and 500 tails or approximately you may say 490 heads and 510 tails if you conduct the experiment suppose it is a biased suppose you have a biased coin then if you conduct it a long term number of times then roughly the ratio of the heads to the tails will be actually the amount of bias that will be there for example if a head is ah say three times as likely to occur than a tail that means it is heavily biased in favor of head then roughly if you conduct the experiment thousand times then maybe around 750 times you will have a head and 250 times you will get a tail now this long term behavior is what encourages us to study the subject of probability theory

so for example if i ah talk about the weather

so every day there are weather predictions

so the weather predictions are based on the long term behavior

so like we say tomorrow it is likely to touch the temperature between twenty to twenty two degree of celsius in delhi then it means that over the last hundred years or hundred fifty years that has been observed that this particular day of the year the temperature is between this

so this long term behavior which is called statistical regularity this is what encourages us to study the subject of probability theory because although each ah experiment what will be the outcome at each trial we may not be able to say but in the long term we know what is the proportion of the trials which will result in particular outcome ah now

so now our concern is only to study the random experiments

so when we are conducting a random experiments we do not know what will be the actual outcome but we know that the outcome can be something which i can enumerate

so if we consider this enumeration we make it as a set then that set is called

a sample space
so we give a formal definition the set of all possible outcomes of a random experiment is called the sample space and we usually node we usually use the notation set theoretic notation means some set
so capital s or sometimes we use the notation omega to denote
so let me give examples of some such things
so suppose two coins are tossed now when the coins are tasked then what do we look at we look at whether head has come or a tail has come
so if two coins are there then the possibilities can be both can be heads both can be tails one can be head and one can be tailed now if two coins are there then head and tail can interchange also like first one is head second one is tail first one is tail second one is head etcetera if we count like this then the sample space can be written like this
so please note my symbols here h h
so this ordered pair denotes that on the first coin there is a head on the second coin there is a head then you may have head on the first you may have tail on the second you may have tail on the first and you may have head on the second and you may have tails on both
so this random experiment that means we have tossed two coins and we are looking at the outcomes the possible outcomes are of four types
so this sample space consists of four elements let me complicate it little bit
so suppose we say suppose a coin and a die is tossed together now how to define the sample space now
so we will have outcome for a coin and we will have outcome for the die outcome for the coin can be head or tail and outcome of the die can be one two three four five and six now since i am considering the experiment together then i have to write the sample space also in the joint form
so if i consider the similar set theoretic representation and an ordered pair will denote the first as an outcome for the coin and the second one has an outcome for a die then i can write it like this
so first one you can have coin occurrence as head and then you can have number one on the dice you can have head on the coin two on the die and
so on head and six then you can have tail one and
so on tail and six
so here you can see that there are twelve elements there are two outcomes on coin and six possible outcomes on the die
so two into six you have 12 possible outcomes in the sample space ah suppose i consider say what is the time of the day ok
so for example we are observing some event ok
so at what time of the day that event occurred
so now if you say time of the day how are you going to observe it that means you look at the time in a clock now the standard clocks they will have three hands one for the hour one for the minute and one for the second and when we observe we observe them in the integer values for example hour will be 1 2 3 up to 12.
similarly if you observe for the minute then minute we will say one minute two minute three minute like that
so starting from zero because if it is at the full hour then it is zero then up to fifty nine similarly when you observe second then second will be again zero one two up to fifty nine now this is i am talking as an observer see if i say time of the day then time can be from twelve midnight to next twelve midnight or twelve mid noon to another twelve mid noon that means you can have a twenty four hour period and its a continuous thing but i am talking about when we observe and we report then we report in terms of integer values for these hours of hand

