

okay friends today we are going to discuss about linear programming problem that is lpp you are well versed with linear equation and linear in equation in one and two variables and also we have discussed in details how to solve system of linear equation system of linear equation in one variable and two variables algebraically and graphically now we discuss about how to use the concept of linear equation and linear in equation in two variable in the field of linear programming problem

so linear programming is nothing but the process of optimization so in day to day life we have to deal with various problem that needs optimization or we

have to find its maximum value or minimum value

so linear programming problem is one of the process of mathematical optimization process

so we can say linear programming also called linear optimization is a method to use to achieve the best outcomes such as maximum

profit or lowest cost these are example in a mathematical model whose requirements

are represented by linear relationship it is special case of mathematical optimization let us take some example from day to day life situation or from real world

so in a military operation the military effort to inflict maximum damage to enemy and minimum loss in an industry manager wants to maximize profit under the constant manpower capitals and available resources similarly a solid class person wants his saved money for investment to maximize his

profit under minimum tax liability similarly we have many examples like transportation problem say they are two factory p and q and three warehouses and this factory p produce five unit and factory q produce 6 unit and the warehouse a has capacity to accommodate four item and where has where horse b has capacity to accommodate four and c has capacity to accommodate three now from a to from p to a from p to b and from p to c we have to transport the goods that produce from factory p and q and purpose

is to minimize the transportation cost and transportation cost is there from p to a is given from p to b is given from p to c is given from q to a from q to b from q to c

so what will

be the quantity that will be sent from p to a x_{11} to b x_{12} to c similarly from q to a x_{21}

to b x_{22} to c

so that the transportation cost will be minimum

so we have many such type of problems

so before going to discuss this problem we have to discuss in details about lpp means linear programming problem

so linear programming problem has two

parts first part that is constants constants are represented as linear equation

or linear in equation and this may be of one variable two variable or more than two variable and second part is plan of action plan of action this part is called programming and one and two together is called linear programming

so we have two x n first of all we have to define all the constants in linear equation or linear equation that may be of one two or more than

two variable and second is how to plan

so that we have to maximize or minimize the linear function or objective function

so we can say linear programming is a method for determining optimum value of a linear function subject to constants as linear equation or in equation some definition that is objective function the decision variables

constants optimization problem feasible region and feasible solution

so first of all we have to

discuss these five six technical terms function a linear function z equal to a x plus b y where a b are constant is called objective function which has which have to be maximized or minimized

so first of all we have to define

objective function then second is decision variable x and y are called decision variable x is always greater than equal to zero and y is always greater than equal to zero means x and y have non negative restriction means x and y is never negative then third is constants constants means condition or we can also say hurdles that we have to face when we have to maximize

or minimize the objective function

so it may be linear equation in the form of linear equation linear in equation and condition on and condition on decision variable variables now fourth point is optimization problem optimization problem a problem which sticks to maximize or minimize is called optimization problem and this problem maximize or minimize under certain constants or condition for example suppose in industry if a industrialist wants to maximize his profit then the constants may be number of main power available resources and available capitals

so these are

constants for industrial problem now feasible region suppose when you draw a graph of constants say a and b for these two constants the defined region or common region for these two constants is say this said addition say $0 \leq a \leq b \leq c$ then this $0 \leq a \leq b \leq c$ is called feasible region region satisfied by given constants and this region may be bounded region or maybe unbounded region

so suppose if you draw a graph of given constants and the common region for given constant is like this then this is called unbounded region and this feasible region is unbounded this is unbounded feasible region and all the points in this feasible

region is called feasible solution

so feasible solution α β belongs to feasible region then α β is called feasible solution feasible solution for the given constants constants simultaneously

so these are some terms that we

have to use during the course of discussion every feasible region very important point every feasible region must be convex set

so every feasible

reason must be convex set it means suppose you consider these are some different types of regions

so these are some different

regions and say the defined region defined region are this these are defined

region let us consider two points say one point is

here and one point is here join these two points again one point is here and one point is

