welcome students today we shall learn
some more formula and derive them   these integrals will basically have a certain
way to derive them ah you must note how we derive   and what a kind of technique that we use to derive
them because that will help you when you are   finding some more difficult integral for which
you may not know the formula
 so the technique   ah in itself will be very useful while finding integrals
 so we shall look for   the integrals which will ah contain the
algebraic expressions or the polynomials   as a first example i will take the integral of a particular form d x by x square plus a
square
 so this looks very similar to one of the ah   integral which we have seen d x over x square
plus one
 so first i will show you what technique   we can use in order to derive this integral so
what we do is that if we substitute x equals to a tan t then this will give us dx is equals to a sec
square t and then the unknown integral i becomes a sec square t sorry d t by x square plus a square
 so since x is a tan
t therefore x square plus a square equals to   a square tan square t plus a square though a
square can be taken as common and this will   give you tan square t plus one you know one plus
tan square theta is sec square theta therefore we   can use the trigonometrical identity this will
give you a square sec square t
 so we get here a squared sec square t and d x is a sec square
t
 so ultimately the two terms will get cancelled   here sec square t and sec square t a will
get cancelled with a and this you can write   as outside the integral because it is a scalar
therefore this can be written as integral of d t
 so you see it has been converted into a very
simple form
 so i can write it as one by a integral   of d t is t plus a constant of integration which
could have been written as c by a and then again   a new constant c one
 so i am
directly writing it as a constant c   and t i already know is that ah
from the relationship of a x and   ah t is that that x is equals to a tan t
therefore t will be x by a ah tan inverse x by a
 so this i will get one by a
and t equals to tan inverse   x by a plus c this we have to always keep in
mind that original integral is with respect to x   and therefore final answer should be a
function of x
 so ultimately when we reach to   t by a we have to replace this t into
the substitution which we made here ultimately the formula which

which which becomes is that   it is important formula to note now d d x
upon x square plus a square equals to one by a tan inverse x by a and plus of
course the constant of integration next we consider another example
of integral d x over x square   plus a square square the root
so now we are going to integrate d x over square root of x square
plus a square again as i told you   the way we have used in the previous case ah
this
was d x over x square plus a square
 so can we use   the same technique let us let us see what will
happen if i substitute here x is equals to a tan t
 so this will again similarly we will
get d x is equals to a sec square t d t
 so that ah finally we will
require t is equals to ah tan inverse x by a or we will see that
for whether we need that relationship   at a later stage and how shall we need
that relationship at a later stage
 so the integral now turns out to be d x is a sec square t d t this will become
in the previous ah problem
we have evaluated that x square plus a square   is same as a square sec square t
therefore this
will become square root of a square sec square t which will be same as a sec t
and
therefore finally we shall get sec t dt
 so we can use the formula directly
 so we
already know integration of sec t d t is same as log of mod of sec t plus tan t
modulus
plus constant
 so using this formula here we will find that this is log of sec t plus tan t
plus the constant
now we have to again go back from t to   x and if i substitute this t is equals
to
tan inverse x by a here i shall get x by a   but here i will get a complicated
relationship sec
of tan inverse x by a
 so what i should do is that   i should not substitute this t directly rather
i should use another trigonometric relationship   with sec and tan which is we
know that sec square t can be written
as one plus tan square t   and therefore sec t can be
written as square root of one plus tan square t
 so instead of set t i
will replace that
 so this i will get log of square root of one plus tan square t plus tan
t plus constant and finally now you can easily   see from here is that that the
relationship will
become tan t
 so t is equal to 10 inverse x y a   will become tan of tan inverse x by a
inverse
function operator
 so we will get x by a here   and here tan inverse tan of tangent x by a
squared
 so that will give you x by a square   square root of one plus x square over
a square plus x by a plus constant   which after further some simplification i
can

write here as ah this a square plus x square   plus x whole divided by a and you know log of

m by n goes to log m minus log n therefore that   log a constant that can be written separately so

this finally expression in terms of a on x is only   will be looking like log of x plus

square root of x square plus a squared minus log of mod a plus c so

finally this calculation leads to log of mod x plus square root

of x square plus a square   and this log a is anyway constant

 so adding with

another constant and renaming this with constant   c 1 you know the constant c 1 does not matter what

