

welcome to this lecture on studying the role of determinants in solving linear system of equations

so the topic that we are going to talk about today is solving linear system of equations

so over the past three lectures we have looked at different aspects of determinants starting from where they may arise

so one of the motivating examples was in fact solving system of equations linear system of equations then we also saw how they have a geometrical interpretation ah then we defined a determinant

so that was all in the first lecture in the next lecture we looked at some of the properties which will help in efficiently calculating determinant values then we saw one application of determinants we saw how determinants can be helpful in calculating inverses of square matrices giving conditions ah whether a matrix will have a inverse or not and in that line today we look at how they help in solving linear system of equations

so the idea is again straight forward and it is one of generalizing simple equations that we usually see

so we may see equations like $2x = 3$ and we want to solve for the variable x similarly when we have more than one equation in suppose we have equations which have multiple unknowns x and y or x , y and z in general n equations then we have seen how we can convert these representations into a matrix representation

so we can write down a general equation $Ax = b$ where A is a general n by n square matrix typically if there are n unknowns and any equations ah and then we would like to solve it

so here we are going to look at how we can use determinants to ah address this issue um and what we will see is how ah the determinants or calculating the determinants of the associated matrix ah will give a condition to find out whether there is a solution or no solutions or many solutions right

so that is what our goal is

so just to write that down we may have previously encountered equations like $ax = b$ where a may be some scalar b also is a scalar and x is an unknown that needs to be solved for that needs to be solved for and because these are all scalar values we can say ok if a is not equal to zero then $x = b/a$ right now when we generalize these ah to have let us say equation in two unknowns $ax + by = m$ and $cx + dy = n$ where now x and y are unknowns

so there are two unknowns and two equations how will we find the solutions of this and we know that we can write this down in a matrix representation $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$ and there is x , y and then m and n right

so this plays the role of an a matrix this is a unknown vector

so to avoid confusing the notation let us say this is a vector \underline{x} right with the under bar

so this x is a scalar this is a vector

so let me just make a note of that this is a vector right two dimensional in this case and these are also known constants now A is known and this thing on the right hand side of the equation is also known but \underline{x} is unknown

so this representation of $A\underline{x} = \underline{b}$ let us call that capital B is equal to \underline{b}

so how can we get the value of \underline{x} over here

so it is the same generalization of what we see in one dimensional this is in two dimension and in general we can have a situation in n dimensions

so how do we solve these system of equations these linear system of equations ah what is the role of determinants in this that is the goal of this lecture all

right

so how do you use determinants how do we solve these equations how do we come up with conditions to check their existence or solutions or not

so we will look at the associated concepts and also look at some problems over here okay

so just to carry on with the previous example we have something like $a \ b \ c \ t$ times $x \ y$ equal to m and n

so we have said this is a this is an unknown vector x this is b ok now the goal as we just said is to find out how to solve for x right and

so ah just a point about notation that we are going to use once we the context is clear we are going to say we are going to replace x with just ordinary notation x not to be confused by the scalar x

so in appropriate context we use x to denote the vector value although we will try to be careful to use the x with the under bar to avoid any confusion ok

so now we have this is an example of a two dimensional system that is that there are two unknowns and there are two equations ah just for the sake of completeness let us write down a three dimensional system of equations and then define the associated quantities three by three example a three dimensional example three dimensional example ok

so here we say there are three equations $a_{11}x + a_{12}y + a_{13}z = b_1$ ok this is equation one the second equation may be $a_{21}x + a_{22}y + a_{23}z = b_2$ and the third equation is $a_{31}x + a_{32}y + a_{33}z = b_3$

so this is an example of three equations with three unknowns x y and z each of which are scalars

so how do we write down this in the general matrix representation well we can collect the terms like we have done for a two dimensional system into a square matrix A we are going to call it A we are going to call it capital A and then an unknown vector x with the under bar which is a column vector having the unknown values x y and z and then on the right hand side of equation will be another column vector with the entries b_1 b_2 b_3 which you should call capital b

so we can write this down as a matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and then a column vector here $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and then vector $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

so this may be denoted as $A \bar{x} = b$ and this is capital b

so the equation that we have is $A \bar{x} = b$ and our goal over here is to find x

