

okay friends now we start lecture three on linear programming problem you are all well versed with linear equation and linear equation in one and two variables they can be solved algebraically or graphically we have learned to solve a system of linear equation in two variable graphically in details now graphical method for solution of lpp for this we have two theorems theorem one says that let r be the feasible region for an lpp and z equal to $ax + by$ be the objective function when z has an optimal value that is maximum or minimum subject to the linear constraints described by either linear equation or linear inequalities the optimal value must lie at the corner points that is vertex of the feasible region second theorem states that r is the feasible region for all lpp and z equal to $ax + by$ be the objective function if feasible region r is bounded then objective function z has both maximum and minimum value and this value occurs at the corner points of the feasible region if r is unbounded then maximum or minimum value of the objective function may not exist and if exist it must occur at corner points of the feasible region now corner point method

so before we discuss corner point methods first of all we have to follow this step first step is that is formulation of lpp formulation of lpp consists two parts that is defining objective function that has to be maximized or minimized and second is linear constraints and after formulation of lpp we have to represent linear constraints graphically to obtain the feasible region and that region may be open region or closed design then we have to define or we have to find the corner points of the feasible region that exist at the vertex of the feasible region then obtain the value of z at each corner point if feasible region is bounded region then the objective function have either maximum value or minimum value or both may exist and that is unique and that may also exist on a line segment means joining two corner points and if the feasible region is open region then the optimal value for z may not exist and if exist then it must exist at corner points

so this is the process by which we can apply the linear programming problem concept in day to day life problem now we have some terms that is objective function objective function if a_1, a_2, a_3 and

so on a_n are constants and $x_1, x_2, x_3, \dots, x_n$ are variables called decision variables then linear function $z = a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n$ which is to be optimized is called the objective function it is always non-negative function then constants then equation or inequality on the variable of an lpp are called constraints they may be equal to greater than greater than equal to less than or less than equal to type the value of the variables x_1, x_2, \dots, x_n in an lpp are always non negative

so variables have no negative constants now we discuss some problem that is application of lpp now first problem is one kind of cake requires 300 gram of flour and 15 gram of fat another kind of cake requires 150 gram of flour and 30 gram of fat find the maximum number of cake which can be made from 7.

5 kg of flour and 600 gram of fat make it an lpp and solve it graphically

so from the given data first of all we have to formulate an lpp let x and y be the number of cakes of first and second kind respectively type one and second and number of cake x and y and flour required for cake one is 300 gram and fat required for cake one is fifteen gram again flour required for second cake is 150 gram and fat required 4 second cake is 30 gram according to question we have to find the maximum number of cakes which can be made from 7.

5 kg of flour and 600 gram of fat

so objective function is number of cakes that is $z = x + y$ and we have two constraints that is flour must be less than equal to seven point five kg so $300x + 150y \leq 7500$

5 kg means seven five zero zero gram and second the constants that is fat constant is fifteen x plus thirty y this must be less than equal to six hundred gram and x is greater than equal to 0 and y is greater than equal to 0 means x and y is non negative constants finally we formulate the lpp like this

so maximize z equal to x plus y subject to three hundred x plus one fifty y is less than equal to seventy five hundred that is two x plus y less than equal to fifty and fifteen x plus thirty y less than equal to six hundred that is x plus two y less than equal to forty and x greater than equal to zero y greater than equal to zero

so in this way we formulate the given lpp now we have to solve or we have to optimize this function z this z is called objective function this is called objective function

so we have to optimize this function by using the given constants associated equation

so constants are two x plus y less than equal to fifty say this is first x plus two y less than equal to forty say second

so associated equation four one and two is two x plus y equal to fifty x plus two y equal to forty now draw the these two lines four one x by twenty five plus y by fifty equal to one

so x intercept twenty five y intercept fifty and four second x by 40 plus y by 20 equal to one

so x intercept forty and y intercept twenty now draw these two line one and two say this is 10 20 30 40 50 60 10 20 30 40 50 60.

so equation 1 x by 25 plus y by 50.

so this is 25 and

so join this two points and for second equation this equation is two x plus y two x plus y equal to fifty now x by forty one point is this and y by twenty y by twenty

so join these two points this is x plus two y equal to forty since two x plus y less than equal to fifty

so if you take origin test origin test four one for one

so two into zero plus zero equal to zero less than equal to fifty is true it means origin lies in the solution region of this constant two x plus y less than equal to fifty now for second origin test four second 0 plus 2 into 0 equal to 0 less than equal to 40 again true it means solution region for this constant also includes solution and we have non negative constant that is x greater than equal to zero and y greater than equal to zero

so feasible reason this will be the feasible reason and the corner points of this feasible region is say a b and c now the clear graph for this problem is like this corner points for this boundary reason will be a twenty five zero and b twenty ten and c zero twenty now we have to find the value of objective function at this corner points that is a twenty five zero b twenty ten and c zero twenty

so objective function is z equal to x plus y

so z at a equal to twenty five plus zero equal to twenty five z at b equal to twenty plus ten equal to thirty and z at c equal to zero plus twenty equal to twenty

so z is maximum at b twenty ten

so number of this is maximum

so number of first cake equal to twenty and number of second gig equal to ten

so in this way we can solve the problem by formulation and using the concept of lpp let us take another example a furniture deals in only two items table and

chair he has rupees 10 000 to invest and a space to store almost 60 pieces a turbo table cost him rupees 500 and share cost rupees 100.