you call this

 so ultimately this constant c since   it is arbitrary we can keep it with the same name

as constant c

 so this turns out to be d x over x   square plus a square square the root becomes this

formula now we look for another kind of integral   which are of this form integral of

d x upon x square minus a square

 so we wish to evaluate how to find

out the integral of this kind uh difference of two squares which goes to

 so the first step which we can take here is that   immediately that what if we write it in that product form

 so now what benefit we have got

here is that writing this this   this is linear term and this linear term

and if somehow we are able to write them   ah in terms of linear term alone ah then we can

utilize the logarithmic formula can we do that

 so let us try to do this

 so what i will do is

that i will write this integrand separately

 so our integrand is one by x minus a into x plus a what i will do is that that if i if i look at

the the term carefully

 so numerator is a constant   and if i take the difference of these two

terms that is x plus a minus x minus a i   see that i ultimately get two a

 so what i

do is that i divide and multiplied by two a

 so that it is written as two a over

x minus a multiplied by x plus a well as i told you that this is two a is nothing

but the difference between these two term   and therefore i can write it

as x plus a minus x minus a

 so if you look at it carefully further you can

see that that this can be broken into two parts   one by two a x plus a over x plus a into x

minus a this will give you one over x minus a   minus one over x plus a

 so if you look at the term which we started

with was one over x minus a into x plus a it   is converted into two terms both of which having

denominator as a linear factor and for the linear   case we have logarithmic formula

so therefore if i
write this integral d x because the integrand has been factorized in this way
so this one over
x minus a will be replaced as one over two a integration of one over x minus a
minus one over x plus
a d x and this you simplify a bit time that first use the distributive
property here
for the integral
so it will give you t x over x minus a then minus one over two a d x over x
plus a and then you integrate it well one by two a can be carried forward
together
so one by two
a this will become logarithmic of mod of x minus a minus one by two a
logarithmic of mod of x
plus a and then the constant of integration as c you know one by two a can be
taken as common as
then again log of m minus log of n will be same as log of m by n
so we can write it as one
by two a log of mod x minus a over x plus a and plus constant of inequality
c the integral i is evaluated in this form and the trick which we used here
is ah a denominator which was the quadratic form which could have been
factorized we converted
into ah two ah factors both of which are having denominator which is a linear
function
so we
will apply the same trick and we look for another formula what if we have to
evaluate d x over a
square minus x squared
so now with the previous example we can see that we
can write it as one by two a again here ah the term which is in the numerator
is constant
so the difference should not be taken such a waste as that the variable term
comes there and therefore we must write it as a minus x plus a plus x divided by
a minus six into a plus x which
are the factors of this quadratic term d x i can write it as one by two a and a
minus x
divided by a minus six into a plus x will give you integral of d x over a plus
x plus another
integral will give you d x over a minus x
so that you will get one by two a this
will give logarithmic of mod of a plus x
so far there is no problem but here if
you see this term is negative minus of x
so we have to evaluate it separately and
if i use the idea of substitution here again or if i use the formula
you can see that this will become log of modulus of a minus x divided by minus
one which is the derivative
of this term and plus a constant of integration
so you can check yourself that what
will be the integral of a minus x
so ultimately what we got
here is that this negative sign will make this sign negative is one by two a
log of mod of a plus x minus
log of mod of a minus x and plus a constant of integration which will give

