welcome students we have finished most of the theory regarding definite integrals today we are going to learn miscellaneous examples which will help you in solving complicated problems of competitive exams let us take an example on area

so the area is defined by a set in cartesian plane four x minus one

so the first step to solve this problem is we need to identify the region

so let us first find out what is represented by y square is less than equal to two x to realize that we have to plot first this equation which you know it is a parabola whose vertex is 0 0 and axis is x axis

so you get this is y square equals to two x now we need to find out which region which region is represented by y square less than or equal to two x

so if you take any point outside the parabola maybe here here here or here

so let us take 0 comma 1 and see

so if x is 0 you get 0 into 2 that is 0 and this side lhs you get 1 which is strictly get 1 is strictly greater than 0

so the inequality is not satisfied similarly you can check it for this point say this point is this point is minus one comma zero

so two x will be minus two and y square will be zero at minus one comma zero

so again you can see that the inequality that you get is this similarly for this point also

so if you take any point above parabola or on the negative side of x axis are below this parabola

so all these points are outside the region

so if you take any point inside the parabola you see that this inequality will be satisfied y y square is less than or equal to x

so ah y square less than equal to two x is this shaded area

so this shaded area is y square less than equal to two x let us see what region is represented by y square greater than or equal to four x minus one

so for that we have to plot first the line y equals to four x minus one

so let us plot y equals to four x minus one and then we try to realize what is this region y equals to y greater than equal to four x minus one

so for that first let us plot the line x y this is one by four comma zero and if x is zero y is minus one

so this is zero comma minus one

so this is the line y equals to four x minus one now if you take any point above ah x axis say on y axis above x axis on y axis say this is zero comma one

so four x minus one will be minus one and y is one

so one is greater than minus one

so then equality is true now if you take any point below this line say you take 0 comma minus 2

so x 0 4 x minus one at x equals to zero is again minus one and y is minus two

so minus two is lesser than minus one

so the inequality is not satisfied

so if you take any point above this above this line that means this region the inequality is satisfied

so this y greater than equal to four x minus one is this red y green or equal to y greater than equal to four x minus one is this shaded area now let us plot both the curves on one cartesian plane

so that we can realize which area we are supposed to compute

so we need to now draw y square equals to two x and y equals to four x minus one on one cartesian plane

so y square equals to 2x is this and y equals to 4x minus 1 is

so your y square less than equal to two x is area shaded by green color and y greater than equal to four x minus one is area shaded by red color

so the common area that satis that is satisfied by both the inequalities is this area
 so we are supposed to find out this area for that we need to find out the point of intersection of both the curves
 so we need to solve both the curves
 so y equals to four y square equals to two x four x is y square by two minus one
 so y values will be
 so we get 1 and minus 1 by 2
 so this is y equals to minus half and this is y equals to one
 so the area required if you integrate in the direction of y
 so you have to divide this area by smallest thin strips horizontal strips
 so if say this is d y width of one strip
 so length of the strip will be y value from this line minus y value from parabola
 so length of the strip is y plus 1 by 4 minus y square by 2 this is the length and divide the width
 so this is your elementary area and if you integrate it from minus half to one you get the required area
 so let us integrate this to find out the final required area
 so this is one by four this is equal to i get this
 so minus nine by thirty two
 so this is your final answer now let us take another example we need to find out the area enclosed by
 so find out area enclosed by above to to curves and the lines x equals to zero and x equals to pi by two
 so let us plot y equals to sin x plus cos x first
 so you just try to find out see what is the value of y zero y zero is 1 y pi by 2 is also 1 and if you find out the derivative you see that it is cos x minus sin x
 so to find out whether it is y dash is positive or negative we need to just this is a rough work not actual plot
 so you need to plot cos of cos and sine both this pi by four
 so graph of sine x from 0 to x equals to 0 from 0 to pi by 2 is this and graph of cos x plot will be like this
 so the values there they have same values at pi by four
 so you can see that this is cos x and this one is sine x black one is sine x and red one is cos x
 so you can see that cos x dominates sin x between zero and pi by four
 so this will be positive in this interval and y dash will be y dash that means y dash is this
 so y dash will be negative when sine x dominates cos x
 so this will be negative in this interval and y dash at pi by four is zero
 so the plot of this pi by four is pi by two this is zero if this is one one unit on y axis
 so at zero the value is 1 at 0 the value of this is 1 and at pi by 2 the value is 1 and at pi by 4 y value is 1 by root 2 1 by root 2 by root 2 that is root 2 which is greater than 1 and
 so somewhere here if if i represent root two this is root two say
 so y is increasing y dash is positive from 0 to 4
 so y is increasing
 so we will get this curve and at pi by 4 it is 0
 so the horizontal any the horizontal line which passes through pi by 4 comma root 2 will be tangent and after that since y dash is less than 0 the curve will

