

welcome students in last class we saw introduction to a method which is known as method of integration by parts in that method we saw that a product of two functions can be integrated by using a certain formula we develop the formula we saw an example also

so let us write back the formula

so for two functions $f(x)$ and $g(x)$ the integration of the product is equals to $\int x \text{ integration of } g(x) dx - \text{integration differentiation of } f(x) \text{ which is } f'(x) \text{ integration } g(x) dx$ and then dx

so this we call method of integration parts

so it is one of very useful technique to use this method and we will see in some time that how ah good we can use this method to solve certain integrals if you remember or recall the previous class we considered this function as first function and this function as second function

so it helps in memorizing ah this particular formula

so what it says is that integration of the product of two functions which we will call as first into second as first function integration of the second minus differentiation of first integration of the second and then whole integration

so this is how we recall that integration of the product first function integration of the second minus differentiation of first integration of the second

so i will keep on referring this formula in that manner if i look at some more example

so let us try for the integration of $x^2 e^x$ raised to power x now again i can use the same technique and i call this as first function and this as second function

so the integral will become first x^2 integration of the second function e^x raised to power $x dx$ minus integration differentiation of the first function this is $2x$ integration of the second function $e^x dx$ and then dx

so this entire thing becomes $x^2 e^x$ integration you know it is e^x as i told you we do not need to put a constant here minus two can be taken outside integration $x e^x$ has integration as $e^x dx$

so we reach to this $x e^x$ we have already evaluated

so now we know that we can evaluate it further

so same method as we used in the previous example twice put a square bracket here again you consider this as first function and this is second function

so first function integration of the second minus differentiation of first integration of the second and the whole integration

so this entire thing will take you to $x^2 e^x$ minus twice of x integration of this function will become e^x minus integration $e^x dx$ and then finally $x^2 e^x$ minus $2x e^x$ plus two integration will become e^x and then a constant of integration you could have also added a constant of integration here and finally it will be this

so the integral of this function $x^2 e^x$ comes out to be this let me take another example for you which involves trigonometric functions

so let us evaluate $x \sin 3x$ again we take this as first function and this as second function now i think there should be a question in your mind that i mean why are we choosing this as first function and this as second function

so we will see that in a moment that there should be a proper choice made for first function and second function many times we choose the function as first function which helps us in reducing terms in the integral since the formula goes like first function integration of the second minus differentiation of the first integration of the second

so we have to keep couple of things in mind
so here in this case if i take x as first function then the derivative vanishes
so that is the thing which i am keeping in mind but there are few other things
also which we should keep in mind

so let us first solve this example

so here the integration says first function integration of the second minus
integration differentiation of the first integration of the second for
integration this gives me x integration of $\sin x$ i know is minus of cosine x and
this function is a linear term therefore i can write the derivative in the
denominator

so this becomes minus $\cos 3x$ over three

so we already have talked about this one that integration of f of $a x + b$ $d x$
is capital f of $a x + b$ by $a + c$ provided we know that integration of f
 $x d x$ is capital of $f x$

so we know that integration of $\sin x$ is minus of cosine x therefore it will
become cosine $3x$ divided by the derivative of this fellowships

so i have used this property you can even substitute $3x$ is equals to t and
then you can integrate it its not a problem now minus integration one times
integration of $\sin 3x$ is again minus $\cos 3x$ by three $d x$

so this term goes to minus of $x \cos 3x$ by three minus minus plus one by
three integration of $\cos 3x d x$ a little care should be taken at the
beginning that is why i am writing all these steps ah

so once you get acquainted with this calculation you can skip couple of steps
and you can write them

so minus $x \cos 3x$ by three plus one by three again integration of the
cosine $3x$ i know it is using the same formula i know integration of $\cos x$
is $\sin x$ and therefore this becomes $\sin 3x$ by three plus the constant of
integration

so it becomes minus $x \cos 3x$ by three plus one by nine $\sin 3x$ plus
the constant of integration

so integration of $x \sin 3x$ is this now can we continue with the similar
process let us choose another example this example will motivate us to find out
or to figure out that which function should be chosen as first one is which
function should be chosen as second function

