welcome to this lecture on determinants where we will focus on solving problems ah in this series of lectures we have looked at many concepts related to determinants starting from defining determinants seeing where they come up ah investigating how determinants help in ah finding inverses of matrixes ah solving system of equations and

so on of course through those conceptual ah lectures we have also looked at some problems in terms of examples but what we wanted to do for this lecture was to focus on problems ah my take on this is that ah of course the conceptuals conceptual grounding is important in solving the problems

so that is given at the same time sometimes solving the problems also adds to our understanding of the concepts and in a sense it is this cyclic nature of both understanding the concepts and solving problems and then understanding the concepts again which really makes us familiar with the material

so if i just draw this in a small graphic and say that these concepts are important in solving problems but complementarily what we will see in general is that when we solve problems solving problems also helps to understand and develop our concepts

so it is this overall feedback between understanding concepts and solving problems which really helps our overall grasp of the material

so in this lecture we will start with ah looking at particular examples solving the problems one by one and try to see how both solving the problems and understanding the concepts go hand in hand how just the idea of looking at solving problems what can we do at each step what are our possibilities just discussing the problems in general all right

so let us start

so the first problem example one is a situation where you have to show that a particular determinant equals 0.

so we want to show that when we have a determinant 1 1 1 b c c a a b and a times b plus c b times c plus a in a cyclic fashion c times a plus b

so what we want to show that this is equal to zero ok

so how do we do this well of course what direct way is to evaluate the determinant and then show that ok it indeed goes to zero ah another approach is to try to look at some properties and see whether we could use that to simplify the situation

so it is in fact the letter which is what we are going to do and over there what we see is that ok if you look at the first column it is all once the second column has expressions um bc ca a b ok and third one is expressions like a b plus a c b c plus b a c a plus c b ok

so when we consider this then we can say ok what we see is that if you sum the second and the third columns for all rows we get the same expression let's write it down maybe that provides a way to simplify this

so let us write down that left hand side what we were just noting is that all of these are 1 then this is b c c a a b and this let me just expand it and say ok this is a b plus a c or c a just to maintain the cyclic order ah b c plus a b and c a plus b c all right

so now we see that if we consider a sum of this column and this column then we get a b plus c a plus b c when we do that here also we get a b plus b c plus c and similarly here a b plus b c plus c a

so let's do that ah and our overall understanding is that c 3 we are going to say c 3 plus c two and we know from the properties of the determinants that this does not change the value of the integrals

so one one one b c c a a b then here we have a b plus b c plus c a a b plus b c plus c a and also a b plus b c surface c a

so all rows in this column are identical in fact we can factor out this entire expression and be left with just one one one

so the relevant property of the determinant shows that this can be written as a b plus b c plus c a and determinant is one one one b c c a a b and again one one one all right

so we have ah noticeably made the determinant more narrower by simplifying the terms ah and then now what well now it is relatively straight forward what do i mean by that what i mean is that now we have two columns c1 and c3 which are identical and

so by the property of the determinant we note that the determinant value is zero which is exactly what we wanted to show

so this is equal to a b plus b c plus c a times 0 and why is it 0 because c one and c three are identical and this is what we had to show this is what we wanted to show right

so what may be observed over here is that of course we can expand the determinant but just as we had mentioned previously the goal of studying these properties was to say ok now can we simplify the evaluation of the determinants make it more systematic make it more efficient and

so by the following rearrangement that we illustrate in this example we can see that we can prove certain identities of the determinants such as the one that we looked at now um

so this you know directly says ok this provides a relevance or provides a usefulness for studying the properties that we saw what else can we say about this example

so this provides one illustration we can look at some other illustrations um right

so let us look at some other examples of how we can try to look at the properties of the determinants and try to simplify the evaluations for next example let us look at the following identity example 2 where we have to show that the following determinant 1 1 1 a b c b c c a a b is equal to a minus b b minus c c minus a we have to show that this determinant is a product of these three terms

so how do we go about it of course we could evaluate the determinant ah the goal over here is to see whether using some of the properties is going to make it more efficient to show the identity

so the left hand side here is 1 1 1 a b c b c c a a b ah note that this column has three ones

so we could consider subtracting these second row with the first row and replace this one with a zero because we know that ah doing these row operations such as r two goes to r two minus r one is not going to change the value of the determinant

