welcome to this lecture on the role of determinants in matrix inverses ah what we want to do over here is to see how finding out the determinants of a matrix helps us to check whether first of all it is invertible or not and next to actually compute the inverse

so previously we have seen how to define a determinant how to compute a determinant then we saw how we can look at different determinant properties which help in its evaluation and here we are going to look at an application of the determinants in computing the matrix inverses

so we are going to look at the role of determinants in matrix inverses now one of the things is perhaps good to recall is like what is a matrix inverse why should we care about a matrix inverse why should we care about inverses in general

so if you recall the definition of a matrix inverse it is that if you have a square matrix a another square matrix b such that a times b

so the matrix multiplication of a and b both on the left and the right

so a b is equal to b a is equal to the identity matrix

so that is the definition of an inverse and we denote ah the inverse of a matrix by a power minus 1.

so we have matrix inverse inverse of a matrix a is defined by the expression a a inverse equal to a inverse a is equal to identity and this is the notation notation for inverse right now what is the main idea here of an inverse

so what is the idea of an inverse

so let us forget about matrices or let us look at a simple one by one matrix which is nothing but a scalar

so let us say the number two why do we talk about the inverse of 2 like do we talk about the inverse of 2 is it required well we want to say ok if you have an expression or an equation like 2 times x equal to 1 how do we solve for x for many of us it might be very straightforward ok two x equal to one means x equal to half but what is underlying idea how is it related to ah matrix inverse

so let us look at that

so if you have an equation like two times x equal to one and this two you can think about this as a one by one matrix or a scalar equivalently how do we solve this is there an inverse of two there a way to find inverse of two is it needed right

so essentially what we say is that there is a notion of multiplicative inverse that is that if you multiply two by the number half which is nothing but two power minus one then what we get is one which is in some sense multiplicative identity and

so effectively what we are doing in solving that equation is that we are multiplying both sides by a half and then we are getting x is equal to half

so we have 2 x equal to 1

so we multiply both sides by 2 power minus 1

so that you get 2 minus one into two times x is two minus one and this we know from ah how we define division multiplication that this is one

so that means that x is two power minus 1 or half

so it is this notion of multiplicative inverse which broadly speaking we are expanding to the notion of a matrix inverse right

so this is broadly speaking what we are saying is we want to have an inverse and why we would want a matrix inverse because just like here we have two times x is equal to one we could have a general matrix equation which is a times x is equal to b and here x is not just a scalar but a vector and

so is b and then one way to solve it is to find the matrix inverse a inverse and multiply both sides and then get a solution for x

so we have may be used for matrix equation how we have a equation consider an equation a x equal to b now this is a general n by n matrix

so if we could find a inverse a inverse then you multiply on the ah through the left

so you would say a inverse times a x equal to b which would give a inverse times a x equal to a inverse b and then this is nothing but the identity and identity time of vector x is just the x itself this is a inverse b

so this is why we would like to have a inverse of a matrix for trying to solve these matrix equations and this is a direct generalization of this algebraic concept of finding what ah solution of a equation like two x equal to one or two x equal to three you multiply by the multiplicative inverse which is half um of course we know that if we instead of two if we had the number zero then it is very hard to solve the equation because zero times x equal to one what is the solution ah and what we will see through this lecture through this idea of determinants which by now must have come through how important this is that if we now look at the determinant of the matrix a that holds the key in trying to get ok what is the invertibility does it exist does the inverse exist how do we calculate the inverse

so that is what we want to do in this lecture

so you want to show that if you have a determinant of a matrix being non zero then the inverse exists and how we can define the inverse of a matrix using the determinant and another quantity which we should define shortly

so that is the goal of the lecture and but the idea is very simple is that we are looking for a way to get a condition to try to see how we could then solve matrix equations how we could solve other problems of this nature

so that is the goal of the lecture okay

so goal is to show the use of determinants in checking for invertibility inward to be infinity of the matrix

so that is the one thing and to actually compute it right

so let us begin now how do we come up with a general ah matrix let us say three by three matrix more generally an n by n matrix how do we come up with the inverse of the matrix

so the idea here is going to be a combination of the definition of a determinant and also one of the properties that we looked at ah specifically that the determinant is a sum of um the product of elements of a row or a column and their corresponding cofactors

