

in the previous lecture we have seen  
 lot of miscellaneous examples as application of definite integrals we will  
 continue the same in this lecture also which will help you  
 in solving complicated problems before we start solving certain problems let  
 us take  
 one concept which is still left and let us learn that consider a function  $f(x)$   
 which is continuous on closed interval  $a, b$  and for simplicity we can take that  $f(x)$   
 is positive but the result that i am going to  
 discuss can be extended very easily for any function which is continuous but  
 it may  
 not be positive  
 so let us draw the curve this is  $x$  equals to  $b$  this is  $x$  equals to  $a$  this  
 is  $y$  equals to  $0$  and this is  $y$  equals to  $f(x)$   
 so you have this area represented by  
 the integral  $\int_a^b f(x) dx$  you know that if a function is continuous  
 it attains its bound over the interval  
 so you can see from this plot that this is the  
 maximum value of the function and say this is attained at some point  $x$  equals  
 to  $c$  and say this is the minimum value of the  
 function which is attained at some point  $x$  equals to  $d$  though we do not need  
 these points but  
 still i have just written and say this height is  $m$  this height is small  
 $m$  and this height is capital  $M$  you know that this is the green shaded region  
 is the actual area this is the actual area and we want to find out a bound of  
 this for the  
 given function  
 so from this plot you can see that actual area is always greater than  
 this area which i have shaded with actual area is always greater than this area  
 shaded by black color and what is the area of this rectangle that is very easy  
 to see that the  
 height of this rectangle is small  $m$  small  $m$  and width of the rectangle is  $b$   
 minus  $a$   
 so the area  
 of the small rectangle will be  $m$  into  $b$  minus  $a$  now let us draw this figure  
 again because it is  
 getting very complicated to understand from this  
 so like this you have this is  $b$  this is  $a$   
 so this green shaded region is the actual area  
 so you can see from the figure that actual area  
 will always be less than area of the rectangle shaded by red color  
 so actual area will be always  
 less than area of the rectangle shaded by red color  
 so how to compute this area of the rectangle  
 shaded by red color  
 so this length is  $b$  minus  $a$  and this length is capital  $M$   
 so this  
 area will be always bounded above by and bounded below by this quantity where  $M$   
 is the maximum value of the function attained in the  
 interval  $a, b$  and small  $m$  is the minimum value other function in the interval  $a$   
 comma  $b$  let  
 us see one example how to get these bounds of the definite integral  $\int_a^b f(x) dx$   
 $x$  we  
 have done it for continuous function but you can very easily extend it to this  
 logic for any

function which changes its sign

so let us take some example and see how to find out the bound of the definite integral

so example is say 0 to 2  $e$  to the power minus  $x$  dx

so this is  $x$  this is  $y$  axis then  $e$

power minus  $x$  will be this curve say this is  $x$  equals to zero this is  $x$  equals to

two

so actual area is this to find out the bounds let us draw the rectangle so because function

value is taking maximum value at zero

so this will be always lesser than so

what is the height of this this height is governed by function value at zero function

is power minus  $x$

so the side is one and this width is two

so upper

bound of this area will be two lo for lower bound we have to draw this rectangle

and since the function is decreasing throughout

so lower bound will be given

by area of this rectangle which is at  $x$  equals to two function value that is is  $e$  power minus two

so this height is this height is  $e$  power minus two

so area

of this rectangle shaded by black color is two into  $e$  to the power minus two so you can see that how this

result helps you in finding out upper and lower bounds of any integral here  $m$  is

the maximum value then maximal of the function and  $m$  is the minimum value of the

function we have assumed  $f$  to be continuous on the closed interval  $a$   $b$  let us finish this discussion and let us

move ahead and solve some more miscellaneous exercises on definite integrals in the sequence let us consider one example on

definite integral given as evaluate minus half to half  $\log$  one plus  $x$  upon one minus  $x$  dx

so let us write the integral as it seems very complicated because we have very heavy functions here greatest integer function logarithmic function whose input argument is 1

plus  $x$  upon 1 minus  $x$  but you see it is of the form minus half to minus half to half which is  $ah$

