welcome students to the second lecture on determinants in the first lecture we have defined what is a determinant for a square matrix and we have investigated certain properties in this lecture we shall continue with examining more properties of a determinant and will try to solve a few problems if you remember in property four we have seen that if a is equal to a one one a one two a one three a two one a two two a two three and a three one a three 2 a 3 3 then if b is equal to k a 1 1 k a 1 2 k a 1 3 a 2 1 a 2 2 a 2 3 a 3 1 a 3 2 a 3 3 then determinant of b is equal to k times determinant of a this we have seen in the last class question is what if all the elements of a are multiplied with k how does it affect the determinant or in other words what is the determinant of b which is is equal to k a 1 1 k a 1 2 k a 1 3 k a 2 1 k a 2 2 k a 2 3 k a 3 1 k a three two k a three three what is going to be the determinant of this matrix if we expand it in a very similar way then we get use determinant is k a 1 1 into k a 2 2 into k a 3 3 minus k a 2 3 into k a 3 2 minus k a 1 2 into k a 2 1 into k a 3 3 minus k a 3 1 into k a 2 3 plus k a 1 3 into k a 2 1 into k a 3 2 minus k a 2 2 into k a a 3 1 what is this if we look at each term has three case one two three here one two three here one two three or in other words we can take k cube to be common and then what i get i am not writing it again you see that this is same as the expression of determinant of a therefore the determinant of b is equal to k cube times determinant of a if all elements of a are multiplied with k in general if a is a n cross n matrix and if all the elements are multiplied by a constant k then the determinant of the new matrix b is going to be k to the power n into determinant of a if two rows or columns of a matrix are identical then the determinant of the matrix is zero ok let me first verify this consider a is equal to a b c a b c x y z

 so the matrix is such that two rows are identical will show that its determinant is zero let us expand it along the first row therefore it is going to be a into b z minus c y minus b into a z minus c x plus c into a y minus b x this we know this is is equal to a b z minus a c y minus a b z plus d c x plus a c y minus b c x now let us look at the terms plus a b z minus epsilon they cancel each other plus a c y minus a c y they cancel each other and minus b c x plus b c x they cancel each other therefore the result is zero since we did not know the result before we had to compute it but once you are an expert and you can see that two rows of a matrix are identical or two columns of a matrix are identical then the determinant of that matrix is going to be zero i have shown it using expanding the row but we know that determinant of a is same as determinant of a transpose

 so if we take the transpose of the matrix two columns the first and second column are going to be this identical and in that case also its determinant is going to be zero property six suppose a is a given matrix and we obtain a new matrix b by interchanging two rows or two columns then determinant of b is equal to minus of determinant of a that is the value will remain same but the sign will change let me illustrate it consider a b c d e f g h and k its determinant is equal to a into e k minus f h minus b into d k minus f g plus c into d h minus e g will come back to this but let us construct b by interchanging two rows let us take the first and third row

 so the third row now becomes the first row second row remains as it is and here we get the first row of a as the third row of b therefore determinant of b is equal to g into e c minus bf minus h into dc minus af plus k into b minus a e let us now compare with the the expression of the determinant of a which we have just obtained and now let us consider the science g e c had a positive sign here i get g c with a negative sign g b f comes with a negative sign here i get gb and gbf comes with a positive sign because minus and minus becomes plus h d c comes as a negative sign h d c comes with a positive sign a hf comes with a