so that is a determinant and if you look at the cofactors of another row or column then that sum goes to zero

so essentially this idea that something is a determinant and something goes to 0 we are going to exploit that the use of these cofactors in forming a general inverse of a matrix i think the ideal place to start over here would be to start with a 3 by 3 matrix and look look at the columns okay

so let us look at a three by three matrix consider a three by three matrix what in general situation we have used a 1 1 a 1 2 a 1 3 a 2 one a two two a two three and a three one a three two a three three all right now the idea is that if we denote a i j as the cofactor of the element a i j where i and j are the indices for rows and columns respectively what we want to do here is to define the idea of an adjoint of a matrix which is the matrix associated by taking the transpose of a matrix where each element is replaced by their corresponding cofactors

so let me write it down and i will say it again

so you will define a joint of this matrix as the transpose of a matrix obtained by replacing each element a with their corresponding cofactors

so a one one a one two a one three a two one a two two a two three a three one

a three two a three three

 so the adjoint is of a matrix is obtained by taking transpose of matrix where each element is replaced by its cofactor all right

 so the joint of a matrix is defined by the transpose of a matrix obtained by replacing each element by the corresponding cofactor

 so in this case in this three by three case we have the matrix a one two a one three a ah two one second row first column second row second column second row third column third row first column third row second column third row third column ok and

 so this is the matrix and its a joint is obtained as the matrix where each element is replaced by its cofactor and a one two is replaced but by taking the transpose it will come over here and a one three ah same here would be a two one because we are replacing the element here with ah the cofactor and then taking the transpose and then a two two a two three same over here this would be a three one a three two and then a three three all right

 so this is the joint

 so if you call this a matrix a then this will be denoted by a joint of a right

 so here is a matrix here is a joint now to understand the idea behind coming up with this adjoint let us just calculate some terms of their matrix product all right and this will be using these properties of what happens when you multiply the row when you sum the product of a rows with the cofactors of a different row right

 so let us take a look at this

 so what would be the matrix associated by what would be the first term in this matrix product this would be a 1 1 times a one one plus a one two times capital a one two plus a one three times capital a one three right

 so this expression you can realize is just the definition of a determinant ok and in fact we could replace this by the determinant

 so this is nothing but the determinant of a what about the second column what about the second entry over here

 so this would be the first row multiplied by the second column

 so it is going to be a one one a two one plus a one two a two two plus a one three a two three

 so if you look at it this expression is nothing but a sum of a product of elements of the first row of the matrix with the cofactors of the second row and which we have seen in the properties evaluates to 0 right

 so this is 0 now there is going to be one more term but for reasons of space i will not be able to write it here but let us just evaluate one more term and let that be the term in the product which is in the second row and the first column

 so let us evaluate this term here what is that going to be that is going to be the second row multiplied by the first column of the joint

 so this will be a 2 1 a 1 1 plus a 2 2 a 1 2 plus a 2 3 a 1 3.

 so if you look at this term what is it going to be this is the sum of product of elements of the second row with the cofactors of the first row

 so this will also as per the property evaluate to zero right now we could do a fourth sum we could do all of them but the general idea and this i encourage you to verify is that all the diagonal elements will nothing be nothing but the determinant and all the off diagonal elements are going to be zeros

 so the final answer over here and this you can do not just for a three by three matrix but for an n by n matrix in general is that we will get here

 so this is a three by three matrix this is a three by three matrix this product will also be a three by three matrix with only diagonal entries which is coming from here a zero which is coming from here another zero which i encourage you to