be decreasing at it reach here

so you have this shape of y equals to sin x plus cos x we get this shape this shape let us plot this curve also diff on a separate cartesian plane then we will combine it let us plot y equals to cos x minus sin x on a separate cartesian plane

so let us plot cos and sin again because we need to refer that this pi by 4 this is pi by 2 this is 0

so cos 0 is 1 because pi by 4 is 1 by root 2 cos pi by 2 is 0

so this is cos x and say this is your sine x this is your sine x

so this is sine x this cos x since modulus is there to open this we need to know where cos x dominates sine x and where sine x dominates cos x

so y dash is cos x minus sin x because cos x dominant sin x in the interval zero pi by four and if you compute y dash you get sin x minus cos x because sine and cross both are positive

so this will be negative in this interval similarly for the other interval that means from pi by 4 to pi by 2 y is minus of because sine x dominates cos x

so we get minus of cos x minus sin x and y dash will be sine x plus cos x which is always positive in this interval

so how this curve this curve will look like ah at y 0 it is again 1 at y by four it is zero and at y pi by two it is from you can put the values of x here and see what is y

so y pi by two it is again one

so if this is this is x equals to zero this is x goes to pi by two and this is x equals to pi by four

so from zero to pi by four y dash is negative

so y is decreasing and y value at zero is one

so and pi by four it is zero

so it is decreasing y is decreasing because y dash is less than 0 from pi by 4 to pi by 2 y dash is positive

so it is increasing and pi by 4 is 0 and pi by 2 it is 1 this one

so you get this curve now let us plot both the curves on a single cartesian plane

so that we can realize what is the area

so y equals to sine x plus cos x and y equals to mod of cos x minus sin x this y axis is x axis this is this is one unit is pi by four is zero this is pi by two

so and this is somewhere is this is root two

so sin x plus cos x is this curve we have seen and mod of cos x minus si sin x is this curve

so this is drawn by black color and this one is drawn by blue color

so the required area is this this is your required area

so to find out this area we need to break it in two parts because if you draw vertical strips thin rectangles say of d x width

so you can see that for this portion which lies between zero and pi by four it starts from one curve when it is it ends uh it finishes on y equals to sin x plus cos x and for this portion it starts with another portion of the curve which is going by different equation

so we need to break it in two parts

so total area required will be elementary area from zero to pi by four

so elementary area is sine x plus cos x minus this which is cos x minus sin x d x plus from pi by four to pi by two d x elementary area is this into d x and what is this is sine x plus cos x minus this equation of this curve is minus of minus cos x plus sin x

so we get zero to pi by four two sin x d x plus pi by four two pi by two two

cos x d x let us integrate it ok
so you got this the integration of this is minus cos x 0 to pi by 4 plus integration of this is sine x pi by 4 2 pi by 2 we get by putting the upper and lower limits we get upper limit is this and lower limit will give you by root 2.

