

welcome to the next lecture on derivatives

so in this lecture we will be learning about the rate of change of quantities
so in this lecture we will see application of derivatives to calculating rate of change of quantities

so here what we have is let t denotes the time and let x and y be two quantities depending on the time t

so x and y are functions of t

so x is some x of t and y is also a function of time t now suppose we are given x as a function of t suppose we are given y as a function of x now the rate of change of x and y are the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$ of x and y with respect to time t

so by rate of change of a quantity we mean the derivative with respect to time t

so if $\frac{dx}{dt}$ that is the rate of change of x is known then the rate of change of y $\frac{dy}{dt}$ can be calculated using the chain rule as follows

so we know that y is given as a function of x

so $\frac{dy}{dt}$ by chain rule can be written as $\frac{dy}{dx} \times \frac{dx}{dt}$

so if y as a function of x is known we can calculate $\frac{dy}{dx}$ and $\frac{dx}{dt}$ is the rate of change of x which is assumed to be known

so then $\frac{dy}{dt}$ can be calculated

so let us look at some examples the first examples suppose it is given that the radius of a circle is increasing at the rate of three centimeter per second find the rate of change of the area of the circle when the radius is 10 centimeter

so what is given is here that the radius of circle this is increasing at 3 centimeter per second and we have to find the rate of change of the area of the circle when radius is 10 centimeter

so what here we have two quantities one is radius another is area

so we know that area of the circle let me write A is given by πr^2 where r is the radius of the circle and then what is given is that the rate at which radius is increasing this $\frac{dr}{dt}$ is equal to 3 centimeter per second and what we have to find is what is $\frac{dA}{dt}$ when r is equal to ten centimeter

so we know that $\frac{dA}{dt}$ is equal to the derivative $\frac{dA}{dr} \times \frac{dr}{dt}$ A is given to be πr^2

so $\frac{dA}{dr}$ is $2\pi r$ times $\frac{dr}{dt}$ and $\frac{dr}{dt}$ is given to be three centimeter per second

so this is $2\pi r$ into three centimeter per second

so $\frac{dA}{dt}$ when r is 10 centimeter is 2π into 10 centimeter times 3 centimeter per second which gives $2 \times 10 \times 3$ is 60π centimeter square per second

so this gives the rate at which the area is increasing the area is increasing at 60π centimeter square per second when radius is 10 centimeter let us look at next problem

so here we are given the volume of a cube is increasing at the rate of 8 centimeter cube per second then the question is how fast is the surface area increasing when the length of an edge of the cube is 12 centimeter

so again lets see what is given and what we want

so let x be the length of an edge of the cube then volume V is x^3 the volume of the cube is the length of the edge cubed and surface area let me write this as A this is equal to $6x^2$ because there are six faces of the cube each is a square of side x

so we are given volume and surface area in terms of the length of the edge of the cube and then what is given is the rate at which volume is increasing

so $\frac{dV}{dt}$ is equal to 8 cubic centimeter per second and we have to find the rate at which surface area that means $\frac{dA}{dt}$ what is $\frac{dA}{dt}$ when x is equal to 12

centimeter

so if you see in this problem we have actually three quantities which are depending on time one is x another is the volume which is x^3 and surface area which is $6x^2$ and we are given $\frac{dv}{dt}$ we want $\frac{da}{dt}$

so if you look at this expression for v equal to x^3 since v equal to x^3 $\frac{dv}{dt}$ will be equal to $\frac{dv}{dx} \times \frac{dx}{dt}$ this is by chain rule and $\frac{dv}{dx}$ is $3x^2$ times $\frac{dx}{dt}$ now we are given $\frac{dv}{dt}$

so we can calculate $\frac{dx}{dt}$ this implies $\frac{dx}{dt}$ is equal to one by $3x^2$ times $\frac{dv}{dt}$ and $\frac{dv}{dt}$ is given to be 8 centimeter cube

so this is $\frac{1}{3x^2}$ times eight centimeter cube per second

so now we know what is $\frac{dx}{dt}$ and we have area surface area is a function of x

so if we know $\frac{dx}{dt}$ we can calculate $\frac{da}{dt}$ equal to $6x$ square this implies $\frac{da}{dt}$ is equal to $6x \times \frac{dx}{dt}$ and $6x$ times $\frac{dx}{dt}$ we have calculated is $\frac{1}{3x^2}$ times eight centimeter cube per second

so this can be simplified and we get x cancels and 3×12 by 3 is 4

so this is 32 divided by x centimeter cube per second

so therefore what we have to calculate what is $\frac{da}{dt}$ when x is equal to 12 centimeter is equal to 32 by 12 centimeter centimeter cube per second