which can be related with even at our functions so if you see  $\log$  one plus  $x$  upon  $\log$  one minus  $x$  if

you assume that it is  $h$   $x$

so  $h$  of minus  $x$  will be  $\log$  of one minus  $x$  upon one plus  $x$  which you can write as minus of  $\log$  of one plus  $x$  by one minus  $x$

so this is equal to minus  $f$   $h$   $x$

which is

so this integrand is an odd function of  $x$  and interval is minus half to half therefore the value will be zero by using the property of definite integrals so

your final integral is minus half to half greatest greatest integer function  $x$  dx so

the greatest integer function takes value 0 between  
so this is the plot for test integer function from  
0 to 1 and then from minus half to 0 that is from minus 1 to 1 actually it  
takes value minus 1.

so  
this will be minus half to 0 we have to break it because function is taking  
different values in  
different intervals we get minus one to  $x$  plus zero to half it is taking  
value zero

so we get  
zero  
so minus  $x$  minus half to zero which gives you minus zero it will be zero then  
minus of this will

be plus and then again one more minus  
so you have minus half that is the final answer  
so you can see

that how for very complicated definite integrals if you use properties it  
becomes much simpler let

us take another example in the sequence evaluate minus  $\pi$  to  $\pi$   $\cos^2 x$   $dx$   
by  $1 + a$  to the power  $x$  is  
positive

so if  $x$  is  
so what should we do because if we try to apply formula of even  
an odd function it will not work because  $\cos^2 x$  is even function but  
this is

this does not satisfy one plus  $a$  power  $x$  either property

so this is not applicable so

what we do we first replace  $x$  by minus  $t$  and see what happens if  $x$  is minus  $t$   
then  $dx$  is minus  $dt$  and  $i$  will be at minus  $\pi$  it will be  $\pi$   
and at  $\pi$  it will be minus  $\pi$

so this is a new limits once you change  
the variable and you get  $\cos^2$  minus  $t$   $dt$  with a minus sign  $dx$  is minus  
 $dt$

so minus  $dt$   $x$

is minus  $c$  to  $\cos^2$  minus  $t$  plus  $a$  to the power minus  $t$

so therefore  $i$  is equal to since you have a negative sign here you can  
change the limits

so you get a positive sign

so you get minus  $\pi$  to  $\pi$  this minus sign

will be ignored as soon as you change the limits interchange the limits and  
you get

$\cos$  of minus axis  $\cos x$

so you get  $dt$  by one plus  $e$  to the power minus  $t$  since  $t$

is dummy variable we can write and let us multiply by upon  $t$  on numer in  
numerator

and denominator

so you get minus  $\pi$  to  $\pi$  and change the variable  $t$  to  $x$  because  
it is dummy

so  $i$  can write it like this now if you add equation one and two if you add  
equation one and two you

get two  $i$  is equal to minus  $\pi$  to  $\pi$  by  $a$  to the power  $x$   $dx$

so this was common

so you get one

plus eight for  $x$  in the numerator also

so it gets cancelled and you can see that you got a very simple integration now using the because  $\cos x$  is even function you can write  $\theta$  twice of  $\theta$  to  $\pi$  and  $\cos^2 x$  you can expand as by using trigonometric properties of  $\cos x$  you can write  $d x$

so value of this will be  $\pi$  because this ah you get here  $x$  that gives you  $\pi$  and  $\cos 2 x$  will be  $\sin^2 x$  by  $2$  and the value of that will be  $\theta$  at  $\theta$  and  $\pi$  therefore finally you get  $\pi$  over here

so  $i$  is equal to  $\pi$  by  $2$  is the answer let us take another example compute  $\int_0^{\pi} (a \cos x + b \sin x) dx$  is equal to  $a \sin x - b \cos x$  so this you can write immediately as  $\pi$  by  $6$  to  $\pi$  by  $3$   $d x$  by  $1$  plus  $\pi$  by  $3$  plus  $\pi$  by  $6$  this is  $6\theta$  this is  $3\theta$  so you will get  $\pi$  by  $2$  a plus this a plus b is  $\pi$  by  $2$ .