so the purpose of writing down the three dimensional system of equation ah was to show when you compare it with a two dimensional system that in general you can write this down for an n dimensional system which has n equations and n unknowns

so ah here is just a case for $n = 3$ we can in general take \bar{x} and b to be their appropriate n dimensional quantities specifically A is going to be an n by n square matrix \bar{x} is going to be a n by 1 vector and b is going to be a n by 1 vector

so in general this is a n by n square matrix this is a n by 1 vector and

so is this an n by 1 vector

so this is the problem set up $A \bar{x} = b$ a linear system of equations how do we look for solutions how do we check for conditions for which this has solutions or not

so the general system is the following $A \bar{x} = b$ now here i want to take this opportunity to define two terms which are frequently used in this context

ah they are opposites of each other

so one of the terms is consistent

so system of equations is said to be consistent if it has a solution

so it could be one or more solutions and it is said to be inconsistent if there are no solutions

so let me write these down but these are terms which are defined given this context of having a system of equations and trying to solve for these unknown values consistency a system of equations such as the one shown here is said to be consistent if a solution exists exist and of course there may be one solution or more than one solution and the analogous definition of inconsistency is a system of equations is said to be inconsistent if a solution does not exist

so let us look at these again

so these are the system of equations referred to here system of equations a system of equations is said to be consistent if a solution exists meaning that x has one solution or more than one solution inconsistency a system of equations is said to be inconsistent if a solution does not exist meaning that there is no x which satisfies these for the given values of the matrices a and b ok

so the goal of saying these ah terms consistency and inconsistency is to give a short form expression for saying for handling or talking about ah solutions of x

so we will say system of equations is consistent or system of equations is inconsistent and that would mean ah short form for saying whether it has a solution or does not have a solution on that node i must mention that a lot of interchangeable terms are used ah

so for example if a system of equations has just one solution we say it has a unique solution unique meaning one solution sometimes they talk about a non trivial solution non trivial meaning that the solution that you get x is not equal to zero

so these are some other terms which are used in the context for our purposes we keep it simple and just use consistency and inconsistency ok

so how do we go about solving this

so a x equal to b we want to know what is the consistency properties what is the inconsistency is it consistent inconsistent how to check right this is what we set out to find how to check whether system of equations are consistent or not and this is where now we talk about the role of determinants especially in deciding whether the matrix a is invertible or not

so our program is as follows we will say ok as we have seen in the previous lecture a matrix can be singular or non singular depending on whether its determinant is 0 or not if it is non-singular that is if the determinant is 0 then it is invertible and if it is invertible then we have a matrix inverse which we call a inverse which you can multiply these equations by and in that case when we multiply the equation by a inverse the left hand side will become ah a inverse times a times x and the right hand side will become a inverse times b and if a inverse times a we know by definition is identity then we have a ready made solution for x then we look at the other case when it is not invertible and then see what happens over there ah

so let us write down what we just said

so if the first case that we will look at is that a is non-singular what does that mean that means that the determinant of a is not equal to zero the immediate implication is a inverse exists ok

so if a inverse exists let us multiply both sides of this equation by a inverse what do we get we get that a inverse a x bar is equal to a inverse times b this we know is identity

so if it is a two dimensional matrix a then this is a two dimensional identity which is one zero zero one in general if it is a n dimensional matrix a it is a

n dimensional identity

so it has n rows n columns and all the diagonal entries are one

so identity times x is x

so this we get x is a inverse times b

so in the case that a is non singular the solution that we get is x is equal to a inverse b right

so this is the first case in which it is non-singular that is the determinant is non-zero

so we have a ready-made solution for this ok now what about the other case well there we again use the tools that we developed in defining the matrix inverse specifically the joint matrix

so in the case the determinant is 0 then we can easily see that the adjoint of a times a is 0 right because previously we had come up with this relation that adjoint of a times capital a is equal to the identity times the determinant of a and

so if determinant is 0 it means that the adjoint of a times a is 0 then let us see what happens

so the second case is if a is singular that is determinant of a is zero

so we had seen that adjoint of a times a is a determinant of a times identity which because this is 0 this is equal to 0 matrix and

