

welcome students

so far we have seen various tools to evaluate indefinite integrals in the very beginning we started with anti-derivative which were developed using the ideas of differentiation of functions then we looked at the idea of integral using method of substitution then for some ah rational functions we looked at the method of partial fractions and then we looked at the method of integration by parts where integrand was written as product of two functions with these tools we have evaluated several examples we have seen how to evaluate them in this lecture we shall use all those tools to evaluate certain integrals and we will be doing these examples which will be involving all of these concepts earlier when we were solving example mostly we were aware that this example is dependent on this particular idea but now we will choose the examples and looking at example we will decide that which method we should use here i must tell you that a particular example may be solved by another method a different method and i may be solving it a with a different method

so it choice is yours ah depending on a particular example you can choose which method you want to select sometimes it is possible that example may be solved by multiple methods

so looking at an example you choose a method you apply it try and solve it

so you practice with the problems and then once you are practiced then you will figure out that looking at example or after solving it for couple of lines you will realize that which method you should apply

so let us look at miscellaneous examples

so we will start with a very simple example to evaluate integral e^x raised to power x plus e^x raised to power minus x dx

so since denominator is having this sum of e^x raised to power x and e^x raised to power minus x we should look at the ideas which we have learnt what what we should do how to find out it is not in the derivative of a function which can be directly found but if we change the form of function a bit and that we know that e^x raised to the power minus x can be written as one over e^x raised to power x and therefore this integral will take the form of e^x raised to power x over e^x raised to power $2x + 1 dx$ now this form looks more comfortable than the earlier one

so if we closely look at a form and convert into a form which looks more comfortable we can easily handle it why this is more comfortable because here it looks like i can use the method of substitution this term has a factor exponential and exponential has the differentiation as exponential themselves and therefore the numerator is having that that as a factor

so the idea of substitution will work and if i make the substitution e^x raised to power x as t then what i will get is that e^x raised to the power $x dx$ as dt

so this example converts into the simple form of dt over $t^2 + 1$ because e^x raised to the power two x is nothing but e^x raised to the power x square and this formula we know using the nd derivative is that nothing but tan inverse t substituting back t into e^x raised to power x we will get tan inverse e^x raised to power x plus constant of integration

so you can see how easily we have solve this by converting it into a problem ah where a substitution worked and then converting into a integral of the known form

so we shall look at some more examples where this conversion can be possible or some more simplification of the functions can be possible

so look at this example find out the integral of say $\cos^2 x$ over $\cos x$ plus $\sin x$ squared dx if i look at this integrand then immediately substitution ah probably will not work but if i use trigonometrical identity which i know for the numerator of the integrand $\cos^2 x$ i can write it as $\cos^2 x - \sin^2 x$

square whole square plus cos raise to power 4 x this further can be written as $1 - 2 \cos^2 x$ here this can be written as $1 + \cos^2 x - 2 \cos^2 x$ plus $\cos^4 x$ and which ultimately is written as $1 - 2 \cos^2 x + \cos^4 x$

so this is what is our numerator now let us look at the denominator and do the same technique here

so denominator $1 - 2 \sin^2 x \cos^2 x$

so as in the numerator everybody i have written in terms of cos here also i want to write it in terms of cos

so i will write it as $1 - 2 \cos^2 x + \cos^4 x$ which will give me $1 - 2 \cos^2 x + \cos^4 x$

so $2 \cos^2 x \cos^2 x \cos^4 x$ now look at these two factors numerator this is your numerator

so numerator $1 - 2 \cos^2 x + \cos^4 x$ and denominator it is having that factor which is there in the numerator $1 - 2 \cos^2 x + \cos^4 x$

so $1 - 2 \cos^2 x + \cos^4 x$

so $1 - 2 \cos^2 x + \cos^4 x$

so this term will get cancelled when we simplify it for integrand i is written as $1 - 2 \cos^2 x$ that is numerator into this term divided by the denominator which is same therefore these two terms will get cancelled

so i do not need to write them i will just write them dx now you can easily evaluate this integral

so first term integral $x - 2 \cos^2 x$

so for the square cos functions we have to convert it into linear function we know that $\cos^2 x = \frac{1 + \cos 2x}{2}$ therefore $\cos^2 x$ is nothing but $\frac{1 + \cos 2x}{2}$