so  $\pi$  by  $2$  minus  $x$  so  $i$  is therefore immediately equal to  $\pi$  by  $6$  to  $\pi$  by  $3$   $d x$  by one plus under root  $\cot x$  which we can write as under root  $\tan x$  by one plus under root  $\tan x$   $d x$  now if ok

so let me do one thing let me just add and subtract here so you get  $i$  equals to  $\pi$  this is  $\pi$  by  $6$   $\pi$  by  $6$  to  $\pi$  by  $3$   $1$  minus so this is this we can break it in two parts so one integral like this then another one  $\pi$  by  $6$  to  $\pi$  by  $3$  one by one plus under root  $\tan x$   $d x$  this is again  $i$

so you get two  $i$  equals two one of this integral is  $\pi$  by  $3$  minus  $\pi$  by  $6$  so therefore  $i$  is  $\pi$  by  $12$  let us take another example evaluate  $\int_0^{\pi/4} \log(1 + \tan x) dx$

so apply this formula this will be so this is equal to  $1 + \tan \pi/4$  minus  $x$   $dx$  this is equal to  $\theta$  to  $\pi/4$   $\theta$  to  $\pi/4$   $\log$  of  $1 + \tan a$  minus  $b$  is  $\tan a$  minus  $\tan b$  upon  $1 + \tan a \tan b$

so therefore we we get  $1 + 1$  minus  $\tan x$  upon  $1 + \tan x$   $dx$  which is equal to zero to  $\pi/4$   $\log$  of if you take lcm and add it you get two  $\pi$  one plus  $\tan x$   $dx$

so this is your  $i$  and earlier  $i$  was  $\theta$  to  $\pi/4$   $\log$  of  $1 + \tan x$   $d x$  now if you sum both one and say this is two this was your earlier equation one

so if you sum one and two you get twice of  $i$  is equal to zero to  $\pi/4$   $\log$  of two upon  $1 + \tan x$   $d x$  plus  $\log$  of  $1 + \tan x$   $d x$

so you get two  $i$  equals to zero to  $\pi/4$   $\log$  of two by one plus  $\tan x$  into one plus  $\tan x$   $dx$  this is equal to  $\theta$  to  $\pi/4$   $\log$  of  $2$   $dx$  which is equal to  $\log 2$   $\pi$  by  $4$  two  $i$  is  $\log 2$  into  $\pi$  by  $4$  therefore  $i$  is  $\pi$  by  $8$   $\log 2$  let us take another example

so find out the value of  $i$  compute  $i$  so let us just write it in expanded form

so you get  $\int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$  two  $\pi$   
one plus cos square x say this is  $f(x)$  and say this is  $g(x)$

so  $f(x)$  that is  $2x$  by one plus cos square x is an odd function this is equal  
to minus of  $f(x)$  therefore by using

property of definite integrals for odd functions this will be zero as we know  
that if  $f(x)$  is odd

then well of the integral is zero if  $f(x)$  is odd

so this is odd finally you arrive at this integral

so your  $i$  is  $\int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$

$x$  now this as we said this is say  $g(x)$   $g(x)$  is an even function because if you  
put minus  $x$  over here you get  $2(-x) \sin(-x)$  by one plus cos square  
minus  $x$

so you get  $2x \sin x$  by one plus cos square  $x$  which is your  $g(x)$  therefore  $i$   
will be

equal to zero to twice of zero to  $\int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$  by one plus cos square  $x$

so we are

using the property that this can be written as  $\int_{-\pi}^{\pi} g(x) dx$  this can be written  
as twice of zero to a  $\int_0^a g(x) dx$  now again what to do

so we apply

this property that zero to a  $\int_0^a f(x) dx$  is zero to a  $\int_0^a f(a-x) dx$

so this is equal to  $2 \int_0^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$  upon  
one plus cos square  $\pi - x$  we get  $i$  as four zero to  $\pi$   $\frac{\pi - x \sin(\pi - x)}{1 + \cos^2(\pi - x)}$   
 $\pi - x$  is  $\sin x$  and  $\cos(\pi - x)$  is  $-\cos x$  but since  
it is square