so what happens when we do that we get 1 a b c

so the first row is unchanged the second row becomes 0 b minus a c a minus b c

so the advantage of using this property here that doing these kind of operations on the row does not change the value of the determinant is that we have replaced this entry here from one to zero similarly we could do the same for the third row r three goes to r three minus r one

so we have one minus 1 0 c minus a c minus a and a b minus b c a b minus b c

so there we have done that we have using these two operations which we know through the property are not going to change the value of the determinant we have made this column from 1 1 1 to 1 0 0 and the advantage is that if you were to expand this determinant we can expand along this column and then we just take the determinant of this smaller 2 by 2 determinant of course we note that this b minus a is there and then here c a minus b c and again we can take a b minus a

common because c is there in both terms
  so even before expanding we can potentially use one more property what is that property let me rewrite this determinant and we can take a look at it
  so this determinant was 1 0 0 a b minus a c minus a b c c a minus b c a b minus b c note that this quantity can be written as minus c times b minus a and this b minus a is the same as over here similarly this quantity can be written as b times or rather minus b times c minus a and note that this quantity is the same as here now from the property that we can pull out this entry from this row without changing the value of the determinant we write this equal to b minus a times 1 a b c 0 1 minus c and we can do the same for this row by pulling out this quantity c minus a from both
  so we have this times c minus a times a determinant where the last row is 0 1 and minus b
  so we are now left with a much simpler 2 by 2 determinant to evaluate when we expand along this row because we have made use of the two properties one where we got these two zeros in the first place by performing some row operations second by pulling out these constant terms from the rows which we know is not going to change the value of the determinant
  so if we are finally left with v minus a c minus a and this two by two determinant which can be evaluated either by directly doing minus b minus minus c or by other methods such as the direct definition here we have v minus a c minus a and minus b minus minus c that's plus c b minus a c minus a and minus and minus
  so it becomes minus b plus c
  so just to make it symmetric we can write this down as a minus b
  so we take a minus sign out from here and that minus sign can go over here into this term
  so this is b minus c times c minus a which is exactly what we have to show in terms of the right hand side
  so now through a sequence of operations primarily based on these two properties where we have taken the replaced the entries here with 0 and second by taking out these constant terms we have been able to show that the left hand side is equal to the right hand side
  so that completes this example
  so now let us look at another example and this example over here is interesting because again it relates a geometrical quantity like an area with evaluation of a particular determinant
  so this has to do in fact with the area of a triangle ah written in terms of its expression in the coordinate geometry
  so the vertices are given as x 1 y 1 x 2 y 2 x 3 y 3 and we know the formula of the area i write that down shortly but we want to show that that is nothing but the same as evaluating a particular determinant
  so let me write that down and then we can see how we evaluate that determinant
  so the example here is the following um and a triangle with vertices at x 1 y 1 x 2 y 2 and x 3 y 3 is has area equal to half x one y two minus y three plus x 2 y 3 minus y 1 plus x 3 y 1 minus y 2.
  show that this expression can be obtained by evaluating the determinant half x one x two x three y one y two y three one one one
  so in other words what we have to show is that the expression which is written in terms of the coordinates of the vertices of a triangle can be expressed in terms of this determinant where the first row has the first vertex followed by 1 and
  so on for the other rows and of course we are talking about an area
  so we should be careful just to take just the positive value of the determinant

although as we had discussed previously then we can also give some geometric interpretation to the sign for our purposes however we just stick to the absolute value of the german

so it is with that expression in mind that we want to show that these two take the same value

so as before we will start from the determinant and try to show whether it is equal to the expression on the right hand side or not

so in other words we will show therefore we have to show that half times x 1 x 2 x 3 y 1 y 2 y 3 1 1 1 is equal to the expression of the area origin up over there which is half x 1 y 2 minus y 3 plus x 2 y 3 minus y one plus x three y one minus y two

so let's see

so let us start from the left hand side

so this is the determinant multiplied by a half x 1 x 2 x 3 y 1 y 2 y 3 1 1 1 right

so this is another determinant which is there whose one column is all ones um how can we handle this as before what we can do is that we can subtract the last row and also the second row with the first row

so that the columns the last columns the ones in the last column there are only one of them which is one and the remaining are zero what do i mean what i mean let me write down here is that i want to say that r two becomes r two minus r one and same with r three r 3 becomes r 3 minus r 1 and we know that with these operations the property is that the determinant value does not change

so we can write down this as x 1 y 1 1 r 2 minus r one

so x two minus x one y two minus y one and one minus one is zero same with the last row

so say x three minus x one y three minus y one n and then this is a natural column to expand the determinant around because it is just a product of these two terms because that is a determinant of because as a cofactor of this entry remaining are 0.