so this is equal to $\frac{8}{3}$ centimeter square per second

so thus the surface area is increasing at the rate of $\frac{8}{3}$ square centimeter per second when x is equal to 12 centimeter

so in this problem we saw that we have two different quantity depending on x and each depends on t and then we had to calculate if we are given the rate of change of one quantity we can calculate the rate of change of the other the third problem that we will do here we have a rectangle and we are given that the length x of a rectangle is decreasing at five centimeter per minute and the width y is increasing at four centimeter per minute now when the length x is eight centimeter and with y is six centimeter find the rates of change of a the perimeter and b the area the area of the rectangle let us see what is given we have a rectangle and let us say its length is x and width is y and we are given $\frac{dx}{dt}$ note that here it is given that the length x is decreasing at five centimeter per minute

so since x is decreasing $\frac{dx}{dt}$ is negative

so this is minus five centimeter per minute and $\frac{dy}{dt}$ the width is increasing at four centimeter per minute

so this is four centimeter per minute now let p and a denote the perimeter and the area of the rectangle respectively

so what we have to calculate is what is $\frac{dp}{dt}$ and $\frac{da}{dt}$ when x is

so to find $\frac{dp}{dt}$ and $\frac{da}{dt}$ when x is equal to eight centimeter and y is equal to six centimeter

so what we know is that the perimeter is equal to two times x plus y two times the length plus width and area is x times y

so therefore if we calculate $\frac{dp}{dt}$ is equal to 2 times $\frac{dx}{dt}$ plus $\frac{dy}{dt}$ and $\frac{dx}{dt}$ is given to be minus 5

so this is 2 times minus 5 plus $\frac{dy}{dt}$ is 4 centimeter per minute

so this gives us minus 2 centimeter per minute thus the perimeter is decreasing at the rate of 2 centimeter per minute now for area a is equal to x times y

so $\frac{da}{dt}$ here area is product of x and y

so by product rule this is equal to $\frac{dx}{dt} \times y$ plus $x \times \frac{dy}{dt}$ $\frac{dx}{dt}$ is minus 5 this is minus 5 y plus $\frac{dy}{dt}$ is 4 x therefore the rate at which area is changing when x is equal to 8 and y is equal to 6 this gives minus 5 times 8 plus 4 times 6 sorry minus 5 times 6 plus 4 times 8

so this is minus thirty plus thirty two

so this is two centimeter square per minute

so the area is increasing at the rate of two square centimeter per minute ok
so the next problem is a spherical balloon is being inflated by pumping in 900 cubic centimeter of gas per second find the rate at which the radius is increasing when the radius is 15 centimeter

so what we have here is we have a sphere

so the volume of the sphere is given by $4/3 \pi r^3$ where r is the radius of a sphere and v is the volume what is given is the rate at which volume is increasing dv/dt this is 900 centimeter cube per second and we need to find what is the rate at which the radius is increasing that means dr/dt when radius is 15 centimeter

so since v is equal to $4/3 \pi r^3$ dv/dt is equal to $4 \pi r^2 dr/dt$

so this is equal to $4 \pi r^2 dr/dt$ we are given that v is dv/dt is 900 centimeter cube per second

so therefore dr/dt is equal to $1/4 \pi r^2$ times dv/dt which is equal to $1/4 \pi r^2$ times 900 centimeter cube per second and then we have to calculate this when r is 15 centimeter

so when r is 15 centimeter dr/dt is $1/4 \pi$ times 15 centimeter square times 900 centimeter cube per second and this gives nine hundred divided by four pi into fifteen into fifteen centimeter per second

so we get this is equal to one by pi centimeter per second

so the radius is increasing at $1/4 \pi$ centimeter per second when r is 15 centimeter ah let us look at another problem

so here we have a ladder five meter long which is leaning against a wall now the bottom of the ladder is being pulled along the ground away from the wall at the rate of two centimeter per second

so the question is how fast is its height on the wall decreasing when the foot of the ladder is 4 meter away from the wall

so lets try to understand this problem we are given a ladder five meter long

so let us say this is wall and this is the ground we have a ladder this length is given to be 5 meter and let us say at this some time x is the distance of this foot of the ladder from the wall and y is the height of the ladder on the wall now what is given is this bottom of the ladder the foot of the ladder is being pulled away from the wall at two centimeter per second

so this x given dx/dt is equal to 2 centimeter per second because it is pulled away from the wall x is increasing with time

so this is with positive sign and we have to calculate to find dy/dt when x is equal to 4 meter

so when the ladder is pulled along the wall this x is increasing and y will be decreasing

so dy/dt will come out to be negative now how do we find this lets see what is the relation between x and y

so we have because we have a right angle triangle here by the pythagoras theorem $x^2 + y^2$ must be equal to five square

