today we are going to learn about definite
integrals as far as history of definite   integrals is considered contribution of
two mathematicians has been extraordinary   one is riemann a german
mathematician another is   lebeg a french mathematician i request all
my students to learn about both of them more   and get motivated let us see what are
the applications of definite integrals
 so there are several applications
of definite integrals for example you can use definite
integrals to compute length of a curve area of planar region area of curved
surface for example surface area of a sphere volume
for example volume of sphere mass etcetera
 so you can see that there are lot of applications
of definite integrals
 so i have taken area as an example to understand what do we
mean by definite integrals
 so as you know   from earlier classes that you can compute
area of simple shapes like triangle rectangle circle etcetera and if you have a
complicated shape   for example if you are asked
to evaluate area of this shape you can break this area into finite number of
simple shapes and then you can compute individu individual
areas of all these simple shapes and sum it up to get the actual area this is
the required area but this concept of breaking
the complicated area into   simple shapes is not always applicable for example
there are several real life problems and there are   several mathematical
problems in which you
will have shapes which we cannot divide into   finite number of shapes whose
area is known to you
so we will see certain examples where you cannot   divide the area into finite
number of shapes whose
area is easily computable
 so area bounded between a line and a curve let us take y equals to 1 and y
equals to x square let us plot them
 so this is
your y axis this is x axis
 so y equals to 1 is a line parallel to x axis and
y equals to x square is a parabola whose vertex is   0 0 and axis is y axis so
if you plot it you get this
 so your required area is i request all students to see whether
they can break it into finite number of   shapes whose area is known to you
try let us see some more examples example two area bounded between two curves y
equals to x square and y square equals to x let us plot them this is your y axis
this is your x axis
 so y equals to x square is a parabola whose
vertex is 0 0 and axis is x axis
 so you have   this parabola and y equal y square equals
to x is again a parabola whose vertex is 0 0   and axis is x axis
 so you have this parabola and
this is the area that is required bounded between   them let us see some more
examples motivating
examples why we are requiring definite integral
 so area bounded between three curves
 so one curve is y equals to root two

x another is y equals to under root two x minus   x square and another is a line x equals
to two let us plot them
 so y equals to   under root two x minus x square is a circle you
can write this equation in the following form
 so you can see that this is a circle
with center one comma zero and radius one
 so you get   the circle radius one comma zero center one comma
zero radius one let us plot y equals to root two x y equals to root 2 x is
parabola whose vertex is
0 0 and axis is x axis
 so you get this parabola   and x equals to 2 is a line since this
coordinate of this point is 2 comma   0.

 so the line will be tangent to the circle
at this point
 so this is the required area   the point to note here is that this parabola
is not going to intersect the circle   anywhere else except zero comma zero
which is
clear if you solve y equals to root two x and   y equals two under root two x
minus x square
if you solve both of them you will see that   the intersect only at zero comma
zero
 so again i
request all my students to see whether they can   divide this area into finite
number of shapes
and add it to get the area of this region   this is the final example most
complicated and let
me tell you why i am giving these examples is that at the end we will be solving
all
these problems and will be seeing   how these problems can be taken care by
definite integrals
 so final example is area bounded between four curves and i have taken four
parabolas y square equals to four x   y square equals to sixteen x y equals to
four
x square and y equals to sixteen x square
 so if you plot them say this is
your y axis this is your x axis
 so y square equals to four x and y square
equals to 16 x they are having vertex 0 0 and   vertex 0 0 and axis as x axis
 so you get these two parabolas and y equals to four x square and y equals to
sixteen x square they are again parabolas but   whose vertex is zero zero
and axis is y axis
 so you get these two parabolas and
 so this is your y square equals to four x sorry y equals to four
x square and this is y equals to sixteen x square
 so this is a region again i request all of you to see whether
you can break it into finite number of   shapes whose area is known to you and
finally you   can can you compute this area by
known methods from earlier classes   and all these examples that i have
discussed
so far we will see at the end and we will solve   solve by using definite
integrals and compute the
required area

so let us define a definite integral a definite integral is defined in this form where a is called lower limit and b is called upper limit and this represents area of the function let us assume
 so that f x is positive
 so if you plot it this is y axis this is x axis is your function this is x equals to a this is x equals to b
 so this definite integral represents this area now the question is how to compute this   value a there are two methods to compute one is by limit of finite sums and another is by using anti derivatives we will see first how this   limit of finite sum evolved over the time let us see how this limit of finite sums has evolved
 so for that again i will consider one example and i will ask you to find out area bounded between y equals to 0 y equals to 1 plus x square x equals to 0 and x equals to 1.