so it is half x 2 minus x 1 x 3 minus x 1 y 2 minus y 1 y 3 minus y 1 all right um and this product time minus this

so this is half x2 minus x1 y3 minus y1 minus x 3 minus x 1 y 2 minus y 1 and what we have to show is that it is in terms of sums which have common x's

so let us expand the x values

so we have x 2 y 3 minus y 1 minus x 1 y 3 minus y 1 minus x 3 y 2 minus y 1 plus x 1 y 2 minus y 1.

so the coefficient of x 1 is going to be y two minus y one minus y three plus y one

so these y ones will cancel

so this is half x one y two minus y 3 plus x 2 y 3 minus y 1 and then just changing the sign over there plus x 3 y 1 minus y 2 and this is what we wanted to show

so again this is an example where we can do the direct evaluation of the determinants but what we are doing over here is that we are just using some properties to simplify the evaluation primarily by reducing some elements of the matrix whose determinant we are evaluating to zero

so that is an easier determinant evaluation ok

so these were some examples we looked at now let us look at a more relatively harder example and we look at it from two ways one is ok how do we solve the problem in the context that is given and also to look at it in a more general or at least sketch how we can look at it in a more general way using the

determinant properties that we are looking at

so this is the problem which has ah been appeared in the j e e mains problem

so let me write it down and i will see how to address it

so the example over here is as follows

so this is based on j e mains problem the problem is that if these alphas and betas are not zero and we define a function f of n which is alpha power n plus beta power n and the following determinant 3 1 plus f 1 1 plus f 2 1 plus f 1 1 plus f 2 1 plus f 3

so these denote f 3 means when you put n equal to 3 and this is alpha q plus beta cube

so those are what are populating the entries of this matrix one plus f two one plus f three and one plus f four ah if this is equal to k times one minus alpha square one minus beta square and alpha minus beta square then

so if that is the case then what is the value of k and the choices are that it is 1 the second choice is that it is minus 1 the third choice is that it is alpha times beta and the fourth choice is that it is 1 by alpha beta

so this problem is from this this question number 65 from the 2014 paper one book e which is accessible it was accessed from the website j e e main dot n i c dot i n web info question papers 2014 dot htm ah and in particular it is the pdf file zero six 0 4 2 0 1 4 e dot pdf

so this is the ah the source of this paper

so it is a j e mains problem right

so this is a general problem which is given now there are of course given the problem there are multiple ways to address it ah let us look at some ways in which may be useful in trying to find out the value of k ah in the first of these ah what we can do is we can say ok ultimately we want to find out the value of k

so maybe what we can do is just to assign some values of alpha and beta and see whether that comes up or not

so that is one way although it may not be the most complete way it may give a solution on the other hand what we could do is to say ok now lets try to solve this in a more general fashion and come up with an expression and then find out the value of k

so we can efficiently hope we hope that to efficiently be able to come up with the value of the determinant ah of course one can also expand it directly but how can we express it in terms of alpha beta and alpha minus beta as it is on the right hand side ah

so therefore ah what are the ways in which we can look at it lets see how we can solve it the first one approach is to just use some values of alpha and beta which are convenient and which will help in finding a quick value of k and then we will try to at least sketch the way we could do it in a more general systematic way using the properties of the determinants all right

so let's see how we can collect

so in the first way in one way we can set values of alpha and beta

so in particular we can say alpha is minus one and beta equal to two values chosen such that they are close to one 1 make it a more efficient evaluation

so what is the determinant it is 3 let me rewrite that 1 plus f 1 1 plus f 2.