so i replace this $\frac{1 + \cos 2x}{2}$ by $\frac{dx}{2}$

so ultimately i get this two gets cancelled with these two minus integration of one is again x and then integration of $\cos 2x$ is nothing but $\frac{\sin 2x}{2}$ plus a constant

so this x gets cancelled with this x ah you can still be i mean write it the same form as it is here or you can write $\sin 2x$ you know $2 \sin x \cos x$ therefore this can be written as $\sin x \cos x$ plus constant

so using trigonometrical identities we saw that this function ah this integrand ah can be written into a very simpler form which further converted again using another technological identity and then simply it is evaluated we shall further look at some more example ah certain other class of examples suppose that we have to evaluate the integral of the kind $\int dx$ over $a + b \sin^2 x$ or for that matter let me call it as $\int dx$ over $a + b \cos^2 x$ where a and b are some constants which are suitably chosen now you can see that this problem can be converted into this problem by choosing $\cos^2 x = \frac{1 + \sin 2x}{2}$ and then simplifying and then choosing some new a and b

so accordingly the problem can be chosen

so we will be looking at how to solve the problem of the kind $\int dx$ upon $a + b \sin^2 x$ ah first thing we should come to mind is that convert into a form where we can use some of our techniques which we have already learnt one of the way will be to divide both the numerator and denominator by $\cos^2 x$

so that in the numerator we will get $\sec^2 x$ and in the denominator we will get $a \sec^2 x + b \tan^2 x$ if i do that ah what i realize here is that that the numerator gets x^2 and if i substitute $\tan x$ as another new variable then $\sec^2 x dx$ will become that dt only problem will be with this $\sec^2 x$ but fortunately we have a relationship that $1 + \tan^2 x = \sec^2 x$

square x is nothing but $\sec^2 x$

so in the denominator we shall replace $\sec^2 x$ as $1 + \tan^2 x$

$\sec^2 x \, dx = \frac{1}{1 + \tan^2 x}$

so $a + a + b \tan^2 x$ using the formula $x^2 = 1 + \tan^2 x$ and then make the substitution $\tan x = t$

so that $\sec^2 x \, dx = dt$

so we get this integral as dt divided by $a + a + b \tan^2 x$ sorry $a + b \tan^2 x$

so $a + b \tan^2 x$ is squared

so this is close to the form dt by $t^2 + a$ only thing is that we need to take $a + b$ as common

so if we take that $a + b$ as common one by $a + b$ as common in the denominator the form will be converted into dt upon $a + b + t^2$ squared ah this is now well known formula if I write $a + b = \alpha^2$

so I can use the formula here

so that it will become $\frac{1}{\alpha} \tan^{-1} \frac{t}{\alpha}$ which after this small substitution will plus constant which after this small substitution $\frac{1}{\alpha}$ will make it square root of $a + b$ divided by square root of $a + b \tan^2 x$ square root of $a + b$ times t divided by square root of $a + b$ plus constant which finally after little simplification you can write it as $\frac{1}{\sqrt{a + b}} \tan^{-1} \frac{t}{\sqrt{a + b}}$

so depending on a and b ah suppose in the problem particular problem it is given that a is having some value b is having some other value

so you can simply figure out that what will be the integral or if form is given in this form then also you can figure out the value of the integral next we choose another example for you

so suppose that we have to find out the integral of $\sin^{-1} 2x$ upon $1 + x^2$

so as there is no other factor present here ah what we should use here is that that there is one way one can use a substitution for x you can put some substitution or the other way is that that you check the idea which you have already used for for the example of inverse function by considering here that this is written as $1 \cdot \sin^{-1} 2x$ upon $1 + x^2$

so let us try and see what happens if I choose in this phase since it is an inverse trigonometric function therefore I must consider this as first function and this one being algebraic function I must consider as second function

so integration by parts I am going to apply here

so if I apply this integration by parts what I am going to get is that $\sin^{-1} 2x$ upon $1 + x^2$ that is first function integration of the second

so integration of one will give me x minus integration differentiation of \sin^{-1}

so I know the differentiation of $\sin^{-1} x$ is $\frac{1}{\sqrt{1 - x^2}}$

so I'll write it as $\frac{1}{\sqrt{1 - x^2}}$

so instead of x it is $2x$ over $1 + x^2$ whole square then multiplied by since it is this factor