check this zero comes from here this is also going to be a determinant of a zero zero determinant of a and this is going to be determinant of a this is a 3 by 3 matrix right
 so this will be determinant of a times an identity matrix the first time let me write down three by three but in general we can understand from the context what is the dimension of the identity
 so there it is
 so we have a matrix and we have ah multiplied it by its a joint which we have defined here and the reason for defining that joint in this way is to is to be able to see this idea that only the diagonal terms are determinant of diagonal terms are zero
 so we can write it down as a constant and in this case it is all important determinant which is the constant times the identity matrix and why is this important well this is important because in looking for a matrix inverse we are looking for something to multiply the um matrix by such that it is equal to identity
 so here we have not got an equal two part but we have got something which is proportional to an identity and
 so ah we can but by using this we can define ah that matrix which when multiplied by ah square matrix will give you identity and what will that matrix be that will be nothing but the adjoint of a divided by the determinant of a
 so the statement that we want to make now the equation that we want to write is that a times a joint of a this we found to be equal to determinant of a times identity and you can also check that if we now instead of post multiplying a joint of a if we say a joint of a times a that will also come as determinant of a times identity and combining these equations we can write that a times the adjoint of a is equal to adjoint of a times a will be equal to determinant of a times i
 so here is a three by three matrix or joint or phasor three by three matrix this is a scalar this is a three by three matrix or in other words we can say a times a joint of a divided by determinant of a is equal to adjoint of a by determinant of a times a equal to i and this we can write down when the determinant is not 0 right and if we compare with a a inverse equal to a inverse a equal to identity which is the defining equation for an inverse we can get that a inverse is equal to adjoint of a by the determinant of a when determinant of a is not zero
 so this is how we can compute the inverse of a square matrix with a non-zero determinant that is just that you define in a joint of a which is nothing but the transpose of a matrix obtained by replacing each element with their corresponding cofactors right and if you divide that by the determinant of a then through a generalization of what we have just seen we can come up with this relation between the joint determinant and the inverse
 so here is this important number of determinant which is now playing a role in defining the inverse of a matrix which is important for the reasons that we outlined at the beginning ok
 so this is what we want to say in terms of the how to get the inverse next we will look at some more ideas behind the inverse of the matrix and how the determinant plays an important role and including specifically saying how we can say like here we are saying that ok if the determinant is not 0 that is when we can define the inverse next we will say that ok we can make that even more stronger statement ok but before we do that i think it is important to point out over here that this adjoint is also a matrix in fact it is a matrix of the same order as the initial matrix and that being said what can we say about the determinant of this adjoint does that make sense

so a natural question which arises now that when we look at matrices is what is the determinant and can we from this come up with the determinant of the joint that is what we want to look at now

so ah note here is what is the determinant of the adjoint right

so what is the determinant of that joint ah and in answering this question we have to state a property which also has independent use independent importance and that is that a determinant of product of two square matrices is going to be the product of their respective determinants the property that we use we use the property that determinant of a into b is equal to determinant of a times determinant of b right where a and b a and b are square matrices ah using some simple examples you can verify that this is the case we will not go into a proof of this property here but we can just check whether it is the case ah you know on surface it might look like a simple property but it is not always the case for example if you have the sum of two matrices it is not necessary for the determinant to ah the sum of their determinants to equal to that value but in this case of the product it happens

so it is in fact a remarkable property and just to see how it works out let us look at some examples

so for example if we take suppose you take the matrix a as one two two one what is its determinant is one minus four

so minus three and let us say we have another square matrix b which is two one one 2 what is the determinant list 4 minus 1 3 um what about their product a b this is two two four one two five ah two f five and four

so the determinant is four sixteen minus twenty five

so minus nine which is equal to

so these are the determinants

so this is an example of saying that therefore we can check the determinant of a b is determinant of a times the determinant of b ok

so we are going to use this property to find out this joint determinant of that joint and going back to the definition of the inverse and the situation that we derived that's where we're going to apply this property ok that the determinant of a product because we have just seen that the product of a and the joint of a is equal to determinant of a times i right and

so now we are going to take the determinant of both sides and because we can decompose the determinant of a product of matrices into the product of the determinants that is how we are going to get this idea

so first in the three by three case that we were looking at we use the property in here

so we have the fact that a times a joint of a equal to determinant of a times the identity right

so take the determinant over here and the fact that we had the determinant of a times adjoint of a is going to be the determinant of determinant of a remember is just a scalar right times an identity ah

so in the three by three case the right hand side is going to be just determinant of a cube right because this matrix is nothing but the diagonal matrix with the determinant of a in the diagonals three times derivative of a cube this is for three by three case and in general it is going to be determinant of a power n for n by n case and what about the left hand side well using the property that the determinant of a product of matrices is the product of the determinants you will get the determinant of a times the determinant of a joint of a right