so you will get again  $\cos^2 x$  now if this is one this is two by  
adding one and two you will see that this term this term will be cancelled

so by adding 1 and

2 you get twice of  $i$  is equal to  $4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$  now let  $\cos x$  is  $t$

so  $-\sin x dx$  is  $dt$

so  $i$  is equal to if  $i$  cancel it

so  $i$  is two

$\pi$  at  $\cos$  zero it is one  $\cos \pi$  is minus one  $\sin x dx$  is  $-\sin x dx$

so you get  $-\sin x dx$

one plus  $t^2$

so this is equal to two  $\pi$  if because there is a minus sign you can  
interchange the limit

so you get minus one to one  $dt$  by one plus  $t^2$  square

so  $i$  is therefore

two  $\pi \tan^{-1} t$  minus one to one two  $\pi$   $\pi$  by 4 minus minus  $\pi$  by 4

so you get  $i$  equals to  $2 \pi \left[ \frac{\pi}{4} - \frac{\pi}{4} \right]$

so you get your final answer for this

integral as  $\pi^2$  let us take one more example evaluate zero to  $\pi$   $\frac{x}{1 + \cos^2 x}$  where  $\alpha$  is given to be lying between  
zero and  $\pi$

so let this integral is  $i$

so  $i$  you can write by using the property

that zero to a  $\int_0^a f(x) dx$  is same as zero to a  $\int_0^a f(a-x) dx$  you can write zero  
to  $\pi$   $x$  is replaced by  $\pi - x$

$\frac{1}{1 + \cos^2(\pi - x)}$

so again  $\sin(\pi - x)$  is  $\sin x$  so

numerator of say one and two both will be same

so if you add

so you get two  $i$  on this side and

this side will be  $\pi dx$  upon  $1 + \cos \alpha \sin x$  which we can write as  $\pi \int_0^{\pi} dx$  by  $\sin^2 x + 2 \cos x + 1$  you can replace by  $\sin^2 x + 2 \cos x + 1$  plus let me write it here  $\cos \alpha$  you can expand it to  $2 \sin x + 2 \cos x + 2$  so therefore  $2$  is equal to  $\pi \int_0^{\pi} dx$  if you divide by  $\cos^2 x$  throughout in the numerator and denominator so you get  $\sec^2 x$   $\int_0^{\pi} dx$  by  $\tan^2 x + 2 \cos \alpha$  so this we can write as  $\int_0^{\pi} dx$  let me let me take  $\sec$  is not  $\sec$  let me take  $\tan x + 2$  as  $t$  so you get  $\sec^2 x + 2 dx$  is equal to  $2 dt$  and the limit  $\tan 0$  will be  $0$  so the limit will be  $0$  at  $\pi$  it will be  $\infty$  so  $\int_0^{\pi} dx$  you get  $\int_0^{\infty} \frac{2 dt}{t^2 + 2 \cos \alpha + 1}$  this we can write as  $\int_0^{\infty} \frac{2 dt}{t^2 + 2 t \cos \alpha + 1}$  so we have to add  $\cos^2 \alpha$  and we have to subtract  $\cos^2 \alpha$  so we get  $t$  this together will give you  $t + \cos \alpha$  whole square and  $1 - \cos^2 \alpha$  is  $\sin^2 \alpha$  so this is of the kind  $\frac{1}{a^2 + x^2}$  so this well of this integral you can write as  $\frac{1}{\sin \alpha} \tan^{-1} \frac{t + \cos \alpha}{\sin \alpha}$  limit from  $0$  to  $\infty$  so finally you arrived at  $\int_0^{\pi} dx$  is equal to  $\frac{\pi}{\sin \alpha}$   $\alpha$  is constant so you can take it out we get  $\tan^{-1} \infty - \tan^{-1} \cot \alpha$  so because the  $\alpha$  lies between  $0$  and  $\pi$  it is not  $0$  and  $\pi$  so  $\sin \alpha$  is non-zero so it is defined this is give you this will give you  $\frac{\pi}{2 \sin \alpha} - \frac{\pi}{2 \sin \alpha} \cot \alpha$  so you get again so your well of the integration is  $\frac{\pi}{2 \sin \alpha} (1 - \cot \alpha)$  so  $\int_0^{\pi} dx$  is final of the integration is  $\frac{\pi}{2 \sin \alpha} (1 - \cot \alpha)$   $\alpha$  is given to be between zero and  $\pi$  this completes the solution of this problem let us take one more exercise on definite integrals  $\int_0^{\pi} x \sin x \cos x dx$  by  $\cos^4 x$  for four  $x$  plus sine power four  $x$  so use the property  $\int_0^a f(x) dx$  is same as  $\int_0^a f(a-x) dx$  so this will give you  $\int_0^{\pi} x \sin x \cos x dx + \int_0^{\pi} (\pi - x) \sin(\pi - x) \cos(\pi - x) dx$   $\cos^4(\pi - x)$  plus sine power four  $x$   $\int_0^{\pi} x \sin x \cos x dx + \int_0^{\pi} (\pi - x) \sin x \cos x dx$  this is equal to  $\int_0^{\pi} \pi \sin x \cos x dx$