 so let us plot this region first
 so this is your y axis this is your x axis
 so y equals to 1 plus x   square is again a parabola whose vertex is 0 comma 1 and whose axis is y axis
 so you get this shape   this is 0 comma 1 and this is x equals to 0 say this is x equals to 1 this is y   equals to zero this is y equals to one plus x square
 so we are looking for this shaded area
 so it is by definite integral the value of the area will be zero to one one plus x square d x
 so let us use the same trick which we were doing earlier and we were dividing the area into   finitely many shapes
 so what we do we divide this   area into two sub areas by taking this midpoint of the interval say this is x equals to half and then we draw rectangles like this and we compute area of these two rectangles say this area of this area of this rectangle is r one and this is r two
 so we will say that area of the two both of the rectangles l two is r   one plus r two r one is area the first rectangle smaller one and the r two is area the bigger one   now if we compute this we get half half is the width of this   rectangle and height is governed by function value at x equals to zero
 so we get one plus zero   then half into function value at half
 so we get one plus one by four this will be equal to one plus one plus one by four which we can write as one plus one by eight which is nine by eight   whose value is equal to sorry one point one two five now if you see   a is the actual area required and l two is a summation of area of these two rectangles
 so while calculating this area we have excluded this area
 so this l 2 is lesser than the area required

l 2 is lesser than the area required now let us try to approximate
the actual area by another method
 so for that we have to draw the figure again this is x this is y and this is
the parabola one plus x square say this is x equals to one this is x equals
to zero this is y equals to zero x axis now we divide again the sub area
this area into two sub areas   this is half this is one this is zero now
instead
of ah taking those rectangles which have taken   we take this as a rectangle and
this
as a rectangle and say this area is   r one bar and this is r two bar
 so if we
add r one bar and r two bar we get say u two and this value will be half into
function
value at half because height of that this   small rectangle is governed by a
function value
at half
 so you get one plus one by four plus   half into function value at one which
is one plus one
 so if we compute we get which is thirteen by eight and this is
equal to one point six two five now this sum that we have computed that
we u2 that we have computed is referred as upper sum and in the last
calculation we have computed l2 we have computed l2 that is referred as   lower
sum now here what we can see
is that u2 is always greater than u2 is always greater than actual area because
this much area   extra has been computed the required area
is this
 so this much area is added
 so u2 is u2 is greater than actual area and l2 is less than actual area the
value of l2 was 1.
125 now let us see how to how to get the actual area
 so what
we see from our computations that if we
 so in once we have computed l two we have taken this rectangle
and once you have computed   ah l two have taken this
rectangle and this rectangle
 so how to increase the accuracy
so what we do we if we divide   this area into further sub areas say
this is one by four this was half   this is three by four this is one
this is zero
 so now you can see that will be getting this rectangle
area of this rectangle   plus area of this rectangle this
rectangle and area of this rectangle
 so some more area will be included hence the value
will be this these two portions are now included   in our approximated area we
say this is l4 this is
another lower sum we refer as l four and l four is   summation of area of these
four rectangles this
will be equal to one by four into height of this   rectangle which is governed
by the function
value zero because function is increasing
 so we get zero one by four into function
value at zero
 so this is one plus   zero then one by four into one plus
function value at one by four

so we get   one by sixteen plus one by four function value at half
 so one plus one by four plus   one by four into function value at three by four
 so one plus nine by sixteen therefore   l four is equal to one by four four plus one plus
four plus nine by 16 which is equal to 1 plus 14 by 4 into 16 which is equal to 32 by 32 plus 7 by 32 which is 39 by 32 whose value is one point two one eight now recall that
your l two was one point one two five
 so what you see that l 2 is lesser than l 4
and l 4 is lesser than a because we are we have   left certain area while we have
approximated the actual area by rectangles now let us compute the approximate area again by dividing this into force of intervals and by taking these rectangles earlier we were having this these two rectangles
so we were having this much area extra
 so now this area will be neglected
 so u4
u4 stands for four intervals u4 will be lesser than u2 but it is actually greater
than the actual area
 so u4 the value of u4 is this time the the height of the first rectangle will be governed by function value at one by four
 so one by four into one plus one by sixteen
plus one by four into one plus one by four   plus one by four into one plus nine by sixteen
this is half this is three by four this is one   plus one by four into one plus one
 so this time
the height is governed by the function values at   this point this point this point and
this point
 so we have this u four   and value of u four is one by four again
four plus one plus four plus nine plus sixteen by sixteen
 so we get 1 plus 30 by 4 into 16 which is equal to 47 by 32 which is equal to 1.
46875
 so recall that value of u2 was   1.
625
 so finally from these calculation
what we are getting is that the lower sums satisfy this relationship and upper sums satisfy this relationship
 so if we have n sub intervals what what will
happen
 so every time we increase more number of   points in the between the interval will be
getting a value which is closer to actual area both from upper side and lower side
that is from lower sum and upper sum
 so if we increase the number of sub intervals upper sum decreases and lower sum increases but if we take finite number of sub sub
divisions sub intervals will never be able   to get the actual value
 so what we do if we take