you one by two a log of a plus x over a minus x plus constant of integration
 so integral for
a square minus x square comes out to be of this form let us take example how shall we apply this formula
 so choose ah this example in this following manner suppose that we want to evaluate this integral
 so if you notice here we should make a choice or we should make a substitution in such a way
 so that it should be   converted into a form which we already ah know
is that that ah i can write this integral as thrice of x square d x over x cube squared plus
two squared and further if i notice then the the   derivative of x cube this is the factor in the
integrand turns out to be three x square d x   that's another factor in the integrand
 so that's
becomes is a good choice for us and therefore we   will put x cube to be a negative variable t
so that trace of x square d x equals to d t and this calculation leads to d t over t square
plus two squared which leads to the formula   ah which we developed first d x over x
square plus a square it will be one by a
 so one by two tan inverse x by a
 so here it is
t therefore formula is modified accordingly t   by two plus constant integration now
what is the problem was in x
 so we must   convert it back into x and therefore it
should go to one by two that is one by e   t is equals to x cube
 so it gives me tan inverse
x cube by two plus constant of integration
 so knowing the formula helped us in evaluating this integral
 so whenever we are writing we must   mention that which formula we are writing for example here we are using the formula
 so while evaluating we should mention that
as we know d x by x square plus a square   is one by a multiplied by tan inverse x by a plus
constant now if i take another example simple and   quick example is to integrate x square
over one minus x raise to power six then again you can easily see here that i can
do i can write it in the similar fashion as i   did in the previous example is that i can
write it as x square over one minus x cube   squared
 so the suitable choice becomes x cube
equals to t which will take you to thrice of   x square d x over d d x equals to d t which
will take you x square d x to d t by three   making the substitutions here
one over one minus t squared   d t by three this leads to one by three
constant comes outside integration of   d t over one minus t square you know the
formula of d x over a square minus x square

so this goes to log of a plus x over a minus x plus constant
so this will lead to one by three   log of here a is one x is t
so i have
to modify the formula accordingly   one plus t over one minus t plus constant
of integration and then finally i have to   convert into x 1 by 3 log of 1 plus x
cube over 1 minus x cube plus constant
so the integral i comes out to be this one
so oh sorry i missed this one by two a here
so this will make in this case
since a is one
so it will be one by   three into two that will make it one by six
so now we shall further look at
some more formula of similar kind one of them is another important formula
so to find out the integral of the kind i   is equals to integration d x over square root
of a square minus x square you have already seen   a similar expression for ah formula
d x over one minus x square ah   which was for sin inverse x
so you
should expect something similar here if you look at this expression carefully again what substitution can be made
so the idea which we are going to use
here is that we are going to substitute   x is equals to a sin t why that x is equals to
a sin t because in that case i will get here ah the expression as 1 minus sin square t which will take you to   the cos square t and then square root will take help us in evaluating those terms
so let us make the substitution this will give
you d x is equals to a sin t will give you cos t   d t
so the integral i turns out to be a cos t d t divided by square root of a square
minus x square will become a square into ah minus   a square sin square t
so i will take a square
s common it will become one minus sin square t and you know one minus sin square t is cos square
t taking the square root
so this term will become   a cos t
so the same as the numerator a cos t
so ultimately i will get here only d t   which will lead me to integral t plus c x by a plus constant the integral
comes out to be equals to and that   should be expected ah because it is the similar
formula to one minus x square
so as i did for the   case of a square plus x square if you want you
can take a square as common and then convert x   y a into t and then again you write and then you
will figure out that you will get the same formula   further we shall look for another
example d x over square root of x   square minus a square this is different than
previous case x square minus a square
so in this case   the substitution which we made earlier will not

work we have to figure out some other substitution notice the relationship of second
tan
 so we can see that that if i put   x is equals to a sec t then this should help
us because then it will become a square sec   square t minus a square
 so sec square t minus one
will appear there which can be converted into ten
 so this should help us
 so let us look at what
expression i should get from here x square minus a   square is equals to a square sec square t minus a
squared ah a square sec square t minus 1 and this   relationship i know already it is tan square
t therefore this is a square tan square t and since a is goes to a t
 so this will also give
us d x equals to a differentiation of sect is   set t tan t and then d t
 so making the
substitution there let us see how this integral ah evolves
 so this becomes a replacing d x by a section tan t d t divided by square root
of x   square minus a square x square minus a square is
a square tan square t this will become a tan t
 so this tan t and a are cancelled
and ultimately i am left with   set t d t this formula we already know integration
of sec t d t is logarithmic of sec t plus tan d
 so this we will use logarithmic of sec t plus tan t again as earlier we can see from here is that that set t is
known to us secti can be easily evaluated   from this relationship since x is equals to a
sec t therefore sec t is equals to x by a what   about tan t fortunately we have a relationship
of sec t and tan t
 so we know that 1 plus sorry we have the relationship tan
t is equal to square root of   sec square t minus one which will help us
get this tan t to be square root of x a x square by a square minus one
substituting the value here we get logarithmic of sec t is x by a plus tan
t is square root of x by a square minus one   plus constant after a bit of simplification and
taking this log a as along with this constant   c i can write this as log of mod x plus
square root of x square minus a squared then minus log a plus c
 so i will call this as
a new constant c itself since c is arbitrary
 so it will take care of that
 so integration
of d x over square root of x square minus a   square turns out to be
 so we will take
up now to certain specific integrals   let us look at the integral of the form
d x over a x square plus b x plus c
 so we have to find out what will be the integral of this kind of function
you can easily see that this is   not of the form of either x square plus a
square or x square minus a square which we have   or a square minus x square
which i have shown