so I have to take the derivative of this factor

so differentiation of $\frac{d}{dx}$ of $\frac{2x}{1 + x^2}$

so so far first function integration of the second minus integral

differentiation of the first and then integration of the second

so integration of the second will give you $x \, dx$ now I have to find out what is

the $\frac{d}{dx}$ of $\frac{2x}{1+x^2}$

so this let me find out separately $\frac{d}{dx}$ of $2x$ upon $1+x^2$

so it is of the form numerator over denominator

so this will give me 2 into differentiation of $2x$ will give me 2 minus $2x$ into differentiation of $1+x^2$ will give me $2x$ divided by $1+x^2$ whole squared this is $\frac{2-2x^2}{(1+x^2)^2}$ which will give me $2-2x^2$ which ultimately i can write 2 into $1-x^2$ divided by $1+x^2$ whole squared

so the derivative this term i now have the value i will write it here $x \sin^{-1} \frac{2x}{1+x^2}$ minus integration i have this term now here $\frac{1-x^2}{(1+x^2)^2}$

so the denominator i will evaluate separately $\frac{1-x^2}{(1+x^2)^2}$

so this is the term here i will write it as square root of $1+x^2$ whole squared minus $4x^2$ divided by $1+x^2$ whole squared which i can write as $\frac{1-x^2}{(1+x^2)^2}$

so if i open the whole square i will get $1+x^2$ raised to power 4 plus $2x^2$ minus $4x^2$ will make it $1-x^2$

so that i will get $\frac{1-x^2}{(1+x^2)^2}$

so the numerator will become $1-x^2$ whole squared over $1+x^2$ whole squared and ultimately i will get it as $\frac{1-x^2}{(1+x^2)^2}$

so this term in the denominator becomes $\frac{1-x^2}{(1+x^2)^2}$

so i will make this substitution for this denominator term

so this i will get as $\frac{1}{1-x^2} \cdot \frac{1}{(1+x^2)^2}$ this multiplied by the differentiation here which i have already got $2 \cdot \frac{1-x^2}{(1+x^2)^2}$ and then ultimately this $\frac{dx}{dx}$

so i will put it x here and then finally $\frac{dx}{dx}$ here

so look at it carefully some terms are getting cancelled for example this $1-x^2$ is getting cancelled with $1-x^2$ $1+x^2$ getting cancelled with one of the $1+x^2$

so ultimately this becomes $x \sin^{-1} \frac{2x}{1+x^2}$ over $1+x^2$ $\frac{dx}{dx}$ again i can take this $1+x^2$ as a new variable t

so that $t = 1+x^2$ $\frac{dx}{dt}$ is $\frac{dx}{dt}$

so $\frac{dx}{dx}$ will be $\frac{dx}{dt}$ by 2 which i can directly write at a logarithmic you can evaluate it

so $x \sin^{-1} \frac{2x}{1+x^2}$ minus half logarithmic of $1+x^2$ plus constant

so this way after evaluating these terms we can get these results sometimes it becomes very handy when you use some substitution for the values in inverse trigonometric functions for example one can look at here ah by choosing let us say x is goes to $\tan \theta$ and see what happens with this particular example i will choose another example for you to show that how it evolves if we use that kind of substitution of ah another variable

so choosing the example of integration of $\cos^{-1} \frac{2x}{1+x^2}$ minus $\frac{dx}{dx}$

so this $\cos^{-1} \frac{2x}{1+x^2}$ minus one t

so instead of going the way i have solved it previous example i will make a substitution as x is equals to $\cos \theta$

so that $\frac{dx}{dx}$ is equals to $-\sin \theta \frac{d\theta}{d\theta}$

so the integral i takes its form as $\cos^{-1} \frac{2 \cos \theta}{1+\cos^2 \theta}$ minus one $\sin \theta \frac{d\theta}{d\theta}$ now this term is $\cos^{-1} \frac{2 \cos \theta}{1+\cos^2 \theta}$ but i know the formula trigonometrical relationship $2 \cos^2 \theta - 1 = \cos 2\theta$

theta therefore $\cos^{-1}(\cos 2\theta)$ will be nothing but twice of theta $\cos^{-1}(\cos 2\theta)$ is nothing but twice of theta and then minus of sine theta $d\theta$