so total value is four times four times one minus one by root two
so required area is four times one minus one by root two
so there is one important point to notice in this example if you see the region this region can be divided into two portions and this region which i have shaded now even with blue color and this one by green
so they are symmetrical
so instead of computing both the areas separately you can just compute this and twice of this will give you the required area fine let us take another example find all possible values of b
so that area of the bounded region enclosed between parabolas y equals to x minus b x square and y equals to x square by b is maximum where it is given that b is positive
so you can see that this is a very simple curve y equals to one by b square is a parabola whose vertex is zero and axis is y axis y equals to one by b x square
so b is for since b is positive if b is negative it would have been inverted
so this is a parabola let us plot y equals to x minus b x square this we can write as minus b if we take common we get x square minus x by b which we can write as plus one by minus one by four b square
so therefore this can be written as plus one by four b
so y minus one by four b is equal to minus p x minus one by two b this
so this equation is of this kind b is positive
so this parabola is inverted whose vertex is at one by two b comma one by four b and it passes through you can see that x equals to zero y equals to zero and x equals to x equals to 1 by b y 0
so this is inverted parabola whose axis is x equals to two by b and vertex is one by two b one by four b now let us plot both the parabolas on a single cartesian plane this is your y equals to x square by b and the other one is this
so your area that is enclosed between them is this one
so we need to find out this point of intersection of both
so let us solve both the equations
so we get x
so x is therefore zero if you solve this you get x equals to zero and x equals to b by 1 plus b square
so this this is x equals to b by one plus b square
so the area required is elementary area
so elementary area is x minus b x square minus x square by b into d x x goes from 0 to b by 1 plus b square let us integrate this
so so required area x square by two minus b by three x cube minus x cube by 3 b 0 to b upon 1 plus b square this is equal to b square by 2 minus whole cube minus b cube by 3 b one plus b square cube minus this b square
so if we take common plus b square cube we get b square upon one by three plus b square cube b square by one plus one by b square by three into one plus b square cube is common
so we get one plus b square
so value of this is required area is b square by 2 into 1 plus b square square minus this cubic will be cancelled and by this one
so you get square here
so b square upon 3 plus b square square this is equal to 3 minus 2 by 6 b square upon 1 plus b square square

so required area is b square by six one plus b square by whole square
so we have computed the required area now the question says that find ah for which value of b the area is maximum
so let us compute that
so your area is ah b square by one plus b square square one by six
so b is positive it is given
so for which value of b area is maximum for that let us differentiate this we get this by applying rules of differentiation
so one plus b square is common here
so what we get is two b plus two b cube minus four b cube
so we get one by six one upon one plus b square power four 2 b minus 2 b q
so finally we get 1 by 3 p one minus b square by one plus p square this is cancel it is cubic not four cube
so we have got b a by d b as one by three b one minus b square upon one plus b square cubic now d a by d b equals to zero will give you the required points where this can be maximum or minimum
so we get b equals to zero b equals to minus one b equals to plus one
so these two values we have to ignore because b is positive
so the only possible value that is allowed is of b that at which we are we are supposed to compute maximum minimize b equals to one now how to find out maximum
so we are not going to find out the double derivative let us see how to find out where the x whether the area is maximum or minimum at b equals to one
so d a by d b d a by d b is one by three b one minus b square by one plus b square cubic
so and you can see that if b is greater than one then d a by d b is negative and once b is less than one between zero and one d a by d b is positive
so a increases when b increases and a decreases when a takes value greater than one
so if you plot this around b if this is the area you plot the area
so b these one this is b
so when b is greater than one d a by d b is less than zero
so this is decreasing and when b is less than one it is increasing
so at one at b equals to one area is maximum with this we finish some examples on area let us see some more examples on definite integrals miscellaneous from different computative exams this is the problem it seems to be a very complicated problem on definite integrals but if you apply certain properties of definite integral it becomes very simple
so we know that x log n power n is n log m
so we get this here you can write this as 6 minus x whole square
so this will give us and 2 gets cancelled we get
so finally we got this integral and see that we are going to use this property of definite integral which says that this value is equal to a plus b minus x dx this value of the integral is same as this value now apply this
so you get 2 to 4 log of a is 2 b is 4
so you get 6 minus x 6 minus x dx by log x will be replaced by 6 minus x plus log 6 minus 6 minus x
so i is therefore two to four log six minus x d x by log six minus x plus log x
so this you can see that this if you say this is 1 and you say this is 2
so both represents our initial integral and if we add it the integrand will become 1 because numerator will be log x plus log 6 minus x and denominator will be log x plus log 6 minus x
so both will get cancelled
so we will add it and we get 2 i equals to 2 to 4 log x plus log six minus x by log x plus log six minus x d x and this will get cancelled