and its as you notice a symmetric matrix

so you do not have to evaluate all of them just half of them plus the diagonal should be ok one plus f three one plus f four

so let us write it down in general for alphas and betas and then we will replace those values

so this is three one plus alpha square plus beta square excuse me i think this should be 1

so then this is just 1 plus alpha plus beta and this is 1 plus alpha square plus beta square okay this is 1 plus alpha plus beta 1 plus alpha square plus beta square 1 plus alpha cube plus beta cube 1 plus alpha square plus beta square 1 plus alpha cube plus beta cube and finally 1 plus alpha 4 plus beta 4 you know there are all these exponents and then just imagining how to expand this determinant in general see whether it simplifies may not be most straightforward thing to do anyway let us put alpha equal to minus 1 and beta equal to 2 then what do we get you can get this is 3 1 plus alpha
 so alpha plus one is zero
 so that is just beta two and then here alpha square is one one plus one two plus beta square four
 so that's six
 so this two goes here its a symmetric matrix this one is also six and is also six we already evaluated these one plus alpha cube is also ah zero because alpha cube is minus one beta cube however is eight
 so there is an 8 here and then again alpha power 4
 so this is 1 1 2 2 plus 2 power 4 is 16
 so that's 18 all right
 so that is the determinant that we get if we set these values um then we can now expand this
 so we could use some properties to try to simplify it or we could do it directly
 so either way we could solve it
 so in this particular case let us just expand it about this row
 so this is 3 into 6 times 18 minus 64 because that is 8 times 8 minus 2 2 times 18 is 36 minus 48 plus 6 times 16 minus 36
 so 6 times 18 is 8 4
 so 108 minus 64.
 36 minus 48 is minus 12
 so this is plus 24 and 16 minus 36 is 20
 so minus 120
 so this is 3 times 44 plus 24 minus 120.

 so this 3 times 44 is actually 1 32.
 plus twenty four minus one twenty
 so there is a twelve at twenty four
 so this evaluates to thirty six now let us look at the right hand side expression which was k times 1 minus alpha square 1 minus beta square alpha minus beta square and what is its value when alpha is minus 1 beta is 2.
 this becomes k into 2 square minus 1 square and alpha minus beta
 so minus 3 square
 so 2 square is 4 times 1 times 36 excuse me 4 times 9 which is 36
 so this is k into 4 into 1 into 9
 so 36 k
 so the left hand side is 36 k 36 and the right hand side is 36 k
 so this together would imply that k equal to 1 which is the first option
 so by using particular values of alpha and beta we get that k is equal to one of course this is not a general way to show what the value of k is ah for that we will have to solve the determinant in general but as a quick way to say ok ah given the fact that we only want to find k n in general it seems it should hold for all alpha and beta 1 can have a leap of faith and just try to check the value of k of course more systematically how do we do that let us see now
 so what we have to essentially do is to say ok we want to find k from the equality 3 1 plus alpha plus beta 1 plus alpha square plus beta square 1 plus

alpha plus beta 1 plus alpha square plus beta square 1 plus alpha q plus beta cube 1 plus alpha square plus beta square 1 plus alpha q plus beta cube 1 plus alpha power 4 plus beta power 4 and this we say is equal to k 1 minus alpha square 1 minus beta square and alpha minus beta square

 so how do we do this

 so how do we find k from here

 so effectively we have to evaluate this entire determinant now there was this one property that you could write down ah if you can represent the rows of a determinant by a sum of two terms you can write down the whole determinant as a sum of those two determinants ah i think that is a property which has the potential to simplify the problem we have to of course look at it and ah maybe we will just sketch how to do it ah to check the general way to solve it ah and also one thing that can be noticed here is that this three which is the first row first column entry can be written as one plus one plus one and this these two ones can be taken as alpha power zero and beta power zero

 so in fact it is like one plus f of zero ah and

 so we have like each element decompose as three terms

 so how do we expand the determinant how is it possible to use this property let's try that let's look at that

 so let's see what the left hand side is

 so we have the thought in mind that we want to just expand the determinant 1 plus alpha square plus beta square will do it row wise it may seem that there are a lot of options but on further considering those options we realize that many of them evaluate to zero

 so of course it may not be the most optimal way to solve it but definitely ah seems like a better way to do it then just directly evaluating the determinant now how do we get this alpha power 4 i mean i am sure they will also cancel out because there is no alpha power you know higher values expression of alphas and betas but how do we do it its relatively cloudy

 so let us just expand this now we can replace this is the sum of three determinants obtained by splitting the first row entries ah

 so we can get that this is one 1 1 and then the same rows as here

 so these are same rows plus the second one one alpha alpha square and then the same rows plus 1 beta beta square and then same rows right