so if the determinant is not zero then we can write just cancel one determinant of a on both sides that we can get that the determinant of adjoint of a is equal to the determinant of a whole square for the case of three by three and in

general it is just determinant of a power n minus 1 for n y

so we have introduced a new matrix which is the joint of a and immediately because we are talking about determinants it is natural to ask what is the determinant of the matrix and here we see that the determinant of that joint is related to the determinant of a it is n minus one ok

so um this is the adjoined determinant and just thinking about it like if n is equal to 3 then it reduces to what we have seen if n equal to 2 it is determinant itself ah for n equal to 1 it is 1 i think that is just a problematic case or a special case which comes out because for a one by one determinant which is a scalar it is really hard to define a cofactor

so i think ah i think this this sort of expression should be taken for n greater than one ok

so this is the determinant of an adjoint of the matrix all right

so now that we have seen that you know we are trying to find out ah about the inverses and determinants and at joints and this is the new matrix that we defined in terms of the adjoint and now we can look at how we proceed from here in terms of noting this importance of a determinant in trying to find out the inverse of a matrix now we just state the same things in a more formal fashion

so in many places we have said that the determinant of a is not zero

so note determinant of a not zero condition invoked or used many times in this lecture in this lecture

so based on this we can define a singular matrix as

so this is a new matrix new word that we are using here we are defining a singular matrix as a matrix with a 0 determinant and analogously we are going to define a non-singular matrix what is going to be a non-singular matrix a non singular matrix as a matrix with a non zero determinant

so a singular matrix will be one with a zero determinant and non singular matrix will be one with a non zero determinant

so in a sense we are sort of highlighting the importance of the determinants in defining classes of matrices either being singular or non-singular depending on whether their respective determinants are 0 or not 0 ok and the theorem that we want to state over here is that a square matrix a or a is invertible if and only if it is non singular

so let me write down the statement and then we can look at the proof

so we are going to say that the theorem is square matrix a is invertible if and only if is non singular meaning that it has a non-zero datable how will we look at the proof well we look at both ways both the if and only if part first we will say that if a is invertible then it is non singular that is determinant is nonzero and the other way around that if it is non singular matrix then we can show that a is invertible let us look at this part first

so if a is invertible that means that there exists a matrix b such that a times b is equal to b times a equal to identity and now taking determinants taking the determinant we can get that determinant of a b is equal to determinant of identity

so what is the determinant of the product of matrices it is nothing but determinant of a times the determinant of b and what is the determinant of identity identity is a diagonal matrix each element is one and

so this is equal to one now this already says the determinant of a is not zero why because if it was zero then it this relation would not hold and we are guaranteed this relation holds by these sequence of steps we start with the fact that a is invertible

so this means just by the definition that a is non-singular right

so the implication part is just by defining or show or by starting from the fact that a is invertible using just a series of steps including the definition

