

till now we have seen what is a definite integral and how to evaluate it there are two methods to evaluate a definite integral one is by method of limit of sums and another is by method of anti-derivatives

so we have seen that a definite integral $\int_a^b f(x) dx$ is equal to $f(b) - f(a)$ where capital $f(x)$ is anti derivative of small $f(x)$ there is a remark over here we know that anti derivatives are

not unique and $f(x) + c$ will also give you the function of x so we can evaluate the function the integral as $f(x) + c$ which is equal to so what we see that this constant c is removed we are getting the same value whether we are using capital $f(x)$ as an anti derivative or $f(x) + c$

as an anti derivative of a small of x

so during definite integral the constant c can be ignored as it does not affect the value of the integral let us solve one more example to

refresh our mind let us take integral

so as you know that definite integrals can be interpreted as area if this is your x axis this is your y axis then and this is line x

equals to zero this is line x equals to one then you can plot this function as so this will be half and this will

be one zero comma one zero comma half

so this integral represents this area this is the graph of one upon one plus x square now if i want to find out this integral by the method of anti-derivatives

so i will be looking for a function whose derivative is one by one plus x square and i hope

all of you remember what is the anti derivative of one upon one plus x square

so you know that derivative of $\tan^{-1} x$

is one upon one plus x square therefore the value of this

definite integral will be which is equal to by the formula earlier explained

which is equal to $\tan^{-1} 1 - \tan^{-1} 0$ is $\pi/4$

and $\tan^{-1} 0$ is zero

so well of this integral is $\pi/4$ given any definite

integral of this kind we can write the value of this integral in this form only if it is easy to

find out anti-derivative or small $f(x)$ that is it is easy to find out a function capital $f(x)$

such that whose derivative is small $f(x)$ but it is a point to note here for all students that it is not easy to compute anti-derivative of $f(x)$ always then what should we do

so i will answer this

question in next few problems

so let us take one more example and see

what will happen if we are not able to find out the anti derivative easily

and let us take this definite integral now it is not easy to find out an

antiderivative of this function integrand

so we try to go for a trick and the

trick is let us take that $\tan x$ is t

so $\sec^2 x dx$ will be dt

so this is now your dt

so the integral will look like

this integral will look like $\int \tan^2 x \sec^2 x \, dx$ and $\int dt$ now since you have changed the variable of the integration

so limits will change

so when x is 0 $\tan 0$ is 0

so t is also 0

so lower limit x equals to

zero goes to two equals to t equals to zero when x is $\pi/4$ $\tan \pi/4$ is

one

so new value of upper limit is one now you know the anti-derivative of t that will be square by two

so value of the integral will be one minus zero equals to one the trick that we applied is known as method of substitution in general if you consider a problem of this kind where this function is itself a function of another function say $g(x)$ and you have $g'(x) \, dx$ if you have this kind of integral then what you can do is you can assume $g(x)$ as your new variable

u then $g'(x) \, dx$ will be du

so the integral will be equal to $\int f(u) \, du$ and limits will change because you are integrating now with respect to

u

so when x is a u is $g(a)$

so x equals to a goes to

so the lower limit will be change to $g(a)$

similarly the upper limit will be changed to $g(b)$ and it might be possible that for this integral you can very easily find out the anti derivative that means capital $F(x)$

so this method is known as method of substitution let us solve some more complicated

problems and see how to explore this method to solve even complicated problems

so let us take this integral now one remark over here that there can be several ways to solve this problem by choosing different substitutions but one of them will be giving you the simplest solution

so for example if you take $1 + x^2$ as u this may be one of the simplest possible solutions

substitutions possible for this integral

so let us see what will be the limits so

x equals to 1 will give you u equals to 2 x equals to 2 will give you u equals to 5

so this integral will be one goes to one by two and

four goes to one by nine now differentiate this

so that you find out the

relationship between like this integrand and du

so for that let us differentiate this

so you get

$2x \, dx = du$ so $x \, dx = du/2$

so $\int \frac{1}{1+x^2} \, dx = \int \frac{1}{u} \cdot \frac{du}{2}$

so this whole expression is now replaced by $\frac{1}{2} \int \frac{1}{u} \, du$

two du and you know what is the anti-derivative of a constant

so you get here u one

by two two one by nine minus two one by nine minus one by two minus two by nine plus one that is seven

by nine

so you can see that it seems initially very complicated to find out the anti-derivative the anti-derivative of the original integrand but by this substitution

