welcome students to the third lecture on determinants in the last two lectures we have defined what is a determinant for a square matrix and also we have defined or we have seen several properties of a determinant in this lecture we shall compute the determinant of several matrices by using those properties
 so that the original matrix gets simplified and computation of the determinant becomes easier consider example one find the determinant of a is equal to one x x square x square one x x x square one of course we can compute its determinant from the first principle that is by expanding along one row or one column but we shall exploit some properties of determinants
 so that we can make it simple since we know that by adding two rows we do not change the value of the determinant
 so we will try to use that property here say for example i change the row 1 as the summation of row one and row two therefore determinant of a is equal to the determinant of by adding row one and row two we find one plus x square x plus 1 x square plus x row 2 remains the same row three also remains the same now i can see that we have one plus x square one plus x x square plus x in the first row but in the third row we have x we have x square and we have one
 so what happens if i replace the first row as the sum of first row and third row then determinant of a is equal to determinant of 1 plus x plus x square now my adding this to this 1 plus x plus x square and adding 1 to this 1 plus x plus x square other rows remain the same now what we find that in the matrix all the elements of the first row are one plus x plus x square therefore determinant of a is equal to determinant of say 1 plus 6 plus x square into determinant of 1 1 1 x square 1 x and x x square 1 now we do very similar thing with respect to the columns we replace column one by the difference of column one and column two
 so we write column one is now c one minus c two therefore determinant of a is equal to 1 plus x plus x square into after subtraction i get 0 1 1 x square minus one one x x minus x square x square one now what we do we subtract column three from column two after doing that we find that determinant of a is equal to 1 plus 6 plus x square into 0 x square minus 1 x into 1 minus x 0 1 minus x x square minus 1 1 x 1 what is the advantage the advantage is that now in the first row we have two zeros and only one non-zero element which is one therefore if i now expand it along the row 1 i get that determinant of a is equal to 1 plus x plus x square into determinant of 0 0 1 x square minus 1 1 minus x x x into 1 minus x x square minus 1 into 1 is equal to 1 times this minus this which is is equal to x square minus 1 whole square minus x into 1 minus x whole square is equal to 1 plus x plus x square now i can take x minus one whole square common from both the terms
 so let me write it as then what is left here we are left with x plus 1 whole square minus x is equal to 1 plus x plus x square into x minus 1 whole square into this gives me x square plus 6 plus one because the two x term will come and one x is subtracted from there this is is equal to x minus 1 into x square plus x plus 1 into another x minus 1 into x square plus 6 plus 1 which is is equal to 1 minus 6 cube whole square
 so by doing the elementary row operations or column operations where we have where we are replacing one column by the difference between two columns or one row by the sum or difference of two rows we simplified the whole thing to get the answer to be one minus x cube whole square ok students
 so let me do a numeric problem find the determinant of root eleven plus root 3 root 20 root 5 root 15 plus root 22 root 25 root 10 3 plus root 55 root 15 root 25 we know that if some row or sub column of a matrix each element can be written as a sum of two quantities then we can write the entire determinant of the sum of two determinants since we have the addition sign here we can think that we can partition it into two parts but we have to be little bit more

careful because root 11 is there and root 22 is there and root 55 is there we can see both of them have the term root eleven

so we will be judicious in separating it

so let me write it as determinant of a is equal to first matrix is root 11 root 22 root 55 remaining things are same root 20 root 25 root 15 root 5 root 10 root 25 determinant of this plus determinant of root 3 root 15 and 3 which we write it as root 3 times root 3.

root 20 root 25 root 15 and root 5 root 10 and root 25 now since all the elements have root 11 in this first one i can take root 11 out and i get the determinant of a matrix similarly all these have root 5 common therefore i can take root 5 out of it and similarly all of these has root 5 common and i can take root 5 out of it

so to simplify the notation let me write it as determinant of a one plus determinant of a two now determinant of a one is equal to after taking out root 11 root 5 root 5 what we are left with is 1 root 2 root 5 2 root 5 root 3 and 1 root 2 root 5.