so we use this by multiplying the system of equations by the adjoint of a

so if you multiply this then we get that adjoint of a times x bar is equal to a joint of a times b

so i am missing a here

so this should be a joint of a times a times x bar right

so that makes sense because the left hand side has a x bar

so we have adjoint of a times a x bar and then adjoint of a times b now from here we know that this term is zero

so the left hand side is zero and then we have a joint of a times b right

so now there are two cases the sub case a small a is if adjoint of a times b is equal to 0 then we cannot say anything

so then we cannot say anything about consistency or inconsistency consistency or inconsistency

so it is a inconclusive result in case that the adjoint of a

so this is the case two b if the adjoint of a times b is not equal to zero then we have a problem because the right hand side is not zero the left hand side is 0.

so in this case it is we say that the system is inconsistent right

so here there are questions about how well posed this is because the left hand side is 0 right hand side is not 0 how can we equate the 2 right

so in this case when a is singular most of the conclusions that we can come up with are the following we the modus of operation is that we say ok we are going to multiply by the adjoint of a just like we did with the a inverse in the previous case of course previously it was a much simpler situation because we know that a inverse exists its a inverse times a is identity and

so we could get a ready-made solution for x here it is a little bit more complicated because here we do not know ah what the inverse is in fact we know that it does not exist

so we cannot do what we did previously

so what we do here is that we multiply by that joint and then in the case that the adjoint of a times the b matrix the constant matrix on the right hand side of the equation depending on whether it is 0 or not 0 then we have the conclusion that we have just written down ok

so this is overall how we use the idea of determinants and in particular its role in determining the matrix inverses to address the issues about consistency and inconsistency in a linear system of equations

so far we have talked about a we have framed we have motivated using two dimensional or three dimensional examples and now we have motivated the problem and then said ok let us consider a general n by n square matrix which is about any equation and n unknowns and then we have come up with a general way to address the issue of whether it has a solution or it has no solution and

so this is sort of the conceptual understanding behind what we want to look at in terms of solving systems of equation next let us look at some examples and try to explore or interpret these issues through these examples ok

so the first example which i want to present is something which we have looked at previously let us continue in this algebraic vein to see what results we get

so this system of equations that we had written in the first lecture had as the matrix a $\begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$ $ax + by = 10$ and this was equal to ten zero

so this plays the role of a this plays the role of x bar and this plays the role of the b matrix all right

so these are to put just some numbers in these values just to get an idea about how do we actually go about the issue

so the first thing that we did is to check whether there are solutions or not and

so for this what we will do is that we look at the determinant of a it's a two by two matrix determinant should be relatively easy let's see if i can ah figure it out

so the determinant of a here is equal to two by two matrix

so we can either do one minus one

so minus one minus four minus five or we could do using um the definition which boils down to nothing but the same expression

so we are multiplying 1×1

so that's minus 1 that is why this term comes and then over here you have minus 4 times 1

so minus 4 is minus 5 and the important thing to note is that it is not equal to 0 and

so this is the first case that we apply and what this tells us is ok this system of equations has a solution in fact when we explore the use of the inverse in trying to find the solution ah we see that we can also construct the solution

so the solution

so this says that this has a solution this implies that this has a solution um how to find the solution what is the solution the solution of x is $a^{-1}b$ right

so using the fact that determinant is not zero we can say that this is a consistent system of equation what is the solution there we have this x bar is $a^{-1}b$

so what is a^{-1} a^{-1} in this case is $\frac{1}{5} \begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix}$ replacing the matrix a with a^{-1}

so $\frac{1}{5} \begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 10 \end{pmatrix}$

so i believe this is the inverse of the matrix we can also check by multiplying this equation with this and it does turn out that it looks to be the identity all right

so this is the inverse and

so the solution

so what is $a^{-1}b$ this is $\frac{1}{5} \begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 10 \end{pmatrix}$ let me just write down $a^{-1}b$ again $\frac{1}{5} \begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 10 \end{pmatrix}$