minute and second
so we may write the sample space like this
so i am using the notation say s here now it is a pair
so i will say it is m n p ok its a ordered triplet m n and p where m can take values 1 2 up to 12 n can take values 0 1 2 59 and p can take values zero one to fifty nine if we consider continuous time then we may write the sample space as say 0 to 24 that means anytime between ah say twelve midnight to twelve midnight
so if you are using the standard terminology then zero to twenty four but here then i can take like even in the second i can split
so depends upon the recording device that you are having then we may write like this also now one should not get confused between these two representations when we actually solve a particular problem then we have to fix our sample space if we consider different sample spaces then the different ways of attacking the problem will be there i will give another example for this suppose i consider a 100 meter sprint race say of olympic standard ok
so if you are considering olympic standard then there are ah 8 to 10 runners let me call them p p 1 p 2 p 8 suppose 8 runners are there
so like how the experiment is conducted that means the all the players assemble at the starting point and then there is a start and the splinters take their run and complete it in a certain specified time now depending upon our interest the sample space can be defined in different ways
so for example suppose i am interested in who is the winner if we record the winner then the sample space can be written as that means either of the eight players can be the winner
so it is p one p two p eight on the other hand suppose we are interested in the winning time if we are interested in then the sample space can be let me write it as s2 since i said it is olympic standard
so interval can be say from nine point five seconds to maybe ten seconds here the recording is in seconds now you can see that for the same problem i am having two descriptions of the sample space now this is an important point when we solve a problem of probability then we should correctly specify the sample space that means your sample space is related to what type of thing you are interested in if we are looking at a particular player being ah winner and then we want to look at the probability of that then we have to look at the possibilities of who can be the winner
so this is the sample space whereas if i am asking a question what is the probability that the winning time will be a world record
so for example we know currently the world record is point five eight seconds that means between nine point five zero to nine point five eight if the time is there then it will become word record
so the time taken here that will determine that the sample space has to be described in this particular fashion of course there are different ways of answering these questions but that we will come later when we discuss the various methods of ah finding out the probabilities let me take another example of the sample space suppose i am considering we record the number of accidents taking place in a city in a year ok in one year how many accidents have taken place now how do you define the sample space here
so maybe in the whole year there was no accident one accident two accidents and
so on now the thing is that what would be the upper limit here what happens that although theoretically we know that the number of accidents will be finite only
so maybe there are suppose it is a small city
so maybe there are 50 accidents in a year suppose it is a very large city like delhi or bombay then the number of accidents may be running in thousands

so maybe ten thousand accidents are there then how do you write like will you write zero one two up two thousand actually we do not need to put a upper limit here we can write it as an infinite valued sample space here ah what happens that when you allot a probability distribution based on certain ah methodology then there will be higher probabilities allocated to the values which are in the beginning and as the value increases the probability becomes very small

so theoretically speaking we can put 0 1 2 3 and

so on add infinitum but practically most of the probability will be concentrated on a finite number of terms here similarly if i am considering say life of a an organism ok again when we say life then the life of most of the organisms will be finite

so you can put an interval like 0 to say 100 suppose i am recording in years months minutes seconds depending upon what kind of organism you are having

so for example if you are considering life of a human being then you may consider say zero to one hundred fifty

so if i say zero to one hundred fifty this upper bound is actually only denoting that practically we do not observe a person living beyond 150 years because normally we observe people living a life of 80 years 85 years 90 years 95 years there are people who are completing 100 years but there will be normally very rare persons who are crossing say 110 because then their names will be coming in the guinness book of world records etcetera and there will be hardly anybody who will be reaching the life of hundred fifty years

so it will be a practical way of putting the sample space otherwise theoretically you may say ok put zero to infinity ok but in a practical sense we may limit our sample space as an interval 0 to 150 if you are recording in the years now i will introduce other terminologies

so first thing that we have seen in probability theory we are concerned about experiments which are non deterministic in nature

so we call them random experiments i have given several examples of that now the set of all possible outcomes of a random experiment we call a sample space and i have explained through some examples that how the various kind of sample spaces can be described now the next thing is that what do we actually study in probability theory

so a ah question ah normal type of question i will say what is the probability that the life of the bulb will be between 20 hours to 25 hours

so when i say i am putting the life of a bulb say from 0 to 1000 hours and i am asking a precise question 20 to 25 hours then if i am considering the interval 0 to 1000 and out of that if i am taking 20 to 25 what it is it is actually a subset similarly if i say if a coin is tossed i am getting a head or a tail