now looking at this matrix since two columns are identical we know that the determinant is going to be zero therefore determinant of a one is equal to zero therefore we can write determinant of a is same as determinant of a two which is is equal to determinant of root three root fifteen root three into root three root 20 root 25 root 15 and root 5 root 10 and root 25 now looking at this we can see that we can take root 3 common out of the first column root five we can take common from the second column same is for column three therefore determinant of a is equal to root three into root five into root five into determinant of one root 5 root 3 2 root 5 root 3 and one root two root five

so let me compute its determinant one into root five into root five minus root 6

so 5 minus root 6 minus 2 into this product to this product which is 5 minus root 6 plus 1 into root 15 minus root fifteen this is is equal to zero

so we are left with minus one into five minus root six is equal to root six minus five therefore determinant of a is equal to five root three into root six minus five

so this is the answer of the overall determinant example 3 find the determinant of a minus b minus c two a two a to b b minus c minus a to b to c to c c minus a minus b we need to compute the determinant now we find that if we add the rows what is going to happen

so let me do two things at one go r one goes to r one plus r two plus r three in the previous example i have done it in two steps first row two was added to row one and then row three was added to row one but now let me do it at one go

so what i am doing i am adding these two rows to the first row then we get that determinant of a is equal to determinant of as i add 2 b and 2c i get a plus b plus c as i have i am adding b minus c minus a and 2c i get b plus c plus a and as i am adding 2 b plus c minus a minus b i am again getting a plus b plus c and other rows remain as it is

so this is the structure that i get what is the advantage the advantage is that i can now take a plus b plus c common from the first row therefore determinant of a is equal to a plus b plus c into determinant of 1 1 1 2 b b minus c minus a to b 2 c 2 c c minus a minus b now what i do i will try to simplify it how i will do that i will subtract column three from column one therefore c one is equal to c one minus c three after doing that operation i get determinant of a is equal to a plus b plus c times one minus one is zero two b minus two b is zero and two c minus c minus a plus b whole thing will become a plus b plus c other columns remains the same what is the advantage the advantage is now is that now we can expand the determinant only taking this term and corresponding

sub matrix i do not need to bother about what is happening with this and what is happening with this therefore determinant of a is equal to a plus b plus c now note that it is the third row first column therefore minus 1 to the power 3 plus 1 into this a plus b plus c into the determinant of this matrix which is 2 b minus b minus c minus a is equal to this gives me one a plus b plus c this gives me one a plus b plus c and this will also give me another a plus b plus c and the sign is positive therefore the answer is a plus b plus c whole cube

 so that is the answer another example find the determinant of x x square y z y y square z x and z z square x y as we can see if we do the row operations by adding or subtracting we can get something

 so what we do we do rho 1 is equal to rho 1 minus rho 2 then determinant of a is same as determinant of what i am getting x minus y what i am getting here is x square minus y square which is x minus y into x plus y and what i am getting here is z into y minus x and other rows remain the same now we can see that we can take x minus y to be common from rho one therefore determinant of a is equal to x minus y into determinant of 1 x plus y minus z because the term there was y minus x

 so minus sign will come y y square z x z z square x y now in a similar way now i will subtract row 3 from row 2.

 that means row two minus rho three now goes to row two therefore determinant of a is equal to x minus y into determinant of 1 x plus y minus z y minus z y minus z into y plus z into x into z minus y z z square x y therefore determinant of a is equal to now i take y minus z from row two and we are left with 1 x plus y minus z 1 y plus z minus x z z square x y now what will do we will subtract this row from this row what will happen it will make this 0 and it will make the make this minus x plus z therefore again doing r 2 is equal to r 2 minus r 1 we have determinant of a is equal to x minus y into y minus z into determinant of one x plus y minus z 0 z minus x z minus x and z z square x y now from this row we can take z minus x common is equal to x minus y into y minus z if i take z minus x common then what is left is 1 x plus y minus z 0 1 1 z z square x y what is this determinant

 so let me first write it as x minus y into y minus z into z minus x into determinant of b where this matrix is b

 so let us calculate determinant of b

 so if i expand along the first column it is one into x y minus z square plus z into x plus y minus minus z is equal to x y minus z square plus x plus y plus z into z is equal to x y minus z square plus x z plus y z plus z square and this cancels is equal to x y plus y z plus z x therefore determinant of a is equal to x minus y into y minus z into z minus x multiplied by x y plus y z plus z x this is the answer another problem what is the determinant of a square plus one a b a c a b b square plus 1 b c c a c b c square plus one this is a very tricky problem because we cannot take anything common from any row neither from any column

 so we do some trick what we do we multiply this first row by a and to take care of that i divided by one by a

 so that the determinant does not change similarly i d i multiply the second row with b and divide it by b and i multiply the third row with c and then again divided by c there

 so that determinant does not change therefore determinant of a is equal to 1 upon a b c into determinant of a into a square plus 1 a square b a square c second row by multiplying with b and third row multiplying with c we get what is the advantage the advantage is that now we can take a common from the first column b common from the second column and c common from the third column therefore determinant of a is equal to a b c divided by this a b c now after