so we get d x which is four minus two is this two

so value of i is there for one now let us take another example x square plus log pi minus x by pi plus x cos x dx

so let us break it in two parts log pi minus x by pi plus x into cos x dx is equal to

so since this integral is of the kind minus a to a f x d x

so we need to find out whether it is even or odd function

so this function is even function because if you replace x by minus x you get x square and cos of minus x is cos x

so you get twice of 0 to pi by 2 x square cos x dx and if you see this function if you put minus x here you get pi plus x by pi minus x is cos of minus x

so cos of minus x will be cos x and here you get plus and minus if you put and by using the property of log you will see that you get a minus sign if you replace x by minus x throughout this integral integrand you get a minus sign outside

so this is an odd function therefore by using the property of the definite integrals you will get zero

so to see it carefully let us do that

so you want to find out whether it is even or odd

so if this is h x h of minus x is log of pi plus x by pi minus x cos of minus x by using the property of cos x and log this can be written as cos x which is

so it this is an odd function therefore minus pi by 2 to pi by 2 h x dx will be 0

so your

so your integral becomes finally you got this expression this is your value of the original integral twice of x square cos x d x this will be equal to

so you need to integrate it by parts first function this is second function this function integral of second is sine x this is this is zero

so twice of zero to pi by two we got

so first function into integral of second zero to pi by two minus zero to pi by two two x sine x dx to pi by four minus zero minus first function into integral of second is minus cos x 0 to pi by 2 minus minus plus 0 to pi by 2 differential of first is 2 and minus cos x dx

so this will be 0 at pi by 2 cos pi by 2 0 at 0 this is 0 because of this

so you get 2 pi by four minus two integral of cos x is sin x

so you get zero to pi by two this is 2 pi by 4 minus 2

so final answer is i have missed here one i square you get pi square pi square

so final answer is pi square by two minus 4 let us take another example under root log log under root log 2 by 2 under root log 3 x sine square x sin x square by sine x square plus sine log six minus x square d x

so since you see that x square is there and x is here in the integrand

so if you take x square equals to t that will ease our computation and limits will be freed from square

so immediately you get x is if you put x is under root log 2 you get log 2 as well of t and upper limit will be t equals to log three two x d x will be d t

so this x you can combine with d x it will replace by half d t sin x square is sin t by sin t plus sine of log six minus t

so your integral is 1 by 2 i say it is i i is log 2 log 3 sine t dt by sine t plus sine log 6 minus t log 6 minus t now by applying the properties of definite integral which says that a to b f x d x is same as a to b f a plus b minus x d x

so we get log 2 to log 3 sine of ln 2 plus ln 3 will be ln 6

so log log 2 plus log 3 will be log 6

so log 6 minus t t by sine six sine log six minus t plus here t is replaced by log six minus t

so you will get sin t from this term

so you can see that if you add these two integrals

so numerator and denominator will be same

so they get cancelled

so you get sine t plus sine log six minus t by sine log six minus t plus sin t d t

so this get cancelled and you get two i equals to one by two log two was two was missing here log three d t this is equal to 1 by 2 log 3 minus log 2 which is equal to

so therefore i is one by four log three by two we have seen certain miscellaneous examples on definite integrals on area and definite ah other kind of definite integrals

so in our next class we shall continue with ah miscellaneous examples and see how to approach such complicated problems thank you you