earlier but ah if you use little algebra this can be converted to one of those forms check the
denominator function a x square plus b x plus c what we can do here is that we can write it in this form
so our first attempt will be to make this x terms as a perfect square
so this is a quadratic term this is a linear term
there should be some constant term which should make it as a perfect square
so if you look at it carefully what you can do is that x square so the formulation should be x square plus two some
number multiplied by x plus square of that number
so this b by a x i can write it as two b by two a x this will help me understand that what should be the next number to
get the perfect square
so this x square two b by two a x
so this is same as b by a
so i have written this number here same
so this means that that ah if i have p square by four a square here it will become a perfect square but that is not this c by a
so i have to add this b square by four a square i have to subtract that b square by four a squared and then i have to
write this c by a
so what i have done here is that i have added and subtracted this number so that this number becomes a perfect square look at it x square twice of x into b
by two a plus b by two a whole square that means that it is x plus b by 2 a whole square this entire term this term i will write it as with a plus sign c by a minus b square over four a square we really do not know what is the sign of this
expression this will be dependent on what is the value of a b and c
so depending on the values of a b and c this will be either a plus sign or a minus sign
so in general i can write this number x plus b by 2 a as some new number capital x and c by a minus b square upon 4 a
square as some number let us say k is squared and since i do not know the sign what will appear
so i will put both the signs
so if it is coming as a positive sign i will keep it as a positive sign if it is coming as a negative i will keep that sign as a negative sign
so ultimately the integral i you can easily see that is small x plus b by two a is equals to capital x this will make d x to be a d
of capital x
so d of small x is same as d of capital x and therefore the expression i will
be converted into d of capital x upon x squared one by a will be outside because this is
the denominator expression this is the denominator expression which is written in

this form this a capital x square plus minus   k square
 so this will be one by
a capital x square plus minus   k squared now look at it carefully this
expression is one of the form d x over x   square plus a square or d x over x
square minus
a square
 so one of the formula can be useful sometimes if sign of a is negative then
the
similar formula can also lead you to the form   of a square minus x square
 so you can also use
that relationship a square minus x square you see   it with the help of an
example and see how it how
it develops
 so let us choose one example for you let us find out the integral of nine x
square plus six x plus five
 so this same expression as i told
you earlier a x square plus b   x plus c a is positive
 so let me write this   nine x square plus six x plus five separately
taking nine as common i should put both this
 so both of them here
 so taking nine
as common you can write it as   x square plus six by nine of x plus five by nine
 so you can easily see that what
you need to do here is that this basically is x square plus two by three
of x plus five by nine further you can write   it as perfect square
 so x square this is two times
one by three of x
 so two times one by three of x   therefore you should write it as one by nine
plus and minus of one by nine and then plus   of five by nine
 so the entire calculation should
lead you to nine x plus one by three whole square   and then five by nine minus
one by nine will
take you to four by nine
 so this is plus sign now four by nine i will write it as two by three whole
square
 so the integral i now can be written as   d x over nine is taken as common x
plus one by
three whole square plus two by three whole square
 so we shall get one by nine and replace this x
plus three by ah some number ah capital x
 so that x plus one by three is equals to capital x so
that d x is equals to d x
 so you will get d   of x over x square plus two by three whole square
 so ultimately i will get one by nine this
is x square plus a square x square plus a   square integral i already know now
it is 1 by
a
 so i will put here 1 by 2 by 3 tan inverse   x by a
 so x by two by three and plus constant
of integration
 so after some calculation i can   see that the number turns out to be one by
six
this three gets cancelled here three two the six tan inverse three by two of
capital x