here join these two points take any these two points in this region again

two points in this region

so one figure one this is figure two this is figure three figure four figure five

so in all these five figures only three four five are feasible reason because convex set means if you take any two points

in the region and if you join that two points the each and every points on the number

on the line must belongs to that reason

so in figure 1 and 2 these

points doesn't belongs to the well different the defined reason that's why

figure 1 and 2 is not convex set whereas figure 3 and 4 5 are convex set

so we have to consider only

figure like three four and five during discussion now graphical method of graphical method of

solving of linear programming problem

so before we start solving of linear programming

problem we have two important theorems or you can say two very fundamental theorems theorem one

says that let r be the feasible region for an lpp and z equal to $a x$ plus $b y$ be the objective

function when z has an optimal value that is maximum and minimum subject to the constants

described by linear equalities this optimal value must occur at a corner point of the feasible

region

so in this region these are corner points these are called corner points

means vertex of

the feasible region are called corner points these are called corner points

and theorem second

says that let r be the feasible region for an lpp and z equal to x plus $b y$ be the objective

function if r is a bounded then the objective function z has both a maximum and minimum value on

r and each of these occurs at a corner point of r remark if r is unbounded then a maximum or a

minimum value of the objective function may not exist if it exists it must occur at corner point

of r

so let us discuss this theorem on a diagram

so again consider two constants like this and one constant say this and if this constant define this half plane and this

constants define this half plane and this constant define this half plane

so common reason for all these constants will be this and this feasible

region is a bounded region and these points are called corner

points

so theorem one says that every feasible region habits corner points and its

optimal value lies at corner points and theorem 2 says that the optimal value that is if

region is bounded if this region is bounded then this region must have both maximum and minimum value let us consider another reason which is defined by different constants and say this is like this and the common reason for both the constants

is this and the corner points for this reason is say abc

so theorem first said that whether reason is bonded or unbonded must have corner points and its optimal values lies at corner points but

theorem 2 says if reason is bonded then both maximum and minimum value lies at corner points

and if reason is unbonded then there may chance of no maximum minimum value and if it exist then

it must exist at corner points now corner point of a feasible region is a point in the region

which is the intersection of two boundary lines and bounded region can be defined as a feasible

region of a system of linear equation inequalities is said to be bonded if it can be enclosed

within a circle otherwise it is called unbonded now corner point method

so to solve lpp we have two very important method that is first is simplex method and second is corner point method simplex method is useful when the number of variable is more than two and corner point method is very much convenient when the number of variable is a less means one or two

so we discuss only here corner point method steps of corner point method find the feasible

reason of the lpp and determine its corner points either by inspection or by solving the

two equation of the lines what does it mean step one says that by plotting the graph of given inequalities we have to define feasible reason

so whether that

is a may be closed region or may be open reason and steps one also says that by defining

feasible reason we have to find corner point if this will be the reason then this will be the

corner points and if this will be the reason then these three will be the corner points so

step one says that first of all we have to plot the graph of all the constants and define

feasible reason and define its corner points now step two evaluate z equal to x plus b y at each corner point let m and m respectively denote the largest smallest values

so suppose z equal to that is objective function z

equal to ax plus b y and if the feasible reason habits corner point say a b c then we have to find the value of z at

a means z a value of z at b z b and value of z at c that is z c since the reason is bounded

is n

so we must have among these three value one value is the smallest value and one value is largest value

so among z a z b and z c one value must be a smallest value and one value must be largest value

so according to step two we have to evaluate z equal to x plus b at each corner point let m and m respectively denote the largest and smallest value now

third step when the feasible region is bounded m and m are the maximum and minimum value of z

so it means the smallest value will be the minimum value and largest value will be the maximum value when region is bounded

region decision is bounded region and in case the feasible region is unbounded say region is unbounded then corner points of this region a b c say the value of z equal to a x plus b y