 so then we have replaced one determinant with the sum of three determinants now um the idea is to now apply these properties to the second row of each of these three determinants

 so then you will have like each determinant has three more possibilities ah

 so it sounds quite a lot because then for each of them will have to do three more additions but if we just take one step further what we will see is that many of these determinants evaluate to 0 and that is because their rows are identical

 so let us see what comes

 so if you look at just the first of these 1 1 1 and then the second row the second over here which has to come is 1 plus alpha plus beta 1 plus alpha square plus beta square ah one plus alpha cube plus beta cube ok and if we see that if we think about expanding these in three determinants right

 so the third row stays what was before

 so if you do this one one one and then we say ok the first one is going to be 1 1 1 and whatever was here plus 1 1 1 with alpha alpha square alpha q and whatever was here and the last one being one 1 1 beta beta square beta cube and whatever was here right as you see we do not need to really look at different options of this because this already is 0 why because these two rows are identical

so we do not have to look at over here now of course for these two we have to still go one step further because their third row was 1 plus alpha square plus beta square 1 plus alpha cube plus beta cube 1 plus alpha power 4 plus beta power 4 okay um but again out of the three possibilities that we will see here again many of them will become zero why because if you just look at one one one alpha alpha square alpha cube and just look at the first term one 1 1 then it is the same as the identical as the first row

 so this element determinant here which has 1 1 1 alpha alpha square alpha cube 1 1 1 is going to be zero then we look at the second one and that also is going to go to zero why because we have one one one alpha alpha square alpha cube then alpha square alpha cube alpha power four effectively these two rows are the same because you can pull out an alpha here will be one alpha alpha square we can pull out alpha square here and that will also be one alpha alpha square

 so this would be one one one one alpha alpha square and taking out an alpha here and there will be one alpha alpha square taking out an 1 square square

 so these two are the same

 so this is also 0.

 so all we will be left with is 1 1 1 alpha alpha square alpha cube beta square beta cube beta power four

 so so far we are left with just one determinant which is not zero similarly when we do over here we will just be left with one determinant because the terms corresponding to expanding this 1 1 1 will be cancelled out with because this row will be identical same if we just look at the last of these beta that will also cancel out

 so all we will be left with from here would be just one determinant 1 1 1 beta beta square beta cube and then we are going to have alpha square alpha cube alpha power 4.

 so out of the multiple more possibilities that we may have all we are left with in obtaining this branch of the determinants is going to be just this determinant and this determinant similarly going back to what we have seen over here

 so here we have got only two determinants then we will see here we get only two more determinants and here also two more determinants

 so overall we will get only six determinants and let me write those down

 so those determinants are going to be the following

 so after simplifying and noting that many determinants are zero we are left with the following six and these are 1 1 1 alpha alpha square alpha cube beta square beta cube beta power 4 plus 1 1 1 beta beta square beta cube alpha square alpha cube alpha power 4 plus 1 alpha alpha square which is coming from the second set of determinants

 so 1 1 1 beta square beta cube beta power 4 plus 1 alpha alpha square beta beta square beta cube 1 1 1 plus and this is coming from the third of the decomposition that we had done 1 1 1 alpha square alpha cube alpha power 4 plus 1 beta beta square alpha alpha square alpha cube one one one

 so these are the six determinants that we get to evaluate for the sake of clarity let us look at each of these six determinants and see how they reduce to the common factor

 so let us write down the first determinant this is 1 1 1 alpha alpha square alpha cube beta square beta cube beta power four

 so if you notice here that this in the second row the term alpha can be taken out right this is exploiting the property of the determinant ah and in the last row the term beta square can be taken out from each of the terms

so then what we are left with is 1 1 1 1 alpha alpha square 1 beta beta square

so this is the factor and this is the determinant this is for the first determinant let us look at the second determinant this was 1 1 1 beta beta square beta cube alpha square alpha cube alpha power 4

so here again the term beta can be removed from the second row and the term alpha square from the first row excuse me the third row

so this is alpha square beta 1 1 1 1 beta beta square 1 alpha alpha square notice the only difference between this determinant and this determinant is the order of the last two rows and

so in fact we can interchange these rows but as we know from the property we will give a minus sign at the outside

so minus alpha square beta 1 1 1 1 alpha alpha square 1 beta beta square

so this is the factor and this is the determinant similarly we can look at all the other determinants one by one which we should do and remove the common factor