positive sign and aahf comes with a negative sign and then k d b comes with a positive sign here but here it is a negative sign and k a e and k a e comes here with a negative sign and k a e comes here with a positive sign thus we see that all the terms are retained but the sign of all terms have been reversed that is if it was a positive fear it is negative here and if it is a negative here it becomes positive here or in other words we can say determinant of b is equal to minus of determinant of a proved property seven determinant of a b is equal to determinant of a into determinant of b i directly show it for a two cos two matrix i directly show it for a and b of size two cross two and i suggest that you verify the same way for three cross three matrices let a is equal to a b c d and b is equal to m n p q therefore determinant of a is equal to a d minus b c and determinant of b is equal to mq minus n p now a b the product matrix is equal to a m plus b p a n plus b q c m plus d p c n plus dq therefore its determinant is equal to m plus b p into c n plus d cube minus c m plus d p into a n plus b q is equal to a m c n plus b p c n plus a m d q plus b p d q minus a n c m minus a n d p minus b c m q minus b d p q we see that these two terms cancel and let us now consider determinant of a into determinant of b is equal to a d minus b c multiplied by m q minus n p is equal to a d m q minus b c m q minus a d n p plus b c in p now let us compare that terms we obtained as the determinant of a b we have a d m q we have a d m q a d m q minus b c m q minus b c m q minus a d n p minus a d n p plus b c n p plus b c n p and we can see that the remaining terms a m c n and minus a n c m they cancel each other therefore we get determinant of a b is equal to same as the product of the determinants of a and b property eight if all the entries of our row of a can be written as the sum of two quantities then the determinant of a can be written as sum of two determinants let me illustrate consider a is equal to a plus k b plus m c plus n d e f x y z we see that the elements of the first row have been expressed as the sum of two quantities each one of them we claim determinant of a is equal to determinant of a b c d e f x y z plus determinant of k m n d e f x y z this is very easy to see when we expand it along the first row of a because determinant of a is equal to a plus k into determinant of e f y z minus b plus m into determinant of d f x z plus c plus n into determinant of d e x y is equal to we can write a into determinant of e f y z minus b into determinant of d f x z plus c into determinant of d e x y plus k into determinant of e f y z minus m into determinant of d f x z minus n into determinant of d e x y and we can easily say that the first one is actually the determinant of a b c d e f x y z plus this is the determinant of k m n d e f x y z thus the determinant of the matrix a can be written as the sum of two determinants let me now do a few problems find the determinant of one a s square one d b square and one c c square we expand it along the first row therefore determinant of a is equal to 1 into b c square minus c b square minus a into c square minus b square c square minus b square plus a square into c minus b is equal to if i take b c common from the first term it is b c into c minus b minus a into c minus b into c plus b plus a square into c minus b is equal to if i take c minus bit as common bc minus ac plus a b minus a b plus a square is equal to c minus b into let me take c common from this

 so c into b minus a minus a into b minus a is equal to c minus b into b minus a into c minus a which we can write it as a minus b into b minus c into c minus a
 so that is the determinant of this matrix let us explore another property if in a matrix the ith row is replaced by the sum of ith row and jth row then the determinant does not change illustration consider a is equal to a b c m n p x y z let b is equal to the sum of first row and second row that means i am replacing rho 1 with the sum of rho 1 and rho 2 which means that now b is equal to a plus m b plus n c plus p and other rows remain same it is m n p and x y z