taking a common out of the first column we get this b out of the second column we get this and c taken out of the third column we get thus we are in a slightly better position because abc and abc gets cancelled therefore determinant of a is same as determinant of a square plus one a square a square b square to b square plus one b square c square c square c square plus one still we find that none of the rows we can break up as sum of two none of the columns we can break up as sum of two elements

so we cannot simplify it in that way

so what we will do i will rather add row 2 and row 3 with row 1 therefore r 1 is equal to r 2 plus r 3 plus r 1 will give us determinant of a is equal to a square plus b square plus c square plus one here we will get a square plus b square plus c square plus one and here also we will get the same thing other rows remain the same what is the advantage the advantage is that now we can take a square plus b square plus c square plus one common of the first row therefore determinant of a is equal to a square plus b square plus c square plus 1 into determinant of 1 1 1 b square b square plus one b square c square c square c square plus one now let us subtract column three from column one and also column three for column three for column two

so we are doing two at the same time c one goes to c one minus c three and c two goes to c two minus c three therefore after doing these two operations we get determinant of a is equal to a square plus b square plus c square plus one into determinant of now this i am subtracting from this therefore 0 b square minus b square is 0 c square minus c square plus 1 is equal to minus 1.

now i am subtracting this column from this column therefore it is 0 1 minus 1 and we are left with 1 b square into c square plus one this makes our life very simple because the entire first row is 0 except the 1 3 element therefore use determinant can be obtained computing this determinant which is very simple which is one therefore this determinant is equal to one therefore determinant of a is equal to one plus a square plus b square plus c square that is the answer now let me do an interesting problem find the determinant of a is equal to n n p n n c n n plus 1 n plus 1 p n plus 1 n plus 1 c n plus 1 and n plus 2 n plus 2 p n plus 2 n plus 2 c n plus 2 looks formidable but we know how to make it simple because we know that n p n is equal to factorial n for all n and n c n is equal to 1 for all n therefore the a matrix will look simpler if i write it as n n plus 1 n plus 2 n factorial n plus 1 factorial n plus 2 factorial and 1 1 1.

can we take anything common if we look at the second column we see that n factorial is there in all of them therefore we can take n factorial out of the second column and therefore determinant of a is equal to n factorial times the determinant of n n plus one n plus two after taking out n factorial from n factorial we get one after taking out n factorial from n plus one factorial we get n plus one and after taking out n factorial from n plus two factorial we get n plus one into n plus two and of course the third column remains same which is 1 1 1.

now what we do we subtract second row from the third row therefore r 3 is equal to r 3 minus r 2 makes that determinant of a is equal to n factorial into n 1 1 n plus 1 n plus one one now as i am subtracting second row from the third row i get one i get n plus one into n plus two minus one that is n plus one whole square into one now we simplify it further as i subtract 1 from 1 we get 0 now we simplify it further what we do we subtract row 1 from row two therefore replacing row two is equal to rho two minus rho one we get determinant of a is equal to n factorial into n 1 1 1 n 0 1 n plus 1 whole square zero therefore now if we expand along the third column we have only one one rest are zero therefore determinant of a is equal to n factorial into minus 1 to the power 1 plus 3 into n plus 1 whole square minus n is equal to n factorial into n square plus n plus

y

so let us verify it for a simple n suppose you want to verify for n is equal to two therefore we know that a is equal to two factorial to one three factorial three one four factorial four one therefore determinant of a is equal to determinant of 2 2 1 3 6 1 4 24 1 or determinant of a is equal to 2 2 1 3 6 1 1 18 0 this we get by making r 3 is equal to r 3 minus r 2.

is equal to determinant of 2 2 1 1 4 0 1 18 zero and this we get by making r two is equal to r two minus r one is equal to now if i expand along this third column we get only determinant of a is equal to 1 into 18 minus four is equal to fourteen now if you remember we got for general n determinant of a is equal to factorial four into n square plus n plus one putting n is equal to two we have n factorial into n square plus n plus 1 is equal to 2 factorial into 2 square plus 2 plus 1 is equal to 2 into 4 plus 2 plus 1 is equal to 2 into 7 is equal to 40.

therefore we verify the answer with n is equal to 2 therefore that gives us the determinant of the apparently complicated matrix with the permutation of n and combination of n formula inside ok friends with that i stop here today in this class we have solved several problems which i believe will make you understand how to deal with the problem of computing determinants by first executing some appropriate row and column operations which will make the computation of the determinants which will make the computation of the determinant simple in the next classes i shall look at some properties of determinant with respect to computation of area of a triangle and also i will be talking about minors cofactors adjoined matrix etcetera with respect to the determinants of square matrices ok students thank you

so much you