so this is same as minus two integration of sine theta $\theta d\theta$ i can now use an integration by parts because i am assuming this as first function and this being the trigonometric an second function

so it will give me minus of two theta sin theta has integration as minus of cos theta minus integration theta the differentiation 1 and sin theta has integration as minus of cos theta $d\theta$ which will finally give me minus minus plus 2 theta cos theta then minus two

so it will become minus minus plus and then this minus will make it minus two integration of cos theta integration of cos theta is nothing but sine theta and a constant of integration

so these substitutions now they will help us in getting back to the what is the value

so $\cos \theta$ is x and θ is equal to $\cos^{-1} x$

so $2x \cos^{-1} x$ from here minus 2 sin theta

so sin theta will be square root of $1 - \cos^2 \theta$

so $2 \sqrt{1 - \cos^2 \theta}$ that is one minus x^2 when plus a constant of integration

so with the substitution of θ when we are given inverse kinematic functions an with the substitution of one variable into the trigonometric function it sometimes helps to convert the integrand into another simpler form which we can easily evaluate an we will see this further in some more examples let us choose the following example

so let us take this example i is equals to integration of $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$ dx to solve this example we will use as we have used in the previous case only thing which we have to keep in mind is that that we should make the substitution of x in such a way that this entire term converts into a tan function

so that this tan inverse with that time function get cancelled an that i should keep in mind

so look at this form $\frac{1-x}{1+x}$

so i should use a formula where these ones should get cancelled

so if you if you notice and if you check then you have the formula for $\cos 2\theta$ in two different form you can write it as $1 - 2\sin^2 \theta$ or you can also write it as $2\cos^2 \theta - 1$

so if i make a substitution of x is equals to $\cos 2\theta$ i notice that one minus x over one plus x i can write as one minus and then here since i am writing it i have to cancel this one

so i should use this formula

so $\frac{1 - 2\sin^2 \theta}{1 + 2\cos^2 \theta - 1}$

so carefully if you look at it what you ultimately get here is this one gets cancelled an this one also gets cancelled in the numerator you will get sine square theta and denominator you will get cos square theta because two also gets cancelled with two

so you will get sine square theta over cos square theta

so $\frac{1-x}{1+x}$ becomes $\frac{\sin^2 \theta}{\cos^2 \theta}$ which is nothing but $\tan^2 \theta$ and this is what was our m

so making this substitution x is equals to $\cos 2\theta$ which will give you dx is equals to minus of two sine two theta $d\theta$

so making this substitution we find that the integrand i can be written as $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$ is $\tan \theta$ $\tan^{-1} \tan \theta$ multiplied by dx that is minus two sine two theta $d\theta$

so i can take this minus two outside of the integral $\int \frac{1}{\tan \theta} d\theta$ will give me $\ln |\sin 2\theta| + C$ and $d\theta$ this again is of the form of previous problem

so i will assume this as first and this as second function and then evaluate use integration by parts will not solve it completely use integration by parts to evaluate this integral and then ultimately convert from θ to x by using the relationship

so you can see that simplification helps when you solve problems ah by making certain substitutions which will make problem into a simpler form another problem of similar kind i will choose for you is this one suppose that we are only given this function to evaluate $\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx$

so from previous example you should at least get some idea that that what can be the substitution because $1 - \sqrt{x}$ and then $1 + \sqrt{x}$

so i think you can easily guess that \sqrt{x} is equals to $\cos 2\theta$

so that $1 - \sqrt{x}$ upon $1 + \sqrt{x}$ as we did in the previous problem for this will come $\frac{1 - \cos 2\theta}{1 + \cos 2\theta}$ in the numerator i will use the sine function the denominator i will use the cosine function

so that i will get it as $\frac{\sin^2 \theta}{\cos^2 \theta}$ just as in the previous problem we did you substitute it using that the relationship

so for numerator you use $\cos \theta$ is equals to $1 - 2\sin^2 \theta$ and for the denominator $\cos \theta$ you use $2\cos^2 \theta - 1$

so you will get the same term

so here if i take the differential what i will get is that one by two \sqrt{x} dx is equals to $-\sin 2\theta d\theta$ \sqrt{x} is already known to me as $\cos \theta$ therefore i can protect as dx is equals to $-4\sqrt{x} \cos 2\theta d\theta$