is  $\cos x \cos \pi$  by 2 minus  $x$  is  $\sin x$  by  $\cos \pi$  by 2 minus  $x$  is  $\sin x$   
 so we get  
 $\sin^4 x \cos \pi$  by two minus  $x$  is  $\cos x$   
 so we get  $\cos^4 x$   
 so this  
 is your  $i$  if this is one and this is two  
 so if you add one and two you get this  
 term will get cancelled with this term  
 so you get  $i$  you get as two  $i$  if you add one and  
 two you get two  $i$  equals to zero to  $\pi$  by two  $\pi$  by two  $\sin x \cos x$  by  $\cos$   
 $\pi^4 x$  plus  
 $\sin^4 x dx$   
 so  $i$  is  $\pi$  by 4  $\theta$  to  $\pi$  by 2  $\sin x \cos x$  by  $\cos^4$   
 $x$  plus  $\sin^4 x dx$  now divide by  $\cos^4 x$  you get  $dx$   
 so  $i$  is equal to  $\pi$  by four zero to  $\pi$  by two  $\tan x$  and  $\sec^2 x dx$  1 plus  
 $\theta^4 x$  1  $\cos x$   
 will get cancelled from here  
 so you get 3 and then 1 is adjusted with  $\sin x$  you get  $\tan x$  and the  
 one by  $\cos^2 x$  is giving you  $\sec^2 x$   
 so therefore  $i$  is equal to  $\pi$  by four  $\tan^2 x$  let us take that  
 $\tan^2 x$  is  $t$   
 so that this will be your  $dt$   
 so you get 2  $\tan$   
 $x \sec^2 x dx$  is equal to  $dt$   
 so  $\tan x \times \sec^2 x dx$  is one  
 by two  $dt$  then zero is zero  $\tan \pi$  by two is infinity  
 so limits will  
 be  $t$  limits will be from zero to infinity  $\tan x \sec^2 x dx$  is  $dt$  by  
 two this  
 is equal to  $dt$  by two  
 so you get  $dt$  by two by one plus  $t^2$   
 so  $i$  is  $\pi$  by eight zero to infinity  $dt$  by one plus  $t^2$   
 so this is  $\pi$   
 by eight  $\tan^{-1} t$  zero to infinity  
 so  $i$  is  $\pi$  by eight  $\tan^{-1} \infty$  minus  $\tan^{-1} 0$   $\tan$   
 $\tan^{-1} 0$  is zero and  $\tan^{-1} \infty$   $\tan^{-1} \infty$  is  $\pi$  by two  
 so you get  $\pi^2$  by  
 sixteen  
 so your answer is  $\pi^2$  by sixteen let us take one more example evaluate  
 zero to  $\pi$  by four  $\sin x$  plus  $\cos x dx$  by 9 plus 16  $\sin^2 x$   
 so this we can write as  $\sin x$  plus  $\cos x$  by 25 minus 16 plus  
 $16 \sin^2 x dx$   
 so this can be written as twenty five minus sixteen one minus  $\sin^2 x dx$   
 this we can write as zero to  
 $\pi$  by four  $\sin x$  plus  $\cos x dx$  25 minus 16 one you can replace  
 by  $\sin^2 x$  plus  $\cos^2 x$  minus two  $\sin x \cos x$   
 so we can write this as 25 minus sixteen  $\sin x$  minus  $\cos x$  whole square now let  
 $\sin x$  minus  $\cos x$  is  $t$   
 so  $\cos x$  plus  $\sin$   
 $x dx$  is  $dt$  this will change the limits  
 so  $\sin 0$  is 0  $\cos 0$  is minus 1 and  $\sin \pi$  by  
 4  $\cos$  by 4 both values are same  
 so it will get 0  
 so  $x \pi$  by 4  $t = 0$   $t$  is minus 1 hence you have