so assume that that we have to find out the integral of $x \log x$ what if i use
my similar trick

so i would have called i will write it here $x \log x d x$

so if i do not ah do anything new i will follow the similar procedure i will
call this function as first function and this function as second function as i
told you this is vanishing we have to see whether that idea really works here

so this will go to the first function integration of second function minus
integration differentiation of the first function integration of second function
 $d x$

so this ultimately becomes x integration of $\log x d x$ minus integration of \log
 $x d x$ and then $d x$

so the problem of finding this integral i took us to this form and which has
become further more complicated for us at this stage because we really do not
know what is the integral of $\log x$

so at this stage we do not know what is the integral of $\log x$ and therefore we
cannot move further

so making a choice of logarithmic x as second function and x the the choice for
the first function as it was vanishing it did not help us here

so it is not always the case that ah choosing a polynomial function ah which
vanishes will help you it depends on other functions also which are present in

the integrand as a product

so in this case let us try $\log x$ as first function

so let us call $\log x$ as first function and x as second function then see what happens

so $\log x$ first function integration of the second that is integration $x dx$ minus differentiation of the first that is $\log x$ differentiation

so differentiation of $\log x$ is one by x multiplied by integration of the second so $x dx$ and then whole integration this leads us to $\log x$ into integration of the second x^2 by two minus integration one by x multiplied by this integration is x^2 by two dx

so notice carefully this x cancels with x here

so we should get x^2 by 2 $\log x$ minus 1 by 2 integration of $x dx$ and this i know

so this x^2 by two $\log x$ minus one half of x^2 by two the constant of integration which is coming throughout

so this will give me x^2 by two $\log x$ minus x^2 by four and then plus constant of integration

so the evaluation of this integral could be easily done if we choose $\log x$ as first function and x as second function while in this case if we choose x is first function in $\log x$ as second function we reach to a problem where we have to find out the integral of $\log x$ which we do not know at this stage

so making a choice of the function is really important if you make a choice which is wrong then you may reach to the the evaluation of the functions which will become very complicated as we saw in the case of $x \log x$

so it was very simple looking function but the integral of $\log x$ was creating triple for us

so the idea will be mostly to choose the function for which the integration does not ah go on or it does not keeps on becoming longer and longer and similarly ah if we if we look for that kind of idea then we can easily figure out that which function we should choose

so i will give you another example here

so suppose that we have to find out the integration of $x \sin^{-1} x$

so this involves algebraic function and inverse trigonometric function

so with the previous experience we know that integration of $\sin^{-1} x$ this will be troublesome

so therefore we will call this $\sin^{-1} x$ as the first function and this x as the second function

so that the integral becomes $\sin^{-1} x$ into integration of the second is now i can write it directly x^2 by two minus integration differentiation of the sine x is one by square root of one minus x^2 into integration of the second is x^2 by two dx

so this leads to the integration $x^2 \sin^{-1} x$ by two minus one by two x^2 or one minus x^2 square root dx

so if i write it as $x^2 \sin^{-1} x$ by two minus one half and this integrand i write it as integration of minus of one minus x^2 plus one

so what did i do here i have added one and i have subtracted one

so minus one plus x^2 plus one

so one gets cancelled

so i will get x^2 divided by square root of one minus x^2 dx this i can further write as $x^2 \sin^{-1} x$ by two minus one half this integral will be broken into two parts using linearity property

so $1 - x^2$ divided by square root of $1 - x^2$ will be giving me $1 - x^2$ with the negative sign