so $\cos 2\theta \sin 2\theta d\theta$ let us for the time being write it as $2\cos 2\theta \sin 2\theta \sin^2 \theta d\theta$

so we have evaluated this term and this is $\tan^2 \theta$

so the integrand i will be converted into integral of square root of $\tan^2 \theta$ that is tangent of θ multiplied by $-2\sin 4\theta d\theta$

so if you look at the evaluation it does not look that we can immediately evaluate it

so we have to go for further use of trigonometrical relationship

so two times integrals this $\tan \theta$ i can write it in terms of $\sin \theta$ upon $\cos \theta$ multiplied by $\sin 4\theta$ which i will write as $2\sin 2\theta \cos 2\theta d\theta$

so this will become -4 further you know you can write $\sin 2\theta$ as $2\sin \theta \cos \theta$

so that $\sin \theta$ over $\cos \theta$ multiplied by this two twos of four is already there $2\sin \theta \cos \theta$ into $\cos 2\theta d\theta$

so this $\cos \theta$ gets cancelled and what you are left with is $-8\sin^2 \theta \cos^2 \theta \sin \theta d\theta$ now you can solve it however you want to but one of the simplest way probably is that that you convert into $\sin 2\theta$

so it will become if i take -2 integration i write here $4\sin^2 \theta \cos^2 \theta$ and this nothing but $2\sin \theta \cos \theta$ that is $\sin 2\theta$ whole square $\sin^2 2\theta d\theta$ again you use the formula $\cos 2\theta$ is equals to $1 - 2\sin^2 \theta$

so here put θ is equals to 2θ

so that you will get one minus sorry

so that you will get $\sin^2 \theta$ as $1 - \cos 2\theta$ divided by two

so replace this minus two integration of $1 - \cos 2\theta$ divided by two $d\theta$

so ultimately this two gets cancelled and you will get here $\sin 2\theta$ minus $\cos 2\theta$ divided by four and plus a constant of integration and the assumption which you took was $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

so $\sin^2 \theta$ must be written into $\frac{1 - \cos 2\theta}{2}$

so that way you can convert

so that here you will have $\theta = \cos^{-1} \sqrt{x}$ and i mean similarly you can look for this $\sin \theta$ you write it as $\sqrt{1 - \cos^2 \theta}$ and then convert this using this \sqrt{x}

so because $\cos \theta = \sqrt{x}$

so you can finally write it as $1 - x$

so simplify it further to get the final answer and you can evaluate it now here using $\sin 2\theta = 2 \sin \theta \cos \theta$ and then substituting back the values of these θ s

so now i will move to another class of simple problems where a specific kind of function many times written with the exponential function e raised to the power $f(x) + f'(x) dx$

so if we are to evaluate this kind of problem many times it becomes very helpful if the product with exponential is written and we can recognize that the product which is written with exponential can be written in this form $f(x) + f'(x)$ i will show you with the example

so to evaluate this we break it into two parts let us say what happens $e^x + x e^x dx$ consider this as I_1 and this as I_2 and check for I_1

so I_1 is $\int e^x dx$ and evaluate it using by parts considering this as first function and this as second function

so the integral will come out to be $x e^x - \int e^x dx$ differentiation of the first x integration of the second e^x and plus of course constant of integration which finally will appear there

so now you can see that this is nothing but $x e^x$ and therefore I_1 is $x e^x + c$

so the integral I_2 now is written as $\int x^2 e^x dx$ for which the value is $x^2 e^x - 2 \int x e^x dx$ which gets cancelled

so finally the integral I_2 turns out to be $x^2 e^x - 2x e^x + 2 \int e^x dx$

so if a problem is of this kind that it is having exponential function and the product is written with exponential as $f(x) + f'(x)$ it becomes very easy to evaluate and we can directly use the formula that it is nothing but $e^x (f(x) + c)$ i will choose a simple example first very simple example for this application of this property

so choose this integral $\int (1 - x) e^x dx$

so you can think easily figure out that if 1 by x is $f(x)$ then -1 by x is $f'(x)$ and therefore the integral of this should come out to be $f(x) e^x$

so the integral is $(1 - x) e^x + c$

so this is the answer which you can see using the formula in one line you can also verify it by making the evaluation