so the value of z at a is equal to say p and value of z at b is equal to say q and value of z at c equal to say r and if this p is a smallest value a smallest value and say r is largest value feasible region is unbounded and if m is the maximum value of z if the open half plane determined by x y ax plus b y

greater than m has no point in the common with the feasible reason otherwise z

has no maximum value it says that if this z c equal to r is maximum then a x plus b y greater than r have have no point in common with the feasible reason if if a x plus b y greater than r have no

point in common with feasible reason then r is the maximum value that is z max equal to r at c and if a x plus b y greater than r have common points with feasible reason then z equal to a x plus b y have no maximum value it means it says that our graph is like this and this will be the open feasible region and a x plus b y draw ax plus b y equal to r equal to r and if this a x plus b y equal to r have no any common points with this

feasible reason then this r then this r means z c equal to r this r will be the maximum value i think it is clear if this a x plus b y equal to r will pass if this a x plus b y equal to r will passing like this if a x plus b y equal to r is like this

then different points on this points which are common to this feasible reason then this r is not the maximum value of objective function similarly in case of when m is minimum value

so z a equal to p smallest

value and if this p is minimum value then again a x plus b y less than a x plus b y less than p have no any common points with the feasible

reason and if such condition holds then we can say z at a equal to p will be the minimum value and if a x plus b y less than p have common value have common value with feasible reason then z a equal to p will not be the minimum value or we can say minimum value does

not exist now let us consider one example solve the lpp graphically maximize z equal to four x plus y subject to the constants x plus y less than equal to fifty three x plus y less than equal to ninety x greater than equal to zero y greater than equal to zero now first of all define the reason for the

given constant x plus y less than equal to fifty three x plus y less than equal to

ninety solution associated equation equation x plus y equal to fifty three x plus y equal to ninety say this is first and this is second and this is third

so x plus y equal to fifty

so from one x plus y equal to fifty put y equal to zero implies x equal to fifty

x equal to zero implies y equal to fifty

so points on $x + y$ equal to fifty are fifty zero and zero fifty from $2x + 3y$ equal to 90 put y equal to 0 x equal to 30 x equal to 0 implies y equal to 90.

so points on line $3x + y$ equal to 90 are thirty zero and zero ninety now draw the graph of these two lines

so the y axis zero x axis point is fifty zero and zero fifty fifty zero and zero fifty

so each division will be of ten

so we have two points zero fifty

and fifty zero join these two points

so this is the line $x + y$

equal to fifty now for second associated equation points are thirty zero and zero ninety

so this is one point thirty zero and this is one point zero ninety

so join these two points

so this is equation of line $3x + y$ equal to ninety

so this line represent this

equation $3x + y$ equal to ninety now $x + y$ less than equal to fifty

so if you test origin test the defined region this line will represent this half plane and second constant is $3x + y$

less than equal to ninety

so again origin test means zero plus zero equal to 0 is less than 50

so this is true similarly $3 \times 0 + 0$ equal to 0 less than 90 this is again true

so in both constants origin includes in the solution region

so again the solution region for this constant is in this direction now the decision variables x and

y have no negative restriction

so the region must define first quadrant only

this is x greater than equal to zero and this is y greater than equal to zero

so when you consider

all these conditions we will find this will be the feasible solution region this will be the solution region now question is we have to

maximize z equal to $4x + y$

so these four points are corner points

so by

inspection we see that the corner points are a thirty zero b twenty three and c zero fifty and one

corner point is origin

so corner points are zero zero a thirty zero b twenty three and c zero fifty

so value of z at corner points

z at zero equal to 0 z at a equal to $4 \times 30 + 0$ means 120

z at b equal to $4 \times 20 + 30$ means 110 and z at c equal to $4 \times 0 + 50$

equal to 50 now in problem we have given maximize z

equal to $4x + y$

so this will the largest value since feasible region is feasible region is bounded region

so z max equal to 120 at a thirty zero ok friends we discuss some more problem in next session thank you