so this sign will become plus then again minus half multiplied

so minus half integration one over square root of one minus x squared
 so i shall evaluate this integrals into two separate parts i will call this
 integral as i one and this integral as i two
 so now i will write for i one i one is square root of one minus x square d x
 and i two is one over square root of one minus x square d x we know for the case
 of i two
 so this is nothing but sine inverse x for the case of i one we have we may have
 to go for some trick
 so we need to evaluate this integral this integral we do not know how to
 evaluate
 so what we need is that we have to think of some substitution ah well as it
 looks like 1 minus x squared dx and a similar integral of this kind we have
 evaluated earlier
 so it looks like a good choice if i put x is equals to sin theta
 so that i will get here one minus sin square theta which is cos square theta
 so square root of one minus x square will become square root of one minus sin
 square theta which is nothing but cos theta and d x in that case will become cos
 theta d theta
 so ultimately the integral i am looking at will become integration cos theta
 into cos theta d theta because its one minus x square which is cos theta then d
 x which is cos theta d theta
 so i one becomes cos theta into cos theta d theta which is nothing but cos
 squared theta this i think you can guess now that we can evaluate since it is a
 having quadratic term here
 so first of all we have to convert it into a linear form of cosine fortunately
 we know the formula is cos of two theta is equals to two cos square theta minus
 one
 so which will give me cos square theta is equals to cos two theta minus one by
 two
 so i will replace it by that factor and then
 so i will get one half as outside and this will become cos of two theta minus
 one d theta which ultimately will lead me to one half of cos two theta
 integration
 so integration of cosine again that property i will use having a linear
 function here
 so integration of cosine theta i know is sine theta and divided by 2 the the
 differential of this factor minus integration of 1
 so that will i get as theta but what is theta theta is i can solve it from here
 theta is nothing but sine inverse x and therefore this is written as 1 by 4 sine
 2 theta
 so sine of twice of sine inverse x
 so minus of theta by two
 so theta is nothing but sine inverse
 so minus of one by two sine inverse x
 so i will simplify this further for you when we will be looking at
 so our integral ah if you look at it the final integral for this particular
 problem this turns out to be this long calculation which gives you these factors
 so in the next space i shall write it down for you what is my integral i
 so my integral i ultimately is look at x square sin inverse x by 2 x squared
 sine inverse x by 2 minus half of i one minus half of i one which i will
 substitute minus half of i two
 so i two we could easily see it is nothing but the sine inverse x and then plus
 a whole constant of integration now look back at this factor half of sine to
 sign inverse

so it is not reaching to any form which which which we know
 so what we do is that we will write it in the form which we know or which we
 can further simplify
 so you know sign this term is equivalent to $\sin^2 \theta$
 so therefore you can write it as let me simplify it here for you to $\sin \theta$
 $\cos \theta$ $\sin \theta$ you already know is x
 so it will become $2x \cos \theta$ you can write it as $1 - \sin^2 \theta$
 θ which you can write as $2x \sqrt{1 - x^2}$ simply you
 can also simplify this also $\sin^{-1} x$ will be twice of $\sin^{-1} x$
 $\sin^{-1} x$ into $\cos^{-1} x$ and then $\cos^{-1} x$
 can be written as $1 - \sqrt{1 - x^2}$ which is nothing
 but the same as this term
 so ultimately we will get the same expression which we got from here
 so this is $\frac{1}{4} \int 2x \sqrt{1 - x^2} dx$
 so this expression is $\frac{1}{4} \int \sqrt{1 - x^2} dx$
 half of $\sin^{-1} x$
 so the inter integral $\int \frac{1}{\sqrt{1 - x^2}} dx$ which we evaluated using this procedure is pretty
 long ah we got this one
 so when we simplify this we get finally $x \sin^{-1} x - \frac{1}{2} \sqrt{1 - x^2}$
 one half of this is two by four that will make it one by two
 so this will make it $x \sin^{-1} x - \frac{1}{2} \sqrt{1 - x^2} + C$
 of $\sin^{-1} x$ and then minus half of $\sin^{-1} x$ plus constant
 so this is the final integral we evaluated after doing this entire procedure
 so we can with the help of this integration by parts we can and using other
 techniques also which we have learnt already we can figure out the integral of
 certain functions now we come to the important question of choice of first and
 second function which means which function we should call as first function and
 which function we should call as second function when we are applying this
 particular formula
 so for that mostly we look at the combination of the functions it depends on
 problem to problem but as i told you that it should not complicate your integral
 ah because you can see in the formula it says $\int f(x) g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$
 so if i take a form function which will have a integral which is very
 complicated or if it keeps on extending then i will be in trouble because it
 again involves another integral here
 so integral of the integral
 so that may create problems
 so i should be choosing the function wisely ah where the integral because i
 already do not know the integral of the entire product and if i choose something
 which will further make it complicated then i will have trouble
 so i mostly should choose the function which should not become very complicated
 there is a convention which says that that if you have a combination of the
 functions which involves your say inverse trigonometric functions logarithmic
 functions algebraic say polynomial etcetera then trigonometric inverse
 trigonometric and trigonometric and then exponential exponential usually do not
 create much problem
 so they say that this should be the the convention says that that this usually
 should be the ah order of the selection that you should first try to choose ah
 inverse gnetric function as first function if it is present and this should be
 in the order and logarithmic is first function
 so we have seen the case where we have to integrate $x \log x$
 so in this case you know algebraic function was present and logarithmic was
 present
 so according to this order i should have chosen logarithmic first and then

