welcome students to the fifth lecture on determinant as i said towards the end of the last lecture that in this class i will be talking about matrix inverse however before going into discussing matrix inverse let me solve one problem on determinants what is the determinant of a is equal to a a square a cube minus 1 a to the power omega a to the power 2 omega a to the power 3 omega minus 1 and a to the power omega square a to the power 2 omega square a to the power 3 omega square minus 1 where omega is cube root of unity and we know that one plus omega plus omega square is equal to zero if we look at the matrix we see that all the elements in the third column is actually summation of two quantities a cube and minus one a to the power three omega and minus 1 and a to the power 3 omega square minus 1 we know that in such situations we can write the determinant as sum of two determinants and we do exactly that one therefore the determinant of a is equal to determinant of a a square a cube a to the power omega a to the power 2 omega a to the power 3 omega and a to the power omega square a to the power 2 omega square a to the power 3 omega square minus the determinant of a a square 1 a to the power omega a to the power 2 omega and 1 a to the power omega square a to the power 2 omega square and 1.

therefore the determinant of original matrix a i have written as determinant of the difference between the determinant of this matrix and the second matrix let us call them a 1 and a 2.

now determinant of a 1 is equal to if we look at it we can take a common from the first row we can take a omega a to the power omega common from the second row and we can take a to the power omega square common from the third row therefore if we take the terms out what we get a dot a to the power omega dot a to the power omega square into determinant of now if we take a out of the first row it becomes one a a square if i take a to the power omega out of the second row it becomes 1 a to the power omega a to the power 2 omega and if i take a to the power omega square out of the third row it becomes 1 a to the power omega square into a to the power 2 omega square is equal to a to the power 1 plus omega plus omega square into determinant of 1 a s square 1 a to the power omega a to the power 2 omega and one a to the power omega square a to the power two omega square since this quantity is equal to zero therefore determinant of a one is equal to determinant of one a square one a to the power omega a to the power 2 omega 1 a to the power omega square a to the power 2 omega square now determinant of a two is equal to determinant of a a square one a to the power omega a to the power two omega one a to the power omega square a to the power two omega square and one now if we check matrix a 1 and matrix a 2 we find that we can get a two from a one by first interchanging column one with column three therefore this one one one goes here and this column comes as the first column after that if we swap these two then we get this matrix then interchanging column one and column two we know that if we interchange two columns or two rows then determinant remains the same in the absolute value but its sign changes therefore with each interchange there is a change of a sign from plus to minus since we get a two from a one by two interchanges determinant of a 2 is equal to minus 1 square determinant of a 1 or determinant of a two is equal to determinant of a one therefore determinant of a is equal to determinant of a one minus determinant of a two is equal to zero any matrix whose determinant is 0 is called a singular matrix likewise a matrix with non zero determinant is called a non singular matrix the concept of singular and non-singular matrix is very important in computing inverse of a matrix

so if a is a non-singular matrix we can find another non singular matrix actually both of them are square matrix because we are talking about their determinants