so implies that \bar{x} is minus 1 by 5 and minus 1
 so that's minus 10 and minus four minus forty
 so this is two and eight
 so this is the solution right
 so what we have done here is the following first we have checked whether this system of equations has a solution or not and in doing that we first calculate the determinant determinant is not zero
 so it is a non-singular matrix therefore it is a consistent system of equations and then we say ok if it is consistent what is the solution and this is in the construction of the solution and previously we have seen that the solution is a inverse times b and
 so we calculate a inverse multiplied by b and then we get the solution now we can of course check whether this solution of two eight satisfies the two individual equations
 so let us check that let us verify whether \bar{x} equal to 2 comma 2 8 satisfies the equations or not
 so what are the equations
 so now let's write them down in their original algebraic forms where you have two equations in two unknowns
 so $4x - 5y = 10$ right
 so suppose you put x equal to two and y equal to eight then yes we see that $x + y = 10$ because two plus eight is ten suppose you put this here
 so x is two
 so four into two is eight minus eight is zero
 so therefore these this solution satisfies the original equations right
 so this is just a sanity check if you will that we have come up with a ah new for some of us way to find out the solution and we see that if you get the solution then by the direct substitution techniques we can come up with checking or a way to check whether the solution satisfies the equations and yes we find that the solution is ah proper solution of these equations okay
 so so far we have looked at this issue algebraically especially because this is a two dimensional example and it is easy to visualize the geometric aspect of this issue we we are going to look at that ah in particular we are going to interpret these results from a geometric point of view and just to get an alternative understanding a geometrical layer to this issue of consistency or inconsistency of system of equations
 so we look at the same example but from a geometric point of view
 so the geometry of this example
 so these are two ah equations
 so these were $x + y = 10$ and $4x - y = 0$ now looking at these two equations from a geometric point of view these are lines these are equations of lines in a coordinate frame
 so let me write that draw it down
 so this is a coordinate frame let's say this is the x axis this is the y axis $x + y = 10$ is a line like this which has as points here $(10, 0)$ and $(0, 10)$.
 this is a rough sketch but the idea here is to get the general shape of these lines proper $4x - y = 0$ is a line like like that
 so this is $4x - y = 0$ and this is $x + y = 10$ and what are we trying to do when we are trying to find a solution for this we are trying to find a set of values small x small y which will satisfy both these equations
 so from a geometric point of view what we are trying to see is whether these two lines intersect at a point or not why is that because if they intersect at a point then that point is going to satisfy the equation of both the lines
 so this point should satisfy both this equation and also this equation and

based on the previous analysis of the example we say that this is the point two comma eight and we have seen that it lies both on this line and also on this line

so the point of intersection satisfies the equation of both the lines and that is the solution that we are looking for right

so this is a consistent system of equations um let's see

so continuing from this idea that ok we can geometrically visualize these two lines as follows in what cases what two lines will have no solution in the sense that what two lines will have no points of intersection well one possibility is that if the two lines are parallel to each other then by definition they do not intersect and

so that may be a situation where ah the solution is going to be inconsistent that is the system of equations is going to have no solution and it will be labeled as inconsistent

so let us see based on this example whether we can come up with a system of equations which have no solution

so suppose we have in mind that we are going to construct a inconsistent system

so suppose we look at again in the phase plane two lines

so this has original is $x + y = 10$ suppose we have another system another line which is $x + y = 20$ this is x this is y and clearly these are two parallel lines

so if we write down these equations like $x + y = 10$ $x + y = 20$ and we try to come up with a matrix version of these two equations and check in our previous way whether this is going to have a solution or based on our understanding of the geometry because these are parallel lines there should be no point of intersection and

so there should be no solution but let us just check the idea being that ok we know what is happening but let us check the established procedure that we are trying to look into to see whether there is too much

so what you would do then is that we say ok this is something like $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$.

and we want to know whether this system is singular or not

so first calculate the determinant is 0 and we then say ok determinant of A is $1 - 1$

so that is 0 .