algebraic

so you saw that when i saw that example that that if i choose this as first function and this is second function the integral becomes very easy and it was easily hand we were able to handle it easily similarly we saw the case for $x \sin^{-1} x$

so here also the inverse function was he first and this was second

so this also falls in this order you see inverse trigonometric and then algebraic

so inverse trigonometric was chosen first and then algebraic function was chosen as second function and ah this is the the order of function which should be kept in mind while evaluating though for simple functions many times it may not be that difficult to evaluate even if you do not follow the order but if a function becomes very complicated it really will be difficult to evaluate i will show you that that for this particular integral even if you choose this as first function this can be evaluated but then it will be little complicated to evaluate

so now we shall look at the important application of this method which will help us to evaluate certain integrals which will be very difficult for us to integrate using the known methods

so will take one example for you integral of $\log x \, dx$ this example will also help us in evaluating the integral which we started with

so what we do is that we understand it as one times $\log x \, dx$ you know one multiplied by a function makes it same function and then we apply the ordering method that which function should be selected as first function and which function should be selected as second function

so here since log thing is present therefore i should consider it as a first function and this algebraic function one is a constant function z for itself the break function should be considered as second function and if i do that integration by part it will give me $\log x$ integration of one i can write it directly now x minus integration differentiation of $\log x$ it should be one by x again integration of one gives me $x \, dx$ this entire thing will give me $x \log x$ minus x because x gets cancelled

so here i will get one x one dx which will give me x and then finally a constant of integration

so i got this important result $\log x \, dx$ as $x \log x$ minus x

so if i want to go back to the integration of $x \log x$ as i told you that now if i choose this as first function and this as second function and since it is a simple function i can if i do not care about the the ordering

so this will lead me to x integration of $\log x$ minus integration differentiation of x one integration of $\log x$ call this as entire function as i

so integration of $\log x$ now i know integration of $\log x$ is $x \log x$ minus x

so i can substitute this value here

so what i will get now $x \times \log x$ minus x as i told you earlier that constant we can ignore here at this stage integration of $\log x$ is again replaced by $x \log x$ minus $x \, dx$

so it goes to integration this further goes to $x^2 \log x$ minus x^2 minus integration of $x \log x$ and then plus x^2 by two and then finally a constant of integration now if you look at it clearly this term $x \log x \, dx$ is nothing but our integrand itself and therefore i can write it i

so i will get $x^2 \log x$ minus x^2 by two plus x^2 by two will give me x^2 by two minus i plus c

so the expression i got now from here is that that i left hand side equals to $x^2 \log x$ minus x^2 by two minus i plus the constant transfer i to the left hand side