the limit of these lns and uns we will see that both of these values will converge to a single
value and that will be your actual value of the integral that is 0 to 1 1 plus x square
d x let us see one example how we can use this trick to find out actual area the example is
let us take a function let f x be a continuous function on closed interval a b in addition
to this assume that f x is positive the reason behind this assumption
that it is easy to explain an area will lie only on one side of x
axis and assume that f x is increasing
 so first we will do it for increasing function
but this theory can be extended to any continuous function which is not increasing
 so let us
try to find out area of f x which lies between x equals to a and x equals to b and that is above x axis
 so let us draw the picture
since we have assume that f x is positive and increasing in the
interval a b we can assume the graph like this this is your f x this is x
axis this is y axis now divide a b into n sub intervals of equal length
 so say now number
the points say this is x naught this is x n
 so x naught is a x n is b
 so we have divided this
interval into n sub intervals of equal length so what will happen that this by this will give you
the length of each sub interval and say that is h since points are equally spaced
 so x k any point can be computed by this formula where k goes from 1 to n
 so we have the situation what we have done we have divided this area into sub areas by taking
equally spaced points points on x axis now let us define ln lower sums
 so this is x naught this is x one this is
x two this is x n this is x n minus one
 so to define l ln we find out area of these rectangles which lie below the curve
 so ln is this width into this height
 so since the
points are equally spaced
 so height of width of each sub interval width of each
rectangle is h
 so ln is h into f x naught plus h into f x one for the last one the height will be governed
height of the rectangle will be governed by the function value at x n minus one
 so h into f x
n minus one
 so we can write ln as summation f x k into h k goes from zero to n minus one similarly we can define u n for
u n we construct the rectangles like this since the function is increasing u n will
be upper sums will be defined as h into function value at x one because the function

is increasing
 so height of this rectangle   will be governed by the function value at
x one
 so h into f x one plus h into f x two for the last one we will get h into   f x
n
 so we can write this also as
summation of k goes from 1 to n f x k into h
 so what we have seen that ln is summation
k goes from 0 to n minus 1 f x k into h and u n is summation k
goes from one to n f x k into h   from the previous discussion we
have seen that elements always   they are lesser than actual area and u
ns are always greater than actual area and their limiting values   gives you the
actual area
 so if you take the
limit of l n that is if you take the limit of k equals to zero to n minus
one f x k into h this gives you the area of the function which lies
between x equals to a x equals to b
 so you can see that how this integral is related
with summation from this formula you can evaluate   the integral by using either
lower sums or upper
sums value of the integral is not going to change
 so if you you can use uns also to compute the   area of the curve given curve
but
lying between a and b above x axis now let us solve some examples
and see how this theory works
 so find out area between y equals to zero y equals to one plus x
square x equals to zero and x equals to one this is the same curve which you
already
drawn
 so i am not going to explain it again
 so we got this area now divide the interval into n sub intervals
of same size
 so this is your 0
 so x naught is 0   x n is 1 x this is x 1 this is x n minus
1 and
 so on
 so since you are dividing the   interval 0 1 into n sub intervals of
equivalent
so 1 minus 0 by n will be giving you h   and by this formula that x k is x
naught plus
k h x 1 will be 0 plus h is 1 by n
 so you get   1 by n into k
 so x k is
 so x k is k by n
 so x one is one by n x two is two by n and x
n minus one is n minus one by n and x n is one let us write ln for this
 so ln will be h which is the length of each sub
interval
 so ln in ln is summation of   all these rectangles which lie below the curve h
into f x naught
 so that
is one plus zero plus h into   one plus function value at x one
 so one
plus one square by n square plus last   rectangle this one h into 1 plus the

height of the function value at x n   minus 1 because the function is increasing
and the rectangle is lying below the curve
 so we get this
 so ln is let me write it again h into h is common
throughout
 so we get 1 plus this is n times   and then we get 1 square plus 2 square
plus n minus one square by n square since we have proved that h is one by n
 so i can
replace h by one by n we get here the summation   as n plus the semi the value
of the
summation is well known to you you can write this summation of one square plus
two square plus n minus one square as this
 so this is equal to 1 plus
1 by 6 1 minus 1 by n n by n   2 minus 1 by n is ln now take limit of this
ln as n tends to infinity we get 1 plus   1 by 6 into 2 this is equal to 1 plus
1
by 3 which is four by three
 so that is a integral of one plus x square   from zero to one
 so you can see that how this
process of limit of sums can be used to compute   area of uh under area under a
given curve which
lies between say x equals to n and x equals to b   above x axis let us see one
more example
 so that
you be more comfortable
 so compute area between y equals two e to the power minus x y equals
to zero x equals to zero and x equals to one
 so this is your y axis this is your x axis
and e power minus x can be drawn like this
 so this is x equals to 0
this is x equals to 1 this is   y equals to e power minus x this is y equals to
0.