then the claim is that determinant of b is same as determinant of a why because of the property that if the row elements can be expressed as sum of two expressions then i can write the determinant as the sum of two determinants
 so therefore determinant of b actually can be written as determinant of a b c m n p x y z plus the determinant of this other components m n p m n p x y z is equal to determinant of a plus determinant of this matrix which is is equal to 0 since two rows are identical determinant of b is equal to determinant of a a little bit more generalization if the ith row is replaced by the ith row plus some constant times jth row then also determinant of b is equal to determinant of a or if b is equal to a plus k m b plus k n c plus k p m n p x y z then determinant of b is equal to determinant of a this is true determinant of a plus m k m b plus k n c plus k p m n p x y z is equal to determinant of a b c m n p x y z plus determinant of k m k n k p m n p x y z is equal to determinant of a plus we know that if all the elements are multiplied by the same constant then we can take the constant out
 so we can write it as k times m n p m n p x y z and this becomes 0 therefore determinant of b is equal to determinant of a let us now do some more problems find the determinant of one b c a into b plus c one c a b into c plus a one a b c into a plus b is equal to determinant of 1 bc a b plus ac 1 c a b c plus b a 1 a b c a plus a b now i have shown that if one row is replaced by the sum of that row and another row then it does not alter the determinant since what we can say about rows we can also say about columns in a similar way we can say if one particular column is replaced by the sum of that column and another column then it does not change the determinant therefore i can write that use determinant is 1 bc now i am adding column 2 to column 3 bc plus a b plus ac the operation is c two now becomes c two plus c three column sorry column three becomes column two plus column three
 so one c a it is c a plus b c plus b a one a b it is a b plus c a it is c b c b now let us look at these elements it is a b plus a c plus b c it is a b plus a c plus b c and this is also a b plus a c plus b c
 so all the elements of the third column are being multiplied by the same constant
 so use determinant is a b plus b c plus c a if we take out that factor what remains is 1 1 1 bc ca a b then 1 1 1.
 and what is the determinant of this matrix because it has two identical columns its determinant is zero since it has two identical columns therefore the determinant of the original matrix zero let us now solve another problem what is the determinant of x plus y plus 2 z x comma y z second element is y plus z plus two x y and the third row is z x z plus x plus 2 y we need to compute the determinant of this matrix what we will do step one we first add column two to column one c one becomes c one plus c two then what we are getting the matrix becomes x two x plus y plus two z x y two x plus y plus two z y plus z plus two x into y x plus z x and z plus x plus two y now we replace column 1 by sum of column 1 plus column 2 column 3.
 then what we are getting it is two x plus two y plus two z x y it is two x plus two y plus two z y plus z plus two x in y and two x plus two z plus two y x z plus x plus two y now we can see that in the first column all the elements are same
 so i can take it out is equal to therefore two x plus two y plus two z into determinant of one one one x y y plus z plus two x x y z plus x plus two y this is still somewhat complicated
 so what we will do will replace now the second row with minus one times the first row just now we have shown that if one row is replaced by a constant times another row plus that particular row then it does not change the determinant

so what i am doing i am replacing row 2 with row 2 plus minus 1 times rho 1 then what we are getting then its determinant is equal to 1 x y 0 y plus z plus x 0 1 x z plus x plus 2 y of course multiplied by 2 into x plus y plus z as that is coming from there now replace r3 with r3 minus r1 what we are getting therefore determinant is equal to 2 into x plus y plus z into determinant of 1 x y 0 x plus y plus z 0 now i am subtracting rho 1 from row 3 0 0 x plus y plus z now if you observe this is a triangular matrix and we know that the determinant of a triangular matrix is the product of the diagonal elements now these diagonal elements are x plus y plus z and x plus y plus z

so their product is x plus y plus z whole square multiplied by x plus y plus y plus z therefore the overall determinant is going to be 2 into x plus y plus z whole cube

so that is the answer notice that we did not explicitly expand it because the terms are pretty long pretty long expressions are there

so if we multiply this sum of three terms with another three terms

so this is going to be nine terms it will explode computationally but what we did we have used many properties that we have shown in the last class and this class very effectively to compute the determinant in a very simple way let me now take a numeric example what is the determinant of two seven sixty five three eight seventy five and five nine eighty six apparently very complicated but it is not i will look at these two columns and see how i can generate the third column it is 65 here we have 7 into and here it is 2 we know that 7 into 9 is 63 plus 2 is equal to 65 similarly 8 into 9 plus 3 is equal to 72 plus 3 is equal to 75 and 9 into 9 plus 5 is equal to 86

so we can write it as 2 7 9 into 7 plus 2 3 8 9 into 8 plus 3 five nine nine into nine plus five since the column is expressed as the sum of two elements i can write it as the sum of two determinants is equal to determinant of two seven nine into seven three eight nine into eight 5 9 9 into 9 plus 2 7 2 3 8 3 five nine five this has two columns identical therefore this is going to give us zero and this has the column 3 is 9 times the column 2 and therefore use determinant is if i take 9 out it is the determinant of 2 7 7 3 8 8 5 9 9 and therefore then we will find it is it has the same two columns therefore risk determinant is also zero therefore answer is ok student i stop here today in this class we have examined several properties of determinants and also we have solved several problems using those properties which made computation of the determinant simpler in the next class i shall see some more problems and also i shall go ahead with the concept of determinants particularly grammar's rule and solving of equations etcetera thank you you