but capital x is small x of one plus
 so it is x plus one by three   plus constant regression and finally if we want to
further simplify it i can write it as one by six   tan inverse introduce this three by two inside
so this will give me three x plus one by two
 so this will make me three by two x plus
one by two
 so that three x plus one by two   plus constant of integration
 so now ah i will show the
same example which i took again that instead of going the way we had gone we could
have ah taken a different path because ultimately   the idea should be taken from here is that that
we have to make this as a perfect square somehow
 so what i can do here but i must warn you
that initially you should go and you should   solve the problem ah the way ah we have derived
them later on you can use these ideas
 so nine x   square plus six x plus five i can easily
see that this is nothing but three x square
 so three x squared and this is second
term is nothing but twice of three   x
 so i can easily make it as a perfect square
by just adding one here
 so since five is   already there
 so i can break it into one plus
four
 so that this entire term will make me as   three x plus one whole square plus four so
immediately i can write this integral i as   d x over three x plus one whole square plus
four and make a substitution that three x   plus one is equals to t
 so that three d x is
equals to d t
 so that this integral transforms   into d t by three i can take this one by three
outside this makes us t square plus two squared   ah which will further lead to one by three
this formula already we know now is one by a tan inverse t by two t by two plus
constant and t is known to us now
 so this is nothing but one by six tan inverse
three x plus one by two plus c
 so which is the   same result as we observed earlier further
i will show you the example of another kind with a similar idea that can we solve the problem
of the form d x over square root of a x square   plus b x plus c again the idea is same we should
be able to write this a x square plus b x plus c   into either of the form of x square plus minus some k square   as or if a is negative then
it can be written as ah k square minus x square
 so depending
on the form of this expression   this term can be converted into one of this
form just as as as was the previous case

so i will show you with the help of an example suppose that we choose the example as d x over square root of x
square plus two x plus two
so this as i as i can see easily from
here it is x square plus two x if i   add one here this will make a perfect
square and therefore i can write it as   d x over square root of x
plus one whole square plus one replacing x plus one by a new variable t you
can just find out that d x is nothing but   d t
so that this integral converts into d t over
square root of t square plus one and this formula   we know that d x over square
root of x square
plus a square is actually a logarithmic function x
so ah d x over x square plus a square
this formula we have already evaluated   it is logarithmic of mod x plus square root of
x square plus a square and plus constant
so here   the the variable of integration is t therefore
it is t plus square root of t square plus one mod plus constant of integration
and   this t is nothing but x plus one
so therefore
this will give you square root of x plus one   plus square root of x plus one whole square
plus one x plus one whole square plus one   plus c which you can further if you want to you
can write it as x square plus two x plus two
so as constant
so you can see that with the use
of this formula it was it has become very easy   to evaluate ah this integral these
examples have further applications when   we have integrand which is of the form p x plus q
divided by a x square plus b x plus c or p x plus   q divided by square root of a x square plus b x
plus c
so we shall look for these two ah forms
so suppose that we have to
evaluate the integral of the form x plus q over a x square plus b x plus c or we have to evaluate it of the form p x plus
q over square root of a x square plus b x plus c
so the procedure for both of for the evaluation
of both of these integrals is is similar only fact   that that the formulas which will be used they
will be little different because here there is a   square root
so those square root formulas will be
used and here is a square
so those perfect square   formulas will be used
so let us choose one of
them ah let us choose this first example here
so to solve this expression let me
call it as i one and let me call it as   i two to solve i one what we do is
that we write numerator p x plus q as some constant times the derivative of
the expression of the denominator that is
a x square plus b x plus c and plus another   constant b where these are some
unknown constant