so that you will get twice of i and then divide throughout by two
 so you can directly write it as i equals to
 so i have transfer this i to the left hand side
 so that it became two i and divided throughout by two
 so $x^2 \log x$ by two minus x^2 by four plus this constant c by two i
 can write it another new constant c one which further can be understood as the
 constant since it is a constant it does not really matter that what arbitrary
 constant you should choose
 so ultimately the solution should look like of this form
 so for this function if i know the integral of $\log x$ this can be evaluated
 but you can see that how complicated ah the calculation had been if we divert
 from the ordering of choice of the functions i will give as another example for
 you which i shall not solve you look for this example evaluate $x^2 \log x$
 x use both the x techniques which i told you first you consider because here
 ordering says that that $\log x$ should be considered as first function and this
 algebraic x^2 should be considered as second function
 so first you choose it as first function and x^2 as second function and
 then integrate and then in second case what you do is that you choose this
 function as first function this function as second function and then integrate
 and see the difference in the calculation and try to learn that
 so next example for another complicated function is integration of $\tan^{-1} x$
 x
 so for this also i will use the same trick which we used for logarithmic x
 consider this $\tan^{-1} x$ multiplied by $\tan^{-1} x$ write this
 function since it is inverse trigonometric function therefore this should be
 considered as first algebraic function should be considered as second
 so this will be $\tan^{-1} x$ integration of the second will be x minus
 integration differentiation of the $\tan^{-1} x$ will give you one over one plus
 x^2 integration of one will give you x and then finally dx
 so you see how simple it has become $x \tan^{-1} x$ minus integration x over
 one plus x^2 dx with our practice of problem
 so far i think you can easily see that numerator is nothing but the
 differential of the denominator
 so what you do is that you take denominator one plus x^2 as a new variable
 t
 so that you get two $x dx$ is equals to dt that means $x dx$ is equals to dt by
 two
 so this integral can be easily evaluated $x \tan^{-1} x$ minus one half
 integration of dt by t and which leads you to the logarithmic function
 so this will give you $x \tan^{-1} x$ minus half logarithmic of $\text{mod } t$ plus
 constant of integration which finally leads you to the final answer $\tan^{-1} x$
 x minus half logarithmic of $\text{mod of one plus } x^2$ and plus constant c
 so the integration of $\tan^{-1} x$ can be easily found in this fashion
 so give you another homework exercise try to figure out what will be the
 integral $x \tan^{-1} x$ choose the first function and second function yourself
 and evaluate this integral
 so if you have noticed when we were finding out the integral of $x \log x$ we used
 the idea there we wrote this i function and when this integration process
 proceeded we found that that the the relationship of the integral turns out to
 be a certain relationship with the original integral itself this trick actually
 sometimes becomes very handy and we can use it in solving certain problems
 so for those class of functions where ah integrations of both kind of functions
 will keep on giving you ah the functions which which will not be diminishing in
 those cases this idea works pretty well

so look at look at the example i will tell you how it can be worked out
so look at the example of taking the integral of $e^{\cos nx}$ and let me call it as I_1 let me call it $e^{\sin nx}$ here m and n can be any number

so according to the the particular numbers you can solve them as a particular case we are considering them as any numbers m and n which are obviously non-zero

so you know exponential function if you either differentiate or integrate you will get another exponential function cosine function if you differentiate or integrate you will get either cosine functions you will get sine function similarly sine function you will get cosine function

so for these two functions I_1 and I_2 i will look for the solution for one of them and i will choose I_2 as the function which i will solve for you similarly you can solve for I_1

so let us write this I_2 and choose $e^{\cos nx}$ as first function and second functions

so since it is a trigonometric function $\sin nx$ i should choose it as first function and according to my order i should choose exponential as the second function

so the integral will give me $\sin nx \cdot e^{\cos nx}$ divided by $m - 1$ integration differentiation of the first

so it is $\cos nx \cdot e^{\cos nx}$ by m integration of the second dx first function integration of the second differentiation of the first integration of the second dx

so this will give me $e^{\cos nx} \sin nx$ upon $m - 1$ this is n by m integration of $e^{\cos nx} \sin nx \cdot dx$ further let us integrate it again using integration by part considering this as first function and this is second function

so what i will get is that $e^{\cos nx} \cos nx$ divided by $m - 1$ n by m first function $\sin nx$ integration of the second $e^{\cos nx}$ by $m - 1$ integration differentiation of the first

so it is $m - 1$ $\sin nx$ integration of the second $e^{\cos nx}$ upon m and then whole integration

so when we simplify it further what we will get is I_2 equals to $e^{\cos nx} \sin nx$ upon $m - 1$ you can see n by m square n by m squared $e^{\cos nx} \cos nx$ then n by m multiplied by this n by m this minus sign will make this sign as plus and then ultimately this minus sign will make