these limits minus 1 minus 1 to 0 and you get this as dt  
 so the value of the integral is dt by twenty  
 five minus sixteen t square this is equal to minus one to zero one by sixteen  
 dt by  
 five by four whole square minus t square now using this formula dx by a  
 square minus  
 x square you can write this value as  $\frac{1}{2} a^2 \log \left( \frac{a+x}{a-x} \right)$   
 therefore this is equal to one by sixteen one by two is five by four log of 5 by  
 4 plus t upon 5 by 4 minus  
 t minus 1 to 0.

so you get  $\frac{1}{40} \log \left( \frac{5+y}{5-y} \right)$   
 so at 0 you will get  $\frac{5}{4}$   
 by five by four  
 so log one minus log you will get minus one will give you one by four and  
 this this will give you nine by four  
 so this is equal to one by forty log  
 one is zero then you get minus one log 1 by 9 which is  
 so this you can write as  $9^{-1}$  and this minus 1 will get  
 cancelled by  
 using property of log  
 so you get  $\frac{1}{40} \log 9$  which you can write as  $\log 3^2$   
 so  $\frac{1}{20} \log 3$  is the final answer  
 let us take one more example on area sketch the region bounded between y equals  
 to x square and y equals  
 to two by one plus x square find out its area  
 so let us first plot y equals to  
 two by one plus x square though we have plotted it earlier but let us  
 plot it in more detail  
 so compute y dash you get and then differentiation of this will give  
 you two  
 so you get minus four x by one plus x square whole square  
 so if x is positive y dash  
 is negative if x is negative y dash is positive and this is true for all x this  
 is true for all  
 x  
 so when x is positive y dash is less than 0 so this is decreasing when x is  
 negative y dash  
 is positive  
 so this curve is increasing at 0 the function value is 2  
 upon one plus zero  
 so two  
 so how this and at y prime at zero  
 is zero which is clear from here  
 so y prime if you put zero you will get  
 zero from here right  
 so let us plot this  
 so at 0 the function value is 2 0 comma two at zero the tangent is parallel to  
 x axis  
 so the curve is like this for positive x axis it is decreasing and  
 for negative x axis it is increasing so this is the curve y equals  
 to two upon one plus x square now let us plot y equals to x square and  
 y equals to two upon one plus x square so let us plot both curve in the same