he can sell a table at rupees 550 and the chair at rupees 115 assume that he can sell all the items that he buys formulate this problem as an lpp

so that he maximizes his profit solve the problem using corner point method so let number of tables equal to x and number of chairs equal to y

so items and numbers

so item is tables and chairs number x and number y cost

so cost of table is given as rupees 500 and cost of chair is rupees 100 500 and cost of chair is 100 and profit

so cost of table is rupees 500 and he can sell a table at rupees 550

so profit is 550 minus 500 equal to 50 and cost of chair is 100 and he can sell a chair at rupees 115 means profit for chair is rupees fifty

so we have to maximize the profit that is z equal to fifty x plus fifteen y and the constant is the furniture dealer have a space to store at most sixty pieces it means number of table plus number of chair must be less than equal to sixty and he has rupees 10 000 to invest

so investment constant is five hundred x plus one hundred y less than equal to ten thousand and x greater than equal to zero and y greater than equal to zero

so finally we have formulation of this problem is like this maximize z equal to fifty x plus fifteen y this is profit function subject to constants five x plus y less than equal to hundred this is investment constant and x plus y less than equal to sixty this is storage constant and x greater than equal to zero and y greater than equal to zero this is non negative constants

so we have two constants

so linear constants are five x plus y less than equal to hundred and x plus y less than equal to 60 say this is first constant and this is second constant again we take associated equation for one and two that is five x plus y equal to hundred and x plus y equal to sixty

so express this in intercept form that is x by twenty plus y by hundred equal to one and x by sixty plus y by sixty equal to one now draw the graph of these two linear equation

so for first equation x intercept is 20 and y intercept is 100

so y intercept and x intercept

so join these two points this is five x plus y equal to 100 and 60 x plus y equal to 60

so y intercept 60 x intercept sixty say x plus y sixty now origin test four one five into zero plus zero equal to zero less than equal to hundred

so origin lies in the solution reason of one it means we have to consider this half plane again origin test for second zero plus zero less than equal to sixty is again true

so origin lies in the solution region

so this half plane will be solution region and x greater than equal to zero and y greater than equal to zero

so feasible region will be this and a b c are corner points originally also corner points but at origin the objective function is zero

so we dont consider origin as a corner point

so corner points is a b c

so its fair graph is like this

so corner points a twenty zero b ten fifty and c zero sixty now we have to calculate the value of z add this corner points

so corner points are a twenty zero b ten fifty and c zero sixty

so z at a means fifty into zero fifty into twenty plus fifteen into zero

so this is one thousand z equal to fifty x plus fifteen y z at b fifty into ten

plus fifteen into fifty

so twelve hundred fifty and $z = 1250$ into 0 plus 15 into 60 equal to 900

because z is profit function

so we have to maximize it

so maximum value of z is 1250 which occurs at $b = 1050$

so maximum value of z equal to twelve hundred fifty at $b = 10$ fifty number of tables equal to 10 and number of chairs equal to 50 .

now let us take another example this is manufacturing problem a manufacturer produces two types of steel tank he has two machine a and b the first type of trunk requires three hours on machine a and three hours on machine b the second type of trunk requires three hours on machine a and two hours on machine b machine a and b can work at most four eighteen hours and 15 hours per day respectively he earns to profit of rupees 30 and rupees 25 per up trunk of the first type and second type respectively how much chunk of each type must he make is there to make maximum profit

so here we have to again maximize the profit and also to find the number of trunk of each type

so that profit will be maximum

so let x and y be the number of trunk of first hand second type respectively

so type of trunk first type and second type and number of trunk first type say x and number of trunk second time type say y x and y machine a machine b profit now according to problem he has two machine a and b the first type of tongue required three hours on machine a and three hours on machine b three hours on machine a and three hours on machine b the second type of trunk required three hours on machine a and two hours on machine b three hours on machine a and two hours on machine b machine a and b can work at most for 18 hours

so machine a $3x$ plus $3y$ less than equal to eighteen and machine b also work fifteen hours per day that is three x plus two y less than equal to 15 he earns a profit of rupees 30 and rupees 25 per trunk of the first type and second type on first rank he earns rupees 30 as a profit and rupees 25 as a profit on second type of trunk

so profit function total profit that is z is equal to thirty x plus twenty five y and this is called profit function

so objective function here is profit function and we have to maximize it we have to maximize it and subject to constants subject to constants third three x plus three y less than equal to eighteen that is x plus y less than equal to six say this is first and three x plus two y less than equal to fifteen say this is second

so first is machine a constant and second is machine b constant and x y is greater than equal to zero that is number of trunks cannot be negative now associated equation equation for 1 and 2 x plus y equal to 6 this implies x by 6 plus y by 6 equal to 1 and $3x$ plus $2y$ equal to fifteen

so this implies x by five plus y by seven point five equal to one now draw the graph of these two equation

so x by six and y by six

so this is line x plus y equal to six and second line is x by five and y seven point five

so for now origin test four first and second origin test for first and second

so first is x plus y less than equal to six

so zero plus zero for one zero plus zero equal to zero less than equal to six which is true

so origin lies in the solution region of one it means an origin test for second three into zero plus two into y two into zero and zero less than equal to fifteen which is true

so origin lies in the solution region for second it means this half plane will be the solution region

so for both the constants we have common feasible region this and the corner point is a five zero b three three and c zero six

so its clear graph is like this

so corner point a five zero b three three and c zero six now we have to find the optimal value of profit function at these corner points

so corner points corner points a five zero b three three c zero six

so value of z at a z is z equal to thirty x plus twenty five y

so thirty into five plus twenty five into zero equal to one fifty and z at b thirty into three plus twenty five into three equal to one 165 and zrc 30 into 0 plus 25 into 6 equal to 150

so this 165 is maximum value which occurs at corner points b

so maximum value of profit function z equal to 30 x plus twenty five y occurs at b three three

so z max equal to one sixty five at b three three

so manufacturer manufacturer should produce three trunks of each type to get maximum profit of rupees 165 in this way we can use lpp in manufacturing problem also ok friend we shall discuss some more problem in next session thank you you