so throughout the sign i will get as a minus sign n into n square m into m square

so n square over m square integration of $e^{\cos nx} \sin nx \cdot dx$ look carefully what you have obtained here is the same as the integral which you started with

so i can replace it by I_2

so that i will get here on the left hand side i will transfer everybody

so $I_2 - 1$ minus of n square minus m n square by m square will come on the left hand side will become plus n square by m square equals to if i take this m square as lcm here will get m square

so that m $e^{\cos nx} \sin nx$ minus n $e^{\cos nx} \cos nx$ simplifying this will ultimately lead to the I_2 which is nothing but the integration of $e^{\cos nx} \sin nx \cdot dx$ equals to m square m square that cancels here you will get m square plus n square which will come in the denominator here one by m square plus n square m $\sin nx$ minus n $\cos nx$ into $e^{\cos nx}$ raised to power m x $e^{\cos nx}$ is common $e^{\cos nx}$ raised to the power m x common m $\sin nx$ minus n $\cos nx$

so this is what ultimately you will get when you simplify this

so similar procedure you can apply for the case when it is e raised to $r \sin x$ and $\cos x$ and then similarly you will get the similar formula

so what is important here to note is that sometimes you can choose these functions and then if you perform integration by part repeatedly you may get the same function and then you transfer that same function to the left hand side and then simplify that equation solve it for that unknown function and then you can get the integral of that function

so this method becomes very handy for some particular problems next I will solve for another special function which I used in another earlier problem so when I was solving the problem of $x \sin^{-1} x$ for u I obtained this function one this kind of function in fact can be solved in general and found their integral

so let us look at the function of square root of $x^2 - a^2$ similarly I can do for $a^2 - x^2$ and $a^2 + x^2$ I will do for $x^2 - a^2$ and then the rest of them I will tell you the formulas for them

so suppose that that we have to integrate this integral

so what you do is that we use the same technique as we use for the logarithmic function we write it as integration of one times $x^2 - a^2$ if you notice the earlier case it was one minus x^2

so the same technique which I am using here can be used in that case also we used there a substitution $x = a \sin \theta$

so to solve this particular form we use substitution $x = a \sin \theta$ and then we evaluate it but here we are using integration by parts

so consider this function as first function and this function as second function as usual

so that you will get $x^2 - a^2$ integration of one will give you x^2 by two sorry x integration of one will give you x minus integration of square root of $x^2 - a^2$ will give you $2x$ upon $2\sqrt{x^2 - a^2}$ this is the differentiation of first function multiplied by integration of the second will give you x this two gets cancelled with this two and here you will get x^2

so that you will get x^2 root of $x^2 - a^2$ minus integration of x^2 over $x^2 - a^2$ square root $d x$

so further if we look at it we can simplify

so you keep on writing the first term as it is minus if I add and subtract here a^2 then I can get a factor out of it

so I will do this $x^2 - a^2 + a^2$ divided by square root of $x^2 - a^2$ easily you can see $x^2 - a^2$ here can be divided by square root of $x^2 - a^2$

so I will get two integral terms here which will give me first term x^2 root of $x^2 - a^2$ minus sign $x^2 - a^2$ divided by square root of $x^2 - a^2$ integration sorry integration square root of $x^2 - a^2$ $x^2 - a^2$ divided by square root of dx and this this minus goes with this

so it will be minus a^2 can be taken as common integration of one by square root of $x^2 - a^2$ $d x$ look at this I is the same as square root

so therefore this term I will replace with I

so my left hand side is I this function is also I therefore the left hand side become now $2I$ is equals to x^2 root of $x^2 - a^2$ minus a^2 square integration of $1/\sqrt{x^2 - a^2}$ is already known to us and that is the function logarithmic of x plus square root of $x^2 - a^2$ a square and then finally a constant of integration

so finally this integral it turns out to be $x \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$