it is very easy to solve the problem and finally the transformed integral only constant is

remaining whose entity derivative is known to you let us take another example now

so if we take x power 4 plus 9 as our new variable

say t it might give you some approach to solve the problem but it is better to assume this as

t you can try the previous substitution i will be taking this substitution

so if you take

the substitution see what happens you get

so you get two x cube dx by x to the power four plus nine under root this is equal to dt

so this integral is converted into

so x cube dx by under root

x power 4 plus 9 is dt by 2

so you get dt by 2 what will happen to the limits

so if x goes to zero t is under root nine that is three and x goes to two t is twenty

five under root that is five three to five

so you get one by two is constant you can take out and derivative is t

so you get five minus three that is one let us take some more examples and solve it that

will help you in solving these kind of problems

so let us take following integral you can see that if i assume this as our new variable say u

so i get minus

t to the power minus two dt equals to du that means is minus du when t goes to minus one u is zero and when t is minus half u is minus one

so your value of the integral i is minus one goes

to zero minus half goes to minus one four times sine square u du now it is not easy to find out the anti

derivative of sine square u directly

so we need to substitute it by a trigonometric identity that is i we

so we need to replace sine

square u by this

so you get minus 2 θ to minus 1 1 minus

$\cos^2 u$ du which is equal to minus 2 u minus sine two u by two zero to minus one

so this will give you minus one minus zero minus sine of minus 2 by 2 plus zero

so the value of the integral is finally two minus sign two

so we have solved several problems and we have seen that how the method of substitution can simplify several integrals definite integrals apart from that there are several other problems which may not be solvable by only method of substitutions

so there are several other properties of definite integrals which help in solving definite integrals

so we are going to learn those properties and prove it one by one

so properties of definite integrals property one this property says that the changing the variable from x to t does not affect the integral at all

so the the proof of this property is one line provided you assume that x is t

so dx will be dt and x equals to a will be replaced by t equals to a and x equals

to b will be replaced by t equals to b and hence we have the property one let us see property number two $\int_a^b f(x) dx$ is equal to minus of $\int_b^a f(x) dx$ in particular $\int_a^a f(x) dx$ is zero we know that if small $f(x)$ has an anti derivative say capital $F(x)$

then value of the integral is written as this

so we can write it in this form hence we can write it as this hence the property now for this you can just replace p by a in this and see that the value is zero by taking this to the other side right hand side to the left hand side another explanation for this that integration

a to a zero suppose $f(x)$ is positive and the graph is like this and so this

integral will represent area under the curve

so if b will coincide with a it is very easy to realize that the area will be zero if this line b vertical line b is coinciding with vertical

line a the area will be zero hence this property therefore this property is true property three property three says that you can break this integral into definite integrals as summation where c lies between b and a

so if capital $F(x)$ is anti-derivative of $f(x)$ then value of $\int_a^b f(x) dx$ is equal to $F(b) - F(a)$ value of $\int_a^c f(x) dx$ is $F(c) - F(a)$ and value of $\int_c^b f(x) dx$ is $F(b) - F(c)$

so you start from this and you can just write here $F(b) - F(c) + F(c) - F(a)$ and then you can use these two

so you get

$F(b) - F(a)$ you can replace by $\int_c^b f(x) dx$ and this you can write replace by this using this equation you get $\int_a^c f(x) dx$ hence the property three is true let us take property four property four says that $\int_a^b f(x) dx$ is equal to $\int_a^b f(a + b - x) dx$

so this property can be very easily proved by just simple substitution

so if you take x as $a + b - t$ so dx will

be minus dt x equals to a will go to t equals to b x equals to b will go to t equals to a so

this integral will become this limit a will be converted into limit t equals to b this b

will go to t equals to a x will be replaced by $a + b - t$ and dx is minus dt here you

can use property two and you can interchange the limits because you have a negative sign so

that negative sign this negative sign will be cancelled out once you interchange the

limits

so you get $a + b - t \frac{d}{dt}$ since property one says that variable t is dummy

so we can replace t by x hence

property four is established all these properties are very important and we shall see it when we solve several examples by using these properties let us see property five which is a particular case of property four and it says that

$\int_0^a f(x) dx$ is equal to $\int_0^a f(a - x) dx$

so you start with this left hand side again and substitute x by $a - t$

so you get dx as $-dt$ x

equals to $\int_0^a f(a - t) (-dt)$ will give you t equals to a substitute this 0 will go to