so write I as $\int (1 - x) e^x dx$ into $\int 1 \cdot e^x dx - \int x e^x dx$ evaluate this integral

using this as first function and this as second function

so what you will get here is that the this as first function

so $1 \cdot x e^{\text{power } x} \text{ minus integration differentiation of the first is } 1 \cdot x \text{ square with a negative sign therefore it will make it plus } e^{\text{raised to power } x^2} \text{ integral } e^{\text{raised to } x} dx \text{ minus integration } e^{\text{raised to the power } x} \text{ upon } x \text{ square } dx \text{ and plus finally a constant of integration}$

so these two terms gets cancelled which finally gives you $e^{\text{raise to power } x} \text{ upon } x \text{ plus constant the same term which you got here using that formula}$

so that means that this formula ah becomes very handy and helpful when we solve ah the kind of examples ah sometimes it may not be given the problem may not be given directly in that form but after certain substitution we can get it in that form

so let us look at a couple of examples where we can make those substitutions

so take this example $i \text{ equals to integration of } \log \text{ of } \log \text{ of } x dx \text{ sorry } \log \text{ of } \log \text{ of } x \text{ plus } 1 \text{ by } \log x \text{ squared this is squared } dx$

so suppose that we have to evaluate this integral

so now since its

so many logarithmic functions are appearing here a natural choice seems that i should substitute $\log x$ to be some new variable and see what actually happens

so if i put $\log x$ is equals to t immediately i can see that $1 \cdot x dx$ equals to dt but since there is no x appearing in this expression therefore i should try and solve it for that x

so i know from logarithmic that logarithmic and exponential inverse function to each other therefore $\log x$ is equals to t also implies that if i solve it x is equals to $e^{\text{raised to power } t}$ and therefore this gives me dx equals to $x e^{\text{raised to power } t}$ which is sorry $x dt$ which is nothing but x is $e^{\text{raised to power } t}$ and therefore $e^{\text{raised to power } dt}$

so dx is equals to $e^{\text{raised to the power } dt}$ that now let me make those substitutions in the integral

so i becomes $\log \text{ of } \log x$ is t plus 1 by t is squared $\log \text{ of } t$ plus one by t square and then this is $e^{\text{raised to power } t} dt$

so writing it in this form i get it that $e^{\text{raise to power } t} \log t dt$ plus $e^{\text{raise to power } t} \text{ by } t \text{ square } dt$

so so far this expression has been converted into form this form ah but directly i do not see the application of the formula which we have used in the previous problem but what i can do here is that i can again use integration by parts for this factor and since logarithmic is present here and therefore i must choose this logarithmic as first function and this exponential as second function

so if i do that i will get $\log t$ integration of $e^{\text{raised to the power } t} z^{\text{raise to power } t} \text{ minus integration differentiation of } \log t \text{ is } 1 \text{ by } t \text{ integration of } e^{\text{raised to the power } t} \text{ is } e^{\text{raised to power } t} dt \text{ plus integration } e^{\text{raised to power } t} \text{ upon } t \text{ square } dt \text{ now let us club them together and see}$

so $e^{\text{raised to power } t} \log t \text{ minus integration } e^{\text{raised to the power } t} \text{ one over } t \text{ minus one over } t \text{ square } dt \text{ now look at this factor}$

so the problem initially involving logarithmic after use of the substitution and ah integration by part to q to the problem which has the application of the formula which i told you

so $c^{\text{raised to power } 1} \text{ by } t \text{ minus } 1 \text{ by } t \text{ square}$ same problem which i solved in the previous example

so you know that with the help of that example is that this is $f(t)$ and this is $f'(t)$ therefore the integration will be $e^{\text{raised to power } t} \text{ into } f(t) \text{ plus constant}$

so let us write that here $e^{\text{raise to power } t} \log t \text{ minus the integral of this}$

fellow will be e raised to power t into $f t$ is one by t plus a constant putting the values back t is equals to $\log x$ and x is equals to e raised to power t c raised to part is $x \log t$ is obviously \log of t is $\log x$ minus e raise to power t is again x into 1 by t is one by $\log x$ and plus constant c

so this is the the solution or answers for this particular problem which we dealt with

so substitution then application of this prob this formula which we studied ah leads to the solution of this particular problem another simple problem for you which uses the same idea is to find out the integral of e raised to power x 1 plus $\sin x$ over 1 plus $\cos x$ at first glance it does not look like a function and fits derivative but if you look at it carefully then you can see that you can convert it into a certain form and you will see that how we will do that one