so when we are talking about the other matrix b of the same dimension or of the

same order of a such that a multiplied by b is equal to b multiplied by a is equal to identity matrix of order n where a and b are n crossing matrices question is how to obtain a inverse given ah n cross n matrix a in this lecture we shall look at computing a inverse with the help of determinants we shall compute a inverse with the help of determinant of a and add joint of a all of you are familiar with adjoint of a because in the last lecture we have discussed it in detail we know that a into adjoint of a is equal to determinant of a into i n therefore a into adjoint of a divided by determinant of a is equal to identity matrix therefore we often a matrix adjoint of a upon determinant of a such that a times adjoint of a and by determinant of a is equal to identity and similarly adjoint of a upon determinant of a is equal to multiplied by a is equal to identity since a is non-singular determinant of a is non-zero therefore division by determinant of a makes sense and therefore this way we can get hold of a matrix namely adjoint of a divided by determinant of a such that which when multiplied by a either pre multiplication or post multiplication it gives identity now question is can there be two inverses of a matrix answer is no why if possible let b and c b two inverses of a therefore a times b is equal to b times a is equal to identity and a times c is equal to c times a is equal to identity therefore c is equal to identity matrix multiplied by c is equal to b times a multiplied by c is equal to b times a multiplied by c is equal to b times identity is equal to b therefore we get c is same as b or in other words the two inverses of a are equal this shows that we can have a unique identity corresponding to each non-singular matrix say called a and we have seen that we can compute that identity matrix or we can find that identity matrix by computing the adjoint matrix corresponding to a and dividing it by the determinant of a an interesting result what is the inverse of a b where a and b are two n crossing matrices we know that a b multiplied by b inverse a inverse is equal to a b b inverse a inverse is equal to a i a inverse is equal to a a inverse is equal to identity therefore we can say by uniqueness of inverse that a b inverse is equal to b inverse multiplied by a inverse another result if a and b are non-singular matrices then adjoint of a b the product matrix is equal to adjoint of b multiplied by adjoint of a proof consider a b times adjoint of b into adjoint of a is equal to by associativity of matrix multiplication a multiplied by b times adjoint of b multiplied by adjoint of a now we know that b into adjoint of b is equal to determinant of b into identity into adjoint of a is equal to determinant of b into a multiplied with identity multiplied with adjoint of a is equal to determinant of b multiplied by a into adjoint of a is equal to determinant of b into determinant of a into identity since adjoint of a b is such that a b multiplied by adjoint of a b is equal to determinant of a b into identity therefore is equal to determinant of a into determinant of b into identity therefore we find that e b into adjoint of a b is equal to a b into adjoint of b into adjoint of a is equal to determinant of a into determinant of b into identity therefore by multiplying both sides with a b inverse we get adjoint of a b is equal to adjoint of b into adjoint of a let me give an example compute the inverse of a is equal to 1 minus 1 2 0 2 minus 3 3 minus 2 four now determinant of a is equal to one into two into four minus minus 3 into minus 2 if i expand along the first column plus 3 into minus 1 into minus 3 minus 2 into 2 is equal to 8 minus 6 plus 3 into 3 minus 4 is equal to 2 minus 3 is equal to minus 1 therefore a is non-singular since this is not equal to 0 therefore we can compute a inverse by computing first the adjoint of a and then dividing it by the determinant namely minus one now a is equal to one minus one two zero two minus three three minus two four therefore a one one is equal to 4 into 2 minus minus 3 into minus 2 is equal to 8 minus 6 is equal to 2 a 1 2 is equal to cofactor of this is equal to minus 1 whole to the power 1 plus 2 0 into 4 minus

minus 3 into 3 is equal to minus 9 a 1 3 is equal to zero into minus two minus three into two is equal to minus six a two one is equal to minus 4 minus 2 into minus 2 is equal to 0 a 2 2 is equal to four into one minus three into two is equal to minus two and a two three is equal to minus one to the power two plus three multiplied by one into minus two minus minus one into three is equal to minus minus two plus three is equal to minus one a three one in a similar way is equal to three minus four is equal to minus one a three two is equal to minus three minus zero

so minus three but the sign will change therefore plus three and a three three is equal to 2 minus 0 is equal to 2 therefore at joint of a we get by writing the transpose of that matrix is equal to 2 minus 9 minus 6 0 minus 2 minus 1 minus 1 3 2 therefore a inverse is equal to we get by dividing this by minus 1 and therefore that is going to be minus 2 0 1 9 2 minus 3 and 6 1 minus 2 verify that a times a inverse is equal to i three that is identity of order three and a inverse dot a is also i3 this i leave as an exercise for you now the question is what is the determinant of a inverse that means if we have the determinant of a then what is going to be the determinant of a inverse we know that a into a inverse is equal to identity therefore determinant of a into a inverse is equal to determinant of i is equal to one now since determinant of product is equal to product of the determinant therefore determinant of a into determinant of a inverse is equal to one or determinant of a inverse is equal to one upon determinant of a therefore if we know the determinant of a then we know that its reciprocal is going to be the determinant of the inverse example what is the inverse of a is equal to 1 0 0 0 cos alpha sine alpha and zero sine alpha minus cos alpha we need to compute the inverse of this matrix how do we do it