so if we differentiate this with respect to time differentiating with respect to t we get $2x dx/dt + 2y dy/dt$ right hand side is constant

so the derivative is 0 and this implies that dy/dt is equal to minus x by y times dx/dt

so you can see from this equation that if x is increasing dx/dt is positive and then dy/dt will be negative now when x is equal to 4 meter what is y y is equal to square root of 5 square minus 4 square

so this will be 3 meter when x is 4 meter

so therefore when x is 4 meter dy/dt is equal to minus x is 4 meter divided by y is 3 meter times dx/dt which is given to be 2 centimeter per second

so we get this is equal to minus eight by three centimeter per second
so the height is decreasing at the rate of eight by three centimeter per second
ok lets look at another problem here we are given a particle moves along the curve whose equation is given as $6y$ is equal to x^3 plus two we need to find the points on the curve at which the y coordinate is changing eight times as fast as the x coordinate

so we are given equation of the curve and we need to find points
so to find x, y such that $\frac{dy}{dt}$ is equal to eight times $\frac{dx}{dt}$
so from this equation $6y$ is equal to x^3 plus two this implies that $6 \frac{dy}{dt}$ is equal to $3x^2 \frac{dx}{dt}$ which implies $\frac{dy}{dt}$ is $\frac{x^2}{2} \frac{dx}{dt}$ now we need to find this x, y such that $\frac{dy}{dt}$ is 8 times $\frac{dx}{dt}$ for that if $\frac{dy}{dt}$ is 8 times $\frac{dx}{dt}$ this implies that this factor $\frac{x^2}{2}$ must be equal to eight which implies x^2 is sixteen

so x has to be plus or minus four now we need to find the y coordinate also
when x is equal to 4 y is equal to x^3 plus 2 divided by 6 which is 64 plus two divided by six sixty six by six that means eleven

so y is eleven and when x is equal to minus 4 and y will be minus 4 cubed plus 2 divided by 6 this is equal to minus 62 by 6.

or minus thirty one divided by three

so so the required points are four comma eleven is one point and minus four comma minus thirty one by three is another point ok now this rate of change is also used in economics

so let us look at application of rate of change in economics

so suppose x is the number of units of an item produced by some industry

so x is the number of units of an item produced and c of x denotes the cost of producing x units

so this will depend on the number of unit the cost that the company has to spend on producing x unit that is c of x and r of x this denotes the revenue obtained by selling x units of the item

so that means that what is the amount that the company will obtain if they sell x units that is r of x and then of course profit is revenue minus the cost this again depends on the number of units x produced and sold now here there are some terminologies which are used

so marginal cost say m_c of x this is defined to be the rate of change of c of x with respect to x that is when we write marginal cost which is a function of the number of unit x this is equal to the derivative of c with respect to x similarly the marginal revenue this will denote by m_r of x this is by definition the derivative of revenue with respect to x

so again if you see that we have to calculate marginal cost or marginal revenue then we need to take the derivative with respect to x

so for example suppose the total cost c of x in rupees for producing x units of an item is given by c of x is point zero zero seven x^3 minus point zero zero three x^2 plus fifteen x plus four thousand

so this formula is given that the cost incurred for producing x unit is given by this formula now what we have to do is we have to find the marginal cost when 17 units are produced

so all we have to do is we need to take the derivative of this cost function with respect to x

so marginal cost at x is $\frac{dc}{dx}$ which is equal to you take the derivative of this

so x^3 will give three x^2 this is point zero two one x^2 minus derivative of x^2 is two x point zero zero six x plus fifteen

so when x is seventeen we need to calculate the marginal cost when x is seventeen this is point zero two one times seventeen square minus point zero

zero six times seventeen plus fifteen and this is point zero two one times two eighty nine minus this will give point one zero two plus fifteen and this i think if you calculate get 20.

967 this is the marginal cost similarly if revenue is given

so the next example the total revenue obtained by selling x units is given by r of x is $13x^2$ plus $26x$ plus 15 find the marginal revenue when x is equal to seven

so again if you know the definition marginal revenue is the derivative of r with respect to x this is equal to $26x$ plus 26

so therefore when x is seven this is equal to 26×7 plus 26 which is equal to 208 now suppose we want to maximize the profit then what should we do

so we know that profit p of x is the revenue r of x minus the cost c of x now to maximize the profit therefore if we take the derivative p' of x this is equal to r' of x minus c' of x which is the marginal revenue minus the marginal cost

so in order to maximize the profit the number of units x should be such that the derivative p' of x is equal to zero that is marginal revenue of x should be equal to marginal cost of x

so if we equate the marginal cost and marginal revenue

so by equating the marginal revenue and the marginal cost we get the values of x for which the profit is maximum

so we will stop here for this lecture and in the next lecture we will see some more applications of derivatives thank you