$a - x$ equals to a will go to t equals to zero

so you will get t equals

to a and t equals to zero $f(x)$ will be $f(a - t)$ and dx is $-dt$ by using

property two you can interchange the limits

so you get this negative sign will be this

negative sign will be cancelled

so you get this now if you replace t being the dummy variable by

x you get the rhs this proves your property five very important property we will see we will

solve lot of problems by using this property property six property six says something about this integral

so we want to find out value of this integral and

so value of this integral you can write as

$\int_0^a f(x) dx + \int_0^a f(a - x) dx$ by using property three because if this is zero this is a and this is

$2a$

so you can break this integral into two integrals by using property three where this

is a this is c this is b by using property 3 you can write like this now let us see

what can be the value of this integral

so let us substitute x by $2a - t$

so dx

is equal to $-dt$ x equals to a goes to t equals to a x equals to $2a$ goes to t equals to zero

so you get a goes to $2a$ a goes to zero

x is replaced by $2a - t$ and dx is replaced by $-dt$ by using property two we can interchange the limits again

so this negative sign will be

cancelled

so we get $\int_0^a f(2a - t) dt$ and by replacing this t again because it is a dummy variable we get $\int_0^a f(2a - x) dx$

so $\int_0^a f(x) dx + \int_0^a f(2a - x) dx$ is equal to $\int_0^a f(x) dx + \int_0^a f(2a - x) dx$ and value of this is computed as this

so we can replace it

here

so we get final formula that is $\int_0^a f(2a - x) dx$ this is your property six let us deduce another property from this formula that is property seven

so as we have seen that $\int_0^a f(x) dx = \int_0^a f(2a - x) dx$

is equal to zero to a $f(x) dx$ plus zero to a $f(x) dx$ now if $f(x)$ is even then $f(-x) = f(x)$ then equation say one will give you that is property six will be simplified into this and if $f(x)$ is odd then $f(-x) = -f(x)$ in that case this property six will be simplified and you will be getting zero

so this is property 7 it says that this is equal to this if this condition is true and if this condition is true if this condition is true then this property we will see that how these properties are going

to help us in solving some complicated problems now let us prove last property that is property eight property eight says that $\int_{-a}^a f(x) dx$ is equal to there are two possibilities if $f(x)$ is even function then this is equal to zero to a $f(x) dx$ and if $f(x)$ is odd and one of this integral is zero if $f(x)$ is odd

so an even function satisfies the following property that is $f(-x) = f(x)$ and odd function satisfies that $f(-x) = -f(x)$

so let us prove this property

so by using property 3 we can write this integral as this now take this integral and simplify it by using the properties of even n r functions so replace x by $-t$ we get dx as $-dt$ and $-a$ will be replaced x equals to $-a$ and a will be replaced by t equals to a and x equals to zero will be replaced by t equals to zero

so we get $\int_{-a}^a f(x) dx = \int_a^{-a} f(-t) (-dt)$

so this is equal to by

using property 2 we can interchange the limits and this minus sign will get cancelled will get cancelled and we get this since t is dummy variable

so we can replace t by x

so finally we arrived at $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$ sorry we got this as $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$

so $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$ we got as $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$ since the variable is dummy

so we can replace it by x $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$ now if $f(x)$ is even

that is $f(-x) = f(x)$ so $\int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx$ will be twice of $\int_0^a f(x) dx$

and if $f(-x) = -f(x)$ is minus of this we get this value as zero

so let us solve

some simple problems and see how this properties of definite integrals can be utilized in solving prob definite integrals very

easily example one let us solve $\int_0^4 |x-2| dx$ now if i ask you what is the anti-derivative of $|x-2|$ can you answer this it is not easy to find

out the anti-derivative of this

so what we do we use property three and break it in

two parts because $|x-2|$ is equal to $-(x-2)$ when x is less than equal

to 2 and $x-2$ when x is greater than or equal to 2

so we can break it in 2 parts

and write in simple polynomial form

so we can write it like this and integrated simply

so here we are using property three

so this will give you zero to two two to four
 so by applying the formula of definite
 integrals by method of anti derivative we get
 so final answer is four

so you can see
 that how property three can be utilized in finding out a little complicated
 problem let us take another example

so we are going to use a property which says that
 $\int_0^a f(x) dx$ is same as $\int_0^a f(a-x) dx$

so therefore i will be equal
 to $\int_0^{\pi/2} \cos x dx$ four times by this property this this i and this i
 will be same this integral value of this integral and value of this integral
 will be same