so if i say what is the probability that there will be a head

so what is head denoting head is one possible outcome out of the two possible outcomes here

so this is a subset if i consider h of h and t

so in general when i am saying i want to find out probability of something that something is called an event and then event in mathematical terms is nothing but a subset of the sample space

so let me then formally define in probability theory we are interested in finding the probabilities of certain possible outcomes these collections or you can say these collections of outcomes are called events

so an event is a subset of the sample space

so let us consider through various examples here

so if i say tossing of two coins and i say event i consider e as h t and t h that means this event is describing that one head and one tail is observed similarly let us consider say i am considering the amount of rainfall during a

monsoon season
so amount of rainfall is recorded in millimeters centimeters etcetera
so if i am considering for a full season i may consider recording in centimeters ok
so here the sample space can be say 0 to 200 centimeters ok
so this is in centimeters if i say i consider a subset say 50 to 75 then it means that the amount of rainfall is between 50 to 75 centimeter
so here you can see this e is a subset of s
so in general then what i am saying is that any subset of the sample space is an event ah what you can observe here is that we are moving slowly towards mathematical representation of the probability when the i mentioned to you that in the 17th century or 16th century europe some of the mathematicians started discussing the problems of probability
so they were talking like in tossing of a three dice what is the probability that an 18 will be observed in tossing of say ah two coins what is the probability that one tail will be there etcetera this kind of problems they studied since they did not have the mathematical framework of set theory that time
so they were discussing it in a very verbose fashion by describing the event and then writing down lot of possibilities in ah essay type of language and consequently they were trying to find out the solution sometimes they got it correctly and many times they got erroneous answers also
so the reason was that they did not have the set theoretic framework at that particular time the set theory as you know it developed only in the late 19th century by george cantor
so now when set theoretic notations are there then you will see that the definitions and the consequently calculation of the probabilities becomes very simple
so now the first thing i have mentioned we have a sample space which is actually a set the set of all possible outcomes
so we have given a set theoretic notation now the second definition i am talking about that is of an event
so event is nothing but a subset of the sample space
so the that means we are particularly trying to follow the notations of set theory here and you will see that things become very ah nice or you can say comfortable for actual ah definitions of the probability through this now when i talk about the event then we have several kind of events
so as soon as we talk about an event there can be some sort of vague statement that we use in day to day life i am certain that it will rain in the evening or you say oh it is not possible
so that means we are giving some events or you can say we are talking about something with certainty by saying that either it will actually happen or it will not happen now these events are also considered in probability theory
so we call them shear event that means the event that will certainly happen
so for example if we are considering tossing of a die and if i say a number less than or equal to 7 occurs then certainly it will happen of course that means we are saying that the die will actually fall on a certain surface and we observe the upper face of that
so the number will be one two three four five or six because these are the six numbers which are there on the die of course we are excluding the possibility that something supernatural may happen and the dye may vanish etcetera but otherwise we know that if we are specifying anything from the sample space itself then it is a shear event
so if i say in the monsoon season the rainfall is less than ten thousand