 so again similar to the previous
case you need to divide the interval   x equals to 0 but and x equals to 1
the interval 0 1 into n sub intervals
 so again will be 1 minus 0 by and that
will be length of the each sub interval and x 1 x k will be x naught plus k h
here
x naught is zero
 so x k is zero plus k   into one by n
 so x one is one by n x two is 2 by n
and
 so on point to note here that if you write l n   you will be getting h times
function value
but here since the function is decreasing   this time the function value will
not be governed
by the function value at x equals to zero but   function value at x one so
height of this rectangle will   be governed by the function value at
x one because function is decreasing   similarly for the last one it will be
functional
function value at x equals to one
 so ln is h into f x one plus for the second one function

value at function value at x two for the height of the rectangle for
lower sums x two plus h into f x n here your function f x is 0 minus x
 so you
get ln as h into e to the power minus one by n
 so h is common
 so we can write it outside
two by n plus e to the power minus one
 so we got l n as h times e to the power minus one by n plus e to the
minus two by n plus e to the power minus one you can write one more term before
this you get n minus one by n
 so this is a geometric progression and
you can write the summation of this by simple formula you get this which is
equal to let me replace 1 by n by h and
multiply by e to the power h
 so you get multiply and divide
 so you get e to the power h minus 1
and this will give you e to the power minus 1 you know that when n tends to
infinity s tends to 0 h tends to 0 n tends to infinity from this
relationship you can see that s tends to 0
 so limit of l n as n tends to infinity will
be same as limit of ln as h tends to zero and this limit will be equal to
one by e to the power minus one e to the power minus one the reason behind that
is the limit of h by e to the power h minus 1 as h tends to 0 is 1
 so you can see that by this method
you can compute you could compute the value of the integral from 0 to 1 e to
the power minus
x dx that as 1 minus u minus 1 let us see how anti-derivatives can be used
to solve definite integrals till now we have seen how to use limit of sums
and find out the value of the different integrals let us see how
anti-derivatives can be used to
find out definite integrals
 so anti derivatives
 so before we start solving problems
we have to discuss certain concepts
 so let us take a function which
is positive and continuous and let us draw it this is a this is b
 so this function represents area function it
it is it is representing this shaded area and this is known as area function if
i put b in place of x it will be giving you
the area under the curve which lies between x equals to n x equals to b above
x axis
 so by using
this area function we state two important theorems one is known as ah
fundamental theorem
of calculus one and other is fundamental theorem of calculus two
 so let
us discuss first fundamental theorem of calculus
 so it says that if you have a
function if f x is continuous on close interval a b and area function is defined
as a to x f x d x then a dash x equals to f x the other theorem which is known
as
second fundamental theorem of calculus will actually be used in computing the
definite integrals and it says that let f x be a continuous function on a b
close interval a b and capital

f x is an anti derivative of small f x that is f dash x is equal to f x then a
to b f x d x is equal to f x x equals to a to x equals to b
which we write as f b minus f a
 so this theorem can be used to evaluate definite
integrals provided we know the anti-derivatives   let us solve some problems and
see how to use this
theorem to evaluate the different integrals
 so we   already evaluated certain integrals and we will
take the help of those
 so we have seen that the value of this integral which represents
area under the curve one plus x square   which lies between x equals to zero and
x equals
to one above x axis is equal to four by three   and which we got from the
process of limit of sums
now apply the method of anti-derivatives and see   whether you are getting the
same value or not so
by this theorem the value of this integral will be f x from x goes from zero to
x goes to 1 where f
x is anti derivative of 1 plus x square that is   f dash x is 1 plus x square
 so we can easily find out that anti
derivative of one plus x square will be this
 so anti derivative of one plus x square is x
plus x cube by three
 so value of the integral is x plus x cube by 3 x goes from 0 to 1 by applying
the
theorem we see that we get 1 plus 1 by 3   minus 0 which is equal to 4 by 3 and
which is
same as the value that we got by the limit of sums   let us take another example
we have find out the
area lying between x equals to 0 and x equals to   1 of the curve below the
curve e power minus x
which lies above x axis and we have seen that it   is 1 minus e to the power
minus one now if you
apply second fundamental theorem of calculus that is this
 so this will be equal to you
can easily see that d by d x of this   is e o minus x that means minus e power
minus x is anti-derivative of e minus x
 so by the theorem you can write it like this and which is equal to one minus e
to the power minus
one and again you can see that   this value which you have got from the
limit of sums is same as the value which   you have got by using anti
derivatives
now in the next class we will be learning   more about properties of definite
integrals
and solve more complicated problems thank you you