so the third one is 1 alpha alpha square 1 1 1 beta square beta cube beta power 4 and here we can take out beta square from the last row

so that is beta square and then 1 alpha alpha square 1 1 1 1 beta beta square now in this determinant if we interchange these two rows which will induce a minus sign at the outside of the determinant we will get what we are looking for in short a product of a factor this one and then the determinant proceeding similarly for the fourth determinant which is one alpha alpha square beta beta square beta cube one one one we can remove a factor beta from here

so this is beta one alpha alpha square one beta beta square one one one and now here to get this common factor we have to ultimately move the last row at the very top

so in a sense it is two row interchanges one is from the third to the second and then it moves from the second to the first each of these will give a minus sign and a product of those minus sign is a plus sign

so we will get beta 1 1 1 1 alpha alpha square 1 beta beta square again there is a factor here and a determinant

so this was the fourth matrix now we look at the fifth one which is one beta beta square 1 1 1 alpha square alpha cube alpha power 4

so this is removing alpha square from the last row is going to give 1 beta beta square 1 1 1 1 alpha alpha square and what we need to be doing here as before is that we want to move the first row to the last using these two sequential row interchanges each of which gives a minus sign

so overall it is a plus alpha square 1 1 1 1 alpha alpha square 1 beta beta square and finally the sixth determinant is 1 beta beta square alpha alpha square alpha cube 1 1 1 which is equal to alpha times 1 beta beta square 1 alpha alpha square 1 1 1 and here we have to interchange these two rows which will give a minus sign which is minus alpha 1 1 1 1 alpha alpha square the second row remains where it is one beta beta square

so in this way we have reduced the fifth and sixth determinant into a factor times the determinant the factors are different but the determinant is the same and

so we can just to complete the step combine all the factors

so what we get is alpha beta square minus alpha square beta minus beta square plus beta plus alpha square minus alpha into the common determinant one one one one alpha alpha square one beta beta square ok and ah just to complete what we can find out by using either direct expansion or by using some of these properties is that this determinant evaluates to nothing but this entire factor

so that this is equal to alpha beta square minus alpha square beta minus beta square let me just write down alpha square here first plus beta minus alpha and

there is going to be a power of two because this determinant also gives the same factor this can be checked and then we can further simplify this one by noting that the term beta minus alpha can be taken as common from many of these terms
 so these term these terms and this term beta minus alpha plus beta minus alpha square
 so there is a beta minus alpha square and alpha beta minus beta minus alpha plus one and this also can be factorized into one minus alpha square and 1 minus beta square
 so overall this can be written down as 1 minus alpha square 1 minus beta square and alpha minus beta square because squaring does not change the sign and this should help to understand how these steps come about
 so here we have expanded the steps to get overall clarity about these things ah what we emphasize here is that these row operations have helped us to simplify the execution of the determinant but the underlying idea is the same which is that we are using these properties of the determinants to try to simplify the evaluation which allows us to solve ah problems which at first glance seem relatively tricky
 so in this particular case the problem that we had to solve was this one and on the face of it looking at this it may not be clear which properties to use where to use what is the value
 so we illustrated two ways in which we could address this ah sure there are other ways also both of them give the value of k to v equal to one and
 so the goal of looking at relatively complicated problem like this was to illustrate how the simple building blocks of the problems that we have tried to do before give us some insight into how to handle these problems
 so ah to summarize ah this topic of how we can use this properties that we have studied of the determinants the other aspects that we have seen about the determinants i think its important to solve problems ah through the series of lectures we have had we have tried to do a combination of presenting the concepts and also to solve the problems ah in particular in this lecture we focus primarily on solving the problems with the goal that they will solidify our concepts and of course we need to know the concepts to solve the problems but it is a feedback loop where we can apply the concepts solve the problems at the same time solving the problems gives some insight into it expands it adds another layer add some appreciation of the concepts that we are trying to understand
 so in particular it is about determinants that we have focused this problem solving session on and
 so together with these concepts and problems and indeed through the overall series of lectures that we have had starting from the definition starting from the motivating examples to the properties or determinants its applications in solving systems of equations in taking matrix inverses we hope to convey the importance of determinants as well as equip us with tools to apply our understanding of determinants to many situations
 so with that i thank you for your attention for both this lecture as well as for the series of lectures that we have done thank you you