so finally the integral which we have evaluated it will write it for you $\int \frac{dx}{\sqrt{x^2 - a^2}}$ is $\ln|x + \sqrt{x^2 - a^2}| + C$ logarithmic of $|x + \sqrt{x^2 - a^2}|$ plus constant of integration the other formula can also be similarly found and I will write them for you which you should evaluate integration of $\frac{1}{\sqrt{a^2 - x^2}}$ which was close to one minus x^2 is equals to $\frac{1}{a} \sin^{-1} \frac{x}{a} + C$ this is the one which we observed in in the example of $\int \frac{1}{\sqrt{1 - x^2}}$ is integration square root of $x^2 + a^2$ $\int \frac{dx}{\sqrt{x^2 + a^2}}$ equals to $\ln|x + \sqrt{x^2 + a^2}| + C$ square root of $x^2 + a^2$ plus $\ln|x + \sqrt{x^2 + a^2}|$ plus constant

so these three important formulations they they will help you in evaluating the integrals certain integrals depending on what kind of form they are we have already seen in case of the functions which were of the form $\frac{1}{\sqrt{ax^2 + bx + c}}$ with a square root that this can be converted into one of the forms either $\sqrt{x^2 + k^2}$ or even if a is negative then $\sqrt{x^2 - k^2}$ kind of form

so similarly here if you are given a function which is integration of $\frac{1}{\sqrt{ax^2 + bx + c}}$ we can try to convert them into one of these three forms and then we can evaluate these integrals

so these three important formulas they can be helpful for solving the integrals as I showed you I will give you a quick example let us try to find out the integral of square root of one minus four x minus x^2 $\int \sqrt{1 - 4x - x^2} dx$

so you can easily see that you can write one minus four x minus x^2

so you write it as $1 - 4x + x^2$

so this $x^2 + 2x$

so if I add 4 here and subtract 4 here

so this ultimately takes one and this becomes $(x + 2)^2 - 5$

so one minus four will become one plus four that is five minus $(x + 2)^2$ square

so the integral I can write as integration of $\sqrt{5 - (x + 2)^2}$ $\int \sqrt{5 - (x + 2)^2} dx$

so now I can use directly the formula since it is a linear factor I can use it directly that formula by making a substitution here $x + 2 = t$

so that the integral becomes square root of five minus t^2 $\int \sqrt{5 - t^2} dt$ because here you can see $dx = dt$

so this is of the form $\int \sqrt{a^2 - t^2} dt$

so one of our previous formula $\int \sqrt{a^2 - x^2} dx$ that will work here

so I will get integral as $\frac{t}{2} \sqrt{a^2 - t^2} + \frac{a^2}{2} \sin^{-1} \frac{t}{a} + C$ that is $\frac{5 - t^2}{2} \sqrt{5 - t^2} + \frac{5}{2} \sin^{-1} \frac{t}{\sqrt{5}}$ replacing t by $x + 2$ we get $\frac{(x + 2)^2 - 5}{2} \sqrt{5 - (x + 2)^2} + \frac{5}{2} \sin^{-1} \frac{x + 2}{\sqrt{5}}$ that is $\frac{1}{2} (x + 2)^2 \sqrt{5 - (x + 2)^2} - \frac{5}{2} \sqrt{5 - (x + 2)^2} + \frac{5}{2} \sin^{-1} \frac{x + 2}{\sqrt{5}} + C$ and this is same as $\frac{1}{2} (x + 2)^2 \sqrt{5 - (x + 2)^2} - \frac{5}{2} \sqrt{5 - (x + 2)^2} + \frac{5}{2} \sin^{-1} \frac{x + 2}{\sqrt{5}} + C$

so I will directly replace this one $1 - 4x + x^2$ plus five by two $\sin^{-1} t$ is $x + 2$ by root five and then plus constant this should be $\frac{1}{2} (x + 2)^2 \sqrt{5 - (x + 2)^2} - \frac{5}{2} \sqrt{5 - (x + 2)^2} + \frac{5}{2} \sin^{-1} \frac{x + 2}{\sqrt{5}} + C$ that's the answer

so with this we come to end of this lecture

so in today's lecture we learnt different techniques of using integration by parts and we saw that how it can help us in solving certain integrals which we cannot solve by using any known techniques which is known to us till this point in further we will see some more examples thank you