so i get this equal to

so $\int_0^{\pi/2} \cos x dx$ is in the first quadrant so
 therefore we get $\cos x$ under root under root $\cos x$ plus under root $\sin x dx$
 now if we add say

one and say this is two

so by adding one and two will get two i on the left hand
 side and on the right hand side once both the right hand side will be
 added will be getting only four dx that is four into well of this
 integral will be $\pi/2$ four into $\pi/2$

so therefore i is equal to $\pi/2$ final answer let us take another example twice of
 zero to $\pi/2$ $\log \cos x dx$ we can write this integral as by using the
 property which will give us i as now

so we add one and two and we get that $\int_0^{\pi/2} \log \cos x dx$ is equal
 to $2 \int_0^{\pi/2} \log \sin x dx$ plus $\int_0^{\pi/2} \log \cos x dx$ so
 this i is equal to zero to $\pi/2$ $\log \sin x \cos x dx$ now dx
 so if

you multiply and divide by 2 here

so you get i as $\int_0^{\pi/2} \log \sin^2 x dx$ minus $\int_0^{\pi/2} \log 2 dx$

so i is equal to if you take $2x$ equals to t in this integral $\int_0^{\pi/2}$
 will go to $\int_0^{\pi/2} \log \sin t dt$ $\int_0^{\pi/2} \log \cos t dt$ is $\pi/2$ and $\log \sin t dx$ will be 1
 by two dx minus $\pi/2 \log 2$

so we got one by two zero to $\pi/2$ $\log \sin t dt$ minus $\pi/2 \log 2$ now in
 this integral we can write here this as $\pi/2$
 $\int_0^{\pi/2} \log \sin t dt$ and apply the formula $\int_0^a f(x) dx$ is equal to twice of $\int_0^a f(x) dx$

provided $f(a-x) = f(x)$ if you apply that we get i equals to one by
 two into two zero

to $\pi/2 \log \sin \pi/2 dt$ minus $\pi/2 \log 2$

so therefore i is
 equal to zero to $\pi/2 \log \sin t dt$ $\log 2$ and we know that from our
 previous

calculation what is the value of this let me show you

so by previous
 calculation value of value of this by previous calculation value of
 this is i by 2

so we got finally i equals to $\int_0^{\pi/2} \log \sin t dt$ minus $\pi/2 \log 2$
 $\log 2$

so therefore i by 2 is minus $\pi/2 \log 2$

so therefore

i equals to minus $\pi \log 2$

so this was a very complicated problem and you see how this properties of definite integrals helps you to solve this problem let us take another problem zero to one

so again using the property that this is equal to $\int_0^1 (1-x)^n dx$ we can write I equals to $\int_0^1 x^n dx$ x will be replaced by $1-x$ $1-x$ to the power n dx we get $\int_0^1 (1-x)^n dx$ which is equal to zero to one dx hence by integration you get zero to one which gives you

so which is equal to let us take one more problem I equals to zero to π $\sin x dx$ upon one plus $\sin x$

x

so this will be equal to $\int_0^{\pi} \frac{1 + \sin x}{1 + \sin x} dx$ so therefore I is equal to $\int_0^{\pi} dx$

so if we add this integral with this integral we get $2I$ equals to $\int_0^{\pi} \frac{1 + \sin x}{1 + \sin x} dx$

so $2I$ is equal to $\int_0^{\pi} dx$ constant

so you can take out and this can be written as $\int_0^{\pi} \frac{1 + \sin x}{2 + \cos x} dx$ this can be written as zero to π $\sec^2 x$ by two dx upon one plus $\tan x$ by two square now let $\tan x$ by two is t

so $\sec^2 x$ by two dx equals to dt half

so you get $2I$ equals to $\int_0^{\pi} \frac{1}{1+t^2} dt$

goes to zero $\tan \pi$ by two goes to infinity and second square x by two dx is two

dt and here you get one plus t^2

so this will give you minus two upon one plus t in integration of this zero to infinity

so this gives you π is there

so π minus 2π 0 minus 1

so final value of I you get as $2I$ is equal to 2π

so I is equal to π here again we have used this property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and then we have used substitution also and

then there are some trigonometric identities also there with this I stop and later we will

consider some more complicated problems and explore more about definite integrals and

about their applications in finding out area of complicated chips bounded between one curve two

curve three curve four grams and

so on thank you you