centimeter then certainly it is sure even because rainfall cannot be more than ten thousand centimeter during a monsoon season because ten more than ten thousand centimeter means it will be ah flooding the whole of the country itself
so that is not possible
so if we want to use a set theoretic notation then we can use s itself to denote the shear event that means if all possibilities are counted then it is a shear event and the converse of it or you can say complement of this is nothing but the impossible event like if i say if a die is thrown and we say that the number is greater than ten
so in a die you are having numbers one two six and you say ten will occur
so it is impossible
so this is denoting by null set or empty set because that possibility is not there we use the notation phi ok now once we are describing the events by the sets then naturally in mathematics you already have the notations of set theoretic operations
so certainly if an event a is corresponding to set a set b is corresponding to an event b
so a and b are events and they are sets now in the set theory we have union intersection difference complement etcetera
so naturally that will lead to construction of new events
so let me talk about that
so union of two events
so if i say a and b ok
so let a and b be two events ok then a union b in set theory it means that we are consisting of all the elements which are in a rb are both
so here it will mean happening of either a or b or both
so we can say like this occurrence of either a or b are both
so we can also say occurrence of at least one of a and b now this representation is good because we can actually use it to write for more than two also ah see we can talk about union of three also like a union b union c now little bit let us take more mathematical notation i will give some notation called union a i that is i is equal to one to n what is it means i am considering a one union a 2 and
so on union a n that means union of n events
so in mathematically you can find these sets it means that elements are must be in a 1 elements must be in a two etcetera that means all the elements which are either in a one or a two or a three are in a n are in two of them are in three of them etcetera all of them will actually belong to union a i
so this in probability terminology it will mean occurrence of at least one a i for i is equal to one to n let us extend it little bit more we can consider union a i i is equal to one to infinity in place of a finite number of terms i can consider even infinite number of possibilities
so then this will mean occurrence of at least one a i where i is equal to one two and
so on
so this union of two events i am able to describe in terms of events that means in event how it means it means occurrence of at least one of them similarly you have intersection of sets
so that will lead to the interpretation for the intersection of events
so for example if i have a intersection b in set theory it means that the elements which are common to a and b
so in probability it will mean that simultaneous occurrence of both a and b now likewise we can extend this concept to more than two that means a one intersection a two and

so on intersection a n
so it means simultaneous occurrence of a one a two a n and we can further extend it a i i is equal to one to infinity
so this means simultaneous occurrence of a one a two and
so on that means simultaneous occurrence of even all infinite number of these now we have some further terminology for example you know the difference of sets a minus b in set theory what it means the elements which are in a but not in b it has actually another representation also we can also write it as a intersection b complement ah ok complementation notation all of you know if there is a universal set then the elements which are not in b will be called to be in b complement ok ah now in the context of probability theory what is the universal set
so here you can consider the sample space to be the universal set because all the events are in the context of that only
so here it will mean occurrence of a but not of b that means a occurs that is but b does not occur
so this is the interpretation for a minus b similarly if i consider say event a then what is the meaning of a complement a complement will mean that not occurrence of a now in set theory we have a concept of disjoint sets
so if no elements are common then the sets are said to be disjoint
so if we say a intersection b is equal to phi that means if a occurs b will not occur if b occurs then a will not occur
so we call them to be disjoint or mutually exclusive events now if we further extend it i can consider like a intersection b is equal to phi b intersection c is equal to phi a intersection c is equal to phi etcetera that means several events are there and in that each pair is disjoint then such events are called pairwise disjoint events
so if i say ah a one a two a n suppose a one a two a n are any events ok and we are having a i intersection a j is equal to phi for i not equal to j then a 1 a two etcetera they are called pairwise disjoint events are pair wise mutually exclusive events ah if i have a certain sample space and i am considering certain events out of that sample space in such a way that if i consider all of them together then that gives me the full sample space then they are called exhaustive events
so if union of a i i is equal to one to n is equal to the sample space then we say that events a one a two a n are exhaustive
so actually exhaustive meaning is that they exhaust all the possibilities of the sample space
so that is why we call them to be exhaustive events let me just take one example here suppose i consider rolling of a die then my sample space is one two three four five six suppose i define event e as say one three five and f as the event two four six then we observe two things about it e and f are mutually exclusive
so basically in terms of language if i speak then i will say e is the event that an odd number is observed f is the event that an even number is observed
so mutually exclusive means that because if an odd number is observed an even number cannot be observed and vice versa secondly if i consider all the possibilities then they are either in e or f because e union f is equal to s
so e and f are exhaustive ah this nomenclature of mutually exclusive and exhaustive will be the one which will be used for actually the first definition of probability
so in the first lecture i have actually introduced a little bit of history historical context of the probability theory where all it is ah useful why we actually want to study this thing and second is that some basic terminology has

been given ah in my next lecture i will introduce the basic definitions of probability theory ah how the probability is introduced and then ah we will proceed to consider its advantages disadvantages we look at some of the problems so in the next four five lectures i plan to give a basic understanding of ah probability theory which is useful at class eleven and twelve level thank you you