plane so  
 your parabola is  $y = x^2$  is vertex  $(0, 0)$  axis  $y$  axis and the  
 other curve is represented by  $y = 2x + 1$  this is your  $y = 2x + 1$   
 to two upon one plus  $x^2$  this point is  $(0, 2)$  we need  
 to find out  
 these point of intersection  
 so the area that is required is shaded by red colour we want to  
 find out area bounded between both the curves  
 so we need to find out the point of  
 intersection of both the curves so solve both the curves you get let us do it  
 because it will be lengthy  
 so you put  $x^2 = 2x + 1$   
 so you get  $y = 2x + 1$   
 so you get  $y = 2x + 1$   
 $x^2 - 2x - 1 = 0$  this is zero  
 so  $y = 1$   
 so why is minus two and one  $y = 1$  is minus two and one  
 so since  $y$  is always  
 positive  
 so minus two is to be ignored  
 so  $y = 1$  is  $y = 1$   
 so corresponding  
 values of  $x$   
 so if you put  $y = 1$   
 so corresponding values of  $x$  are  $x = 1$   
 to plus minus one  
 so this is your  $x = 1$  to minus one and this is your  $x = 1$  to plus  
 one  
 so your area your area will be minus one to one because you have got let me  
 draw it again  
 because otherwise how to tell you the limits  
 so you have the situation this is  
 your one car the other curve is this and this is minus one this is plus one  
 so this  
 is your required area this is elementary area this would whose width is  $dx$   
 and this is two  
 by one plus  $x^2$  this is  $x^2$   
 so you get  $\int_{-1}^1 (2x + 1 - x^2) dx$  is the elementary area  
 so value of the required area is  
 so required area is red shaded a region  
 so this is equal to  
 $\int_{-1}^1 (2x + 1 - x^2) dx = \left[ x^2 + x - \frac{x^3}{3} \right]_{-1}^1$   
 $= \left( 1 + 1 - \frac{1}{3} \right) - \left( 1 - 1 + \frac{1}{3} \right) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$   
 so your final value will be this is  $\frac{1}{3}$   
 so  $\frac{1}{3}$   
 this is  $\frac{1}{3}$  minus  $\frac{1}{3}$  this is  $\frac{2}{3}$   
 so you get  $\frac{1}{3}$  minus  $\frac{2}{3}$   
 so this is the final answer  
 let us take one more example is the region discuss the curve and identify the  
 region bounded by  $x = 0$  to the  $x = \frac{1}{2}$

equals to  $2^y$  equals to  $\log x$  and  $y$  equals to  $2$  to the power  $x$   
 so if you plot the region  $\log x$  is drawn like this this is  $x$  equals to  $1$ .  
 and then  $2$  to the power  $x$  at  $0$  it is  $1$  and then its when  $x$  increases  $x$   
 increases  
 its value increases  
 so it goes like that  
 so say this is  $x$  equals to  $2$   $x$  is equal to half is somewhere here this is  
 your  
 half  
 so this is this curve is  $2$  to the power  $x$  this is  $\log x$  this is  $x$  equals to  
 half say this is  
 $x$  equals to  $2$   
 so your region of integration is this  
 so to solve this again you need to define elementary area  
 so elementary area is  
 $f(x)$  this is your  $f(x)$  this is your  $g(x)$   $f(x) - g(x)$  try to recall the formula  
 into  $\int dx$  to the power  $x$  minus  $\log x$  into  $\int dx$  and limits of integration will be  
 from  
 minimum to maximum value of  $x$  that is from  $x$  equals to half to  $x$  equals to  $2$   
 so while of the integration  
 will be  $2$  to the power  $x$  by  $\log 2$  minus value of  $\log x$  will be  $x \log x$  minus  $x$   
 limit goes from half to  $2$  so value is therefore ah at  $2$  you  
 will get  $2^2$  by  $\log 2$  minus  $2 \log 2$  minus  $2$  plus  $2$  minus  
 at  
 half you will get  $2^{1/2}$  by  $\log 2$  minus  $1/2 \log 2$  minus  $1/2$  plus  
 half  
 so you get  $4$  minus  $2^{1/2}$  by  $\log 2$  by combining  
 these two terms then  $2$  minus half will be  $3/2$  by combining these  
 two terms  
 then these two terms together will give you this if you take this ah you can  
 write this as  
 $\frac{1}{2} \log 2$   
 so therefore you get minus this is ok you have a minus sign here  
 so you will your  
 minus sign here you get plus again you get a minus  
 so you get  $5/2 \log 2$   
 so this is your  
 final answer  
 so see this is a minus sign so this minus minus will be plus  
 but because  $1/2$  is there so that will again be minus  
 so this term and  
 this term will be clubbed together  
 so  $2$  plus  $1/2$  is  $5/2$   
 so hence this is the final  
 answer with this we stop thanks for listening you