which will be determined later notice carefully   the numerator is the linear term denominator is
a quadratic term therefore if you differentiate   this quadratic term you will get a linear
term
 so these two expressions are comparable
 so linear expression this expression will also
give you a linear expression and another constant   in this case also p x plus q equals to
this is another quadratic
 so we will take   this quadratic term a x square plus b
x plus c the differentiation of this ax   square plus bx plus c
 so let us look at what
will happen what
 so p x plus q will become   a differentiation of this term will give you
twice of a x plus b plus b and ultimately   if i compare since ah these are the polynomials
on both the sides
 so they are comparable
 so i must get p the coefficient of x here   same as the coefficient of x here is
twice of small a times capital a and q   which is the constant term here must be same as
capital a times b plus b in order to satisfy this   relationship this expression must be satisfied
so if you look at the the the equations now you   have two equations in two variables we do
not know what is we do not know what v is   but we got two equations and therefore we can
simply solve them
 so from first equation you can   easily solve a is equals to p by two a and then
you substitute that a here and you can solve b
 so ultimately from these equations you can
know what your a is you can know what your b is   and once you know that what your a and
b is are you substitute them back here   and then replace this p x plus q in this
expression by this expression and break   this integral using the linearity property
into two parts
 so the first part will have
 so i can write the integral i one here this integral i one this will be broken into
these two parts the first part will have capital a   integration d by d x of a x square plus b
x plus c by a x square plus b x plus c d x
 so this will be because this b x plus
q is inside a is constant taken outside   plus b integral one over
a x square plus b x plus c
 so i already know this expression now now if you
notice carefully this denominator term ah has this   numerator term as derivative therefore
if i take this as new variable t   this will come to as long this this integral

will come as a logarithmic function this again   i will convert into a perfect square and get the
integral
 so this integral i one can be evaluated will see this with help of an example that
how this can be evaluated and in fact similar   procedure will be used for the case but in that
case what will happen is that the derivative will   appear here and there will be denominator will be
having a square root and therefore in that case   you will get 1 by root t and that you also you
can you can evaluate
 so that will not be a problem
 so i will show you it with with
help of an example 6 x minus 2   divided by three x square plus two x minus one d x
 so you can see easily this is of the form p   x plus q divided by a x square plus b x plus c so
let us use the idea which we developed now six x   minus two should be written as capital a unknown
terms d by d x of the denominator three x square   plus two x plus one plus b
 so that you know the
differentiation of this will give you six x plus   two a times six x plus two plus b here
 so six
x minus two equals to a times six x plus b this   will give you very easily you can compare the
two terms here this is six a x is equal to six   therefore a must be equals to one and this
is twice of a plus b is equals to minus two
 so since a is one
 so this will
become become b is equals to   ah minus four price of a plus b is equals to
two therefore b is equals to minus four right
 so substituting these values in the
integrand what i will get is that i which is   this expression equals to a which is one
times let me write it first d by d x of three x square plus two x plus one all divided by
three x square plus two x plus one d x later on   you can directly write this term as six x plus
two divided by b
 so b is minus four integral   of one over three x square plus two x minus one
sorry here minus one this is also minus one sorry
 so this term ah after after evaluation you can
see that if i choose this number as t what i will   get here is that integral of dt by t minus four
times this term here three x square plus two x   plus one again using the same formula as
as ah we have done earlier here let us take   first of all choose one by three outside
so this integral can be represented as   d x divided by x square plus
two by three of x minus one this you can easily see x square plus two
by three of x minus one can be written as   x plus one by three whole

square minus four by nine you can now write it and therefore this integral
first will give you logarithmic of t   logarithmic r t plus a constant let me call it
as c one minus four by three the other integral   will give you d x over x plus one by three
whole square minus two by three whole square
 so this formula also you know and you
can simply evaluate the final integral   t is the expression which is three x square
plus two x plus one
 so this will be log of   log of mod of three x square plus
two x minus one minus four by three   this is d x over x square minus a square
 so it
will come one by two a two times two by three log of x minus a
 so x plus one
by three minus two by three   over x plus a
 so x plus one by three plus
two by three and then the constant c one   was already there
 so put it as c one plus c two
which i will write finally as a new constant c
 so finally you will get three x square plus
two x plus one minus one two x minus one   minus this four cancels and this three cancels
logarithmic off one by three minus two by three   will give you again minus of one by three one
by three and two by three will give you one
 so this will get you three x minus one or three
x plus three x plus three plus a new constant c   ah one can further do some ah computation or
simplification to get this expression here
 so ah this is finally we saw that how we
can simply solve this example and we can   get the answer by using that technique and
in the similar fashion we can also take up   the example where the square root of square
root of this term is given
 so we can use that
 so with this we summarize what we learnt today
is that we learnt some more integral of some more   functions which involves ah rational functions of
algebraic expressions we shall in the next class   we shall learn some more new techniques
for evaluating these integrals thank you you