so we first compute the cofactors of a a 1 1 is equal to cos alpha into minus cos alpha minus sine alpha into sine alpha is equal to minus cos square alpha minus sin square alpha is equal to minus one a 1 2 is equal to 0 times minus cos alpha minus zero times sine alpha is equal to zero a 1 3 is equal to 0 times sine alpha minus 0 times cos alpha is equal to zero a two one is equal to zero times minus cos alpha minus zero times sine alpha is equal to zero a two two is equal to minus cos alpha minus zero is equal to minus cos alpha in a similar way a two three is equal to minus one to the power 2 plus 3 into 1 into sine alpha minus 0 is equal to minus sin alpha a three one is equal to zero into sin alpha minus zero into cos alpha is equal to zero a three two is equal to minus one times one into sin alpha minus zero is equal to minus sin alpha and finally a three three is equal to sine alpha zero in a three three is equal to 1 into cos alpha minus 0 is equal to cos alpha therefore adjoint of a is equal to minus 1 0 0 0 minus cos alpha minus sin alpha zero minus sine alpha cos alpha therefore a into adjoint of a is equal to 1 0 0 0 cos alpha sine alpha 0 sine alpha minus cos alpha multiplied by minus 1 0 0 0 minus cos alpha minus sin alpha zero minus sin alpha cos alpha is equal to first row first column is equal to minus one first row second column is equal to zero first row third column is again zero second row first column is equal to zero second row second column is equal to minus cos square alpha minus sin square alpha and second row third column is equal to minus cos alpha sin alpha plus cos alpha sin alpha third row first column is zero third row second column is equal to minus cos alpha sine alpha plus cos alpha sine alpha and the third row third column is equal to minus sine square alpha minus cos square alpha therefore a into adjoint of a is equal to minus 1 0 0 0 minus 1 0 0 0 minus 1 that is minus 1 times identity of order 3 is equal to determinant of a into identity of order 3 therefore also we can calculate the inverse of a is adjoint of a divided by determinant of a is equal to minus one zero 0 0 minus cos alpha minus sin alpha 0 minus sin alpha into cos alpha whole divided by minus 1 is equal to 1 0 0 0 cos alpha sine alpha and 0

sine alpha minus cos alpha
 so this is the inverse of the given matrix i want you to verify that a into a inverse is equal to a inverse into a is equal to identity matrix let me do another example show that the matrix one two 2 2 1 2 2 2 1 satisfies the equation a square minus 4 a minus five times i three is equal to o where o is the 0 matrix of order 3 cross 3 also find out the inverse of the above matrix let me call it a since a is equal to 1 2 2 2 1 2 2 2 1 a square is equal to a into a is equal to 1 2 2 2 1 2 2 2 1 multiplied by 1 2 2 2 1 2 2 2 1 is equal to 9 8 to 8 8 9 8 8 8 9 therefore s square minus 4 a is equal to 9 8 8 8 9 8 8 8 9 minus 4 times 1 2 2 2 1 2 2 2 1 is equal to 5 0 0 0 5 0 0 0 5 is equal to 5 times the identity matrix of order 3.
 therefore a square minus four a is equal to five i therefore a square minus four a minus five i is equal to 0 matrix of order 3 cross 3 therefore a square minus 4 a is equal to five i pre multiplying by a inverse a inverse into a square minus four a is equal to five times a inverse into i or a minus 4 into identity matrix is equal to 5 times a inverse therefore a inverse is equal to a minus 4 times identity matrix divided by 5 is equal to 1 2 2 2 1 2 2 2 1 minus four zero zero zero four zero zero zero four divided by five is equal to minus 3 to 2 2 minus 3 2 2 two minus three divided by five is equal to minus three by five two by five two by five two by five minus three by five two by five two by five minus three by two by five minus three by five
 so once a matrix equation is given like this we could very easily compute the inverse of the given matrix a i want you to verify a into a inverse is equal to a inverse a is equal to identity i finish this lecture with a tricky question what is the determinant of adjoint of adjoint of a when determinant of a is given we know determinant of a into adjoint of a is equal to determinant of a into determinant of adjoint of a is equal to determinant of a into identity of order n where a is n cross n is equal to determinant of determinant of a into identity of n where a is n cross n is equal to determinant of this matrix that is identity multiplied by determinant of a of the order of n cross n is equal to determinant of a whole to the power n therefore determinant of a into determinant of adjoint of a is equal to determinant of a to the power n therefore determinant of adjoint of a is equal to determinant of a whole to the power n minus 1 therefore determinant of adjoint of adjoint of a is equal to determinant of a to the power n minus 1 whole to the power n minus 1 therefore given a matrix a we can compute determinant of a inverse determinant of adjoint of a and also determinant of adjoint of adjoint of a okay friends i stop here today in the next class i shall show some application of determinants particularly in solving systems of linear equations thank you you