so clearly we cannot construct a solution like before in terms of $A^{-1}b$ um as you are seeing let us see what happens when we look at that joint of $A^{-1}b$ what is the joint of $A^{-1}b$ a joint of $A^{-1}b$ is replacing one one minus one minus one

so this is the joint of A because the cofactor of one is one

so let us put here cofactor of this one is minus one and that is put here of course the same entry

so it is a symmetric matrix but in general it need not be

so then we will check whether what is the value of a joint of $A^{-1}b$ times d

so in this case this is b

so what is adjoint of A times b this is $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ and this is going to be $\begin{bmatrix} 10 - 20 \\ -10 + 20 \end{bmatrix}$

so minus 10 and then plus 10 .

so we have a situation where the right hand side of the equation which we obtained by multiplying adjoint of A times $A^{-1}b$ equal to b the right hand side is non-zero but the right the left hand side because $A^{-1}b$ times A is zero

so we know that we can check directly here that is zero

so this will come up with a situation where 0 is equal to something which is not 0

so this does not make sense and that is why we labeled it inconsistent and from a geometric point of view we can see that these are two parallel lines

so there should not be any solution and that also is along the lines of our idea of a consistent system of equations

so the goal of this exercise here was to say ok from a purely algebraic matrix point of view perhaps it is not

so clear why is it the case that we label these as inconsistent here we can look at it geometrically and say ok parallel lines no point of intersection no solution therefore by definition definition they are inconsistent ok ah

so here is an example where there is no solution now remember in the idea of ah defining something as consistent we had to say there could be one solution or more than one solution um and we have looked at an example where there is ah one solution ah can we think of an example where there may be more than one solution and particularly in infinite number of solutions um well going back to this geometrical idea of saying that these are lines in a plane let us consider what happens when the two lines describe the same equation if you have two lines you describe the same equation or i apologize if i say i should have said if the two equations describe the same line then what happens then two lines are on top of each other

so any point x and y which is on the line is going to solve the system of equations

so that is what we mean when we say we have more than one solutions from a geometrical point of view it is that those two define the same line

so let just ah for completeness look at an example of that we are looking at an example of possibly infinitely many solutions

so lets geometrically the idea is that if you have the same equation like $x + y = 10$ and this is an equation of the line $x + y = 10$ we had $x + y = 10$ ah and $x + y = 10$ it may not be directly $x + y = 10$ it could be something like $2x + 2y = 20$ into two or twenty because as we see this is just still the representation of the equation of the line

so we have infinitely many solutions why because any point on this line is going to be solving these two just for the sake of verification if you write this down as a system $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ this is the matrix a what is the determinant of a there that is also 0

so it is not straight forward to come up with the solution what about the adjoint of a like previously we are going to write down the adjoint of a

so what is that joint of one a joint of one the cofactor of 1 excuse me is 2 1 and then minus 1 minus 2 .

so this is a joint of a ah what is adjoint of a times b adjoint of a into b is going to be 0 and 0 .

so this is the case where both the left hand side and the right hand side are going to be 0 .

so we cannot say anything about

so as per the procedure we cannot conclude about consistency or inconsistency from here

so we need something else and

so this makes sense because from the point of view of geometric idea there are

infinitely many solutions

so we can use the geometry of the situation to find out whether it is consistent or not consistent

so in these three cases or three examples we have looked at different versions of the system of equations ah in the first case it was going to be one point of solution and we found that it was indeed one point of solution because that was a point of intersection of the lines then in the next case we saw that these are two parallel lines and

so there was no solution and that was also consistent with what we found in the system of equations being inconsistent and finally in this case we come up with a case where we cannot conclude anything about the consistency or inconsistency and some other things may be required and we know which from the underlying geometrical picture that it is consistent system of equations because there are infinitely many solutions

so let me write down this in terms of a table

so the previous three sub examples previous example three variations and the summary is as follows in the first case one determinant of a was not zero

so then we could say what the solution is a inverse b and the underlying geometrical picture is was one point of intersection second and particular to a was when the determinant was equal to zero

so we cannot come up with that but what we found was that this was parallel lines

so we had found that this was actually inconsistent and geometrically this was parallel lines i believe this was the case ah two b yeah

so this was the case two b that we had looked at corresponding to what we did in the procedure and the case two a was when determinant of a was 0 but we could not make a conclusion but from the underlying geometry we knew that this was the same line