so first of all we will write this trigonometrical expression 1 plus $\sin x$ over 1 plus $\cos x$ and convert into using those same angle formulas

so one plus $\sin x$ i will write one i will write $\cos^2 x$ by two plus $\sin^2 x$ by two $\cos^2 x$ by two plus $\sin^2 x$ by two plus two $\sin x$ by two $\cos x$ by two

so this is one and this is $\sin x$ whole divided by one plus this $\cos x$ i will convert into two $\cos^2 x$ by two minus one

so that this one gets cancelled and what you will get here is that $\cos x$ by two plus $\sin x$ by two whole squared divided by $\cos^2 x$ by two one by two here and introduce this $\cos x$ inside here

so that you will get one half of one plus $\tan x$ by two whole squared if you get ah this term one plus $\tan x$ by two whole square now we expand this function because still we did not reach to where we want to

so now we expand this function in the following as 1 plus $\tan^2 x$ by 2 plus $2 \tan x$ by two and notice that one plus $\tan^2 \theta$ is nothing but $\sec^2 \theta$

so that this becomes half $\sec^2 x$ by two two two gets cancelled plus \tan of x by two

so the trigonometric function which is in the integrand can be written in this form now notice this factor if i consider this as $f x \tan x$ by 2 as $f x$ then derivative of $\tan x$ by 2 is $\sec^2 x$ by two multiplied by one by two

so this factor is $f' x$

so after little manipulation with each to that factor that that i can write integrant i equals to which is this fellow e raised to power x multiplied by this factor

so this factor i have simply computed into this form i will write it as $\tan x$ by 2 plus half of $\sec^2 x$ by 2 and then dx and now i know this is $f x$ this is $f' x$ therefore using the formula previous case e raised to the power x $f x$ plus $f' x$ will give you e raise to power x $f x$ plus constant as the solution

so you see this complicated looking problem and we simplified into certain using certain relationships we can get this one to here and then using that formula we solve it

so that's the that's the answer for it finally we shall look at one more example 1 plus x plus 1 plus x minus 1 by x e raise to power x plus 1 by x $d x$ in order to evaluate this integral you see we need to look at it from a different angle because if we choose x plus 1 by x as t directly it will give 1 minus 1 by x square $d x$ is equals to $d t$ which does not appear here

so let us first break it into two parts keep one separate and keep x minus one by x separate

so that we can write it x plus one by x dx and the second part will write it as x minus 1 by x e raised to power x plus 1 by x dx now we will try to deal with

this second factor first and choose $x + 1$ by x as some new variable t
so that $(x + 1)^2 dx = dt$ which if I look at it carefully is same as $x^2 dx = dt$ here if I look at this factor

so it gives me $x^2 - 1$ upon x

so what I need to do is that I multiply and divide by x

so that I can get that factor but that is not that is that is something which is which is at this moment is not present here

so I will do that I will multiply and divide by that factor multiply by x and divide by x

so if I do that what I will get here is $(x^2 - 1)/x$ multiplied by x is x^2

so look at this carefully leave this numerator x right now you can see that that if you take this as t then that the part in the new part in the integrand times dx is dt and therefore the integral of this fellow is possible

so I should consider it as second function x as first function and apply integration by parts that's the idea which

so will put it in this way $e^{x+1} dx$ and then plus first function x integration of the second

so it becomes e^{x+1} and this entire factor will become $x dx$

so integration of e^{x+1} is the same as integration of ae^{x+1}

so it will be e^{x+1} you need to compute it separately the claim is $(x-1)/x \cdot e^{x+1} dx$ is equal to e^{x+1}/x can be obtained by substituting this

so this you need to separately calculate minus differentiation of the first one and then again integration

so you will get e^{x+1}/x now look at them carefully they are same therefore this cancels with this sorry I missed a constant equation

so this is the same as this fellow and therefore it looks $e^{x+1}/x + c$

so certain substitutions certain change of variables helped us in evaluating these integrals and we learnt and we saw that how we can convert them into a form which is comfortable for us which is which we can easily handle

so ah with this we come to end of this lecture ah practice with more problems and get yourself comfortable with them thank you you