so infinitely many solution infinitely many solutions right

so this is like a systematic exploration of one example to understand especially because we can see the geometry of the situation to understand how we go about analyzing whether a system has a solution or called consistent or inconsistent or how we would analyze it from a general matrix point of view

so the same procedure can be looked at in a more general situation when these are three by three or more generally n by n matrices ah and here we want to highlight how just by checking the determinant of the associated matrix we can make a lot of conclusions next let us look at another example of finding out whether the system of equations has ah solution or no solution

so this particular example is based on a je problem in particular this is from the jee advanced website website <https://www.jeeadvanced.ac.in/sample-questions-2016-p2.pdf>

so let me briefly illustrate what the question is

so briefly

so and for more general one could look up the problem in the website which is stating the full statement

so here we are going to consider a two dimensional system of equations $\alpha x + 2y = \lambda$ ah and $3x - 2y = \mu$ and α, λ, μ are real numbers

so the overall problem has many options and one has more more one or more than one can be true but let us just look at one of them and try to see whether that statement can be said to be true or not

so one particular option

so the question is that we are going to look at is the following true that is that if $\alpha \neq -3$ then the system has a unique solution for

all λ and μ does the question make sense

so this is based on a problem which can be accessed in this pdf file from this web page and this is a sub part of this

so based on problem in advanced website and this is a part of it which we are trying to understand based on what we have done

so we have a system of equations two dimensional and we want to know that if α is equal to minus three then whether it has a unique solution or not

so how do we find that out well we can easily represent this in terms of a matrix α two three minus two and then x y then we have λ μ

so in our notation this is the matrix A this is the matrix \bar{x} and this is b and what we want to check is whether it has a unique solution

so by unique we mean only one

so first of all let us check whether it has a solution or not how do we check that we check that by first calculating the determinant of A

so the determinant of a matrix is $\alpha - 2$ and we note that this is determinant is not equal to zero if α is not equal to minus three because if α is equal to minus three then this determinant is zero therefore if it is not equal to minus three it is not zero now what the statement says is that ok if determinant of A is not zero right because that is what they say is α not equal to minus three then what can we say about the system of equation then we automatically say that it is a consistent system but does it have one solution or not for all values of λ and μ how do we check that well let us just check it by constructing the inverse

so the inverse is going to be $\frac{1}{\alpha - 2}$ and then replacing the matrix by the adjoint

so that is $\frac{1}{\alpha - 2}$ and then λ μ

so what we have here is $\frac{1}{\alpha - 2}$ and $\frac{1}{\alpha - 2}$

so $\frac{1}{\alpha - 2}$ and $\frac{1}{\alpha - 2}$

so as you can see there is only one solution for a given λ μ and α and the α is given to be not equal to minus

so therefore yes there is a unique solution because for any value of λ and μ we can come up with one value of this as long as α is not equal to minus three

so yes

so this statement is true

so there are three other options given in the problem and we can each of them can be checked using ah this method

so this goal of presenting this part of this advanced problem is just to say ok the sort of concepts and the problems and the discussion that you have been having is also something that is tested at a more advanced level and the same tools that we have done i mean straight away the determinant comes into picture ah straight away our understanding of whether ah the system is consistent or not that comes into the picture one can also look at and i encourage you to do

so to look at the geometrical picture of ah this particular problem and try to see ok is it going to be a straight point of intersection sort of situation or whether the lines become parallel and

so on and

so forth

so to summarize the goal of this lecture was to investigate the role of determinants in solving system of equations linear system of equations and we did that for a general n by n case we saw how constructing the system can come from simple two by two or three by three examples and then we defined the words consistency inconsistency and then saw using what we learned about in the inverses matrix chapter how the previous lecture about the matrix inverses could

be used especially the idea that the determinant determines whether it is a single matrix or not because that can give you an idea about whether the inverse exists or not and then that can be used to construct the solution or to say something about whether the solution cannot be constructed

so this again illustrates the importance of determinants in solving linear system of equations

so let me just write that down as a summary statement

so this lecture highlighted the roll of determinants in solving linear system of equations ok

so with that i thank you for your attention and i hope the concepts and the problems that we have discussed here are of use in your understanding of the idea about determinants thank you you