of this invertibility including the fact that you can take the determinant of a product of matrices nothing but the product of the respective determinants we can say that a is a matrix which is non singular the only if part is also relatively simple it says ok if we know it is non singular means that we can define a matrix like the joint of a divided by the determinant and that is where we are using the fact that determinant is not zero that we can divide by the determinant and this as we have seen ah specifically for three by three case but we can check for a general n by n case will define the inverse of the matrix
 so the inverse part or the reverse part is that a is non-singular this means by definition determinant of a is not zero which means we can define a inverse is the adjoint of a divided by the determinant of a and this as we have seen satisfies the properties of what is required from the matrix inverse and therefore a is invertible this satisfies a a inverse equal to a inverse a equal to i therefore a is invertible
 so the statement here was that a is invertible if and only if it is non-singular and non-singular is defined in terms of the determinant
 so there you have it
 so you have or we have a way in which we say that ok if by calculating the determinant of a matrix if it is nonzero then you are guaranteed that it is invertible and not only that is just a statement and in the proof we have come up with a way of defining the inverse
 so this theorem is significant for these two reasons that it both gives a condition for ah checking the inverse of a matrix which is the non zeroness of the determinant and also it defines the inverse all right
 so this can be summarized in this statement that determinants help in checking invertibility of matrices and also in computing the inverse
 so this is the significance of the last theorem significance of theorem
 so determinants are important ok
 so that is how we have seen how determinants help in finding out the invertibility of matrices and in computing the inverse now we are going to look at some examples of calculations of the inverses and how we can of course it is not always the case that just because we know now we can define a inverse in this adjoint matrix and using the determinant we can also come up with by sometime in some cases ultimately it's a question of which is the easier way to define the inverse
 so if it is a general case we can always use that joint definition divide by the determinant that will be the inverse in some other cases if for example it is a purely diagonal matrix ah we can just by inspection also come up with a inverse because ultimately the concept is that if you take a matrix multiply it by its inverse that should give you identity
 so whichever way you can come up with but the importance of determinants is that it gives some method to this intuition that you can formally come up define in a joint divide by the determinant and that will give you the inverse right
 so let us look at some examples
 so one example is as follows let us look at a three by three matrix consider a matrix like x 0 0 0 y 0 0 0 z now looking at this matrix it's a diagonal matrix
 so we what would we multiply a diagonal matrix by
 so that it gives you identity then we could go through defining the joint of this matrix finding the determinant let us do that another way we could also directly write down the solution what would be the direct solution well just by looking at this we see that ok if we can multiply just each of the diagonal elements by their respective inverses
 so if i can multiply x with 1 by x y with 1 by y and z by 1 by z and that will only work if each of them is not 0 then it is possible i can get an identity and

how will i do that well i can do that if i have another diagonal matrix which entries one by x one by y one by z

so what i am saying is that one way to do it would be to say ok if i multiply it by

so this roman numeral one ah is not identity

so maybe i can just say that this is 1.

so if i multiply by 1 by x 0 0 0 1 by y 0 0 0 1 by z you can verify that this product of this matrix and this priority is going to be identity because x zero zero multiplied by this first column x times one by x is one zero zero same if i multiply this by this diagonal matrix there is no term which is not zero same with other columns essentially only the diagonal terms will be picked up but this will work this works when x is not equal to zero y not equal to zero z naught equal to zero

so this is one inspection way

so this might work for some cases like you know there is there is nothing um there is no requirement that always a matrix inverse have to be defined through that joint route sometimes by intuition we can always come up with a thing but the importance of the what we have done now is that it gives a formal way to come up with the inverse

so let us try to define that joint of this matrix come up with the inverse in the way we have defined it right

so first of all what you do you will check ok is it invertible at all for that how do we know well the theorem says if and only if the matrix is invertible if and only if the determinant is 0 excuse me if and only if the determinant is zero

so what is the determinant of this matrix

so this is the second way

so two method two um what is the determinant determinant is its a diagonal matrix

so by the property the determinant is x y z and this is non-singular if determinant is not zero

so x y z the product is not zero and that you can see is the same condition as we see over here that each each of these are non zero then it is not zero and vice versa these are the same condition that we get

so already we we see that this can be ah it is going to be an equivalent kind of derivation ah well what about that joint can we come up with an adjoint of this matrix

so it is going to be ah transpose of the matrix obtained by replacing each element with their cofactors

so what is the cofactor of this it is the determinant of this matrix

so that is y times z what about here well you see if you just black out this row and this column you are left with a matrix which has three zeros in fact one row identically of zero and

so that is going to be zero same over here

so for example if you black out this column and this row all we are left with is these four elements ah number of ways to use the property to say that the determinant of this thing is zero one is that one row is zero second is upper triangular matrix with the diagonal elements are zero there are many ways or you can also directly calculate the determinant in fact if you look at all the calculations you see that all these terms will be zero this will be x times z and this will be x times y it's a diagonal matrix

so the transpose is equal to itself

so this is just to write it down again would be y z x z x y and everything else

is zero ok and the inverse which will be the joint divided by the ah determinants ok that expression you see that we divide y z by x y z

so you will get one by x

so that is nothing but what you got here you divide x z by x y z we will get one over y which is this element we divide x y by x y z will get one over z

so this term which we derive from this more formal method will going to give you the same thing

so both through formally defining the determinant and also by inspection we will come up with the same sort of method

so this ah the purpose of the example is just to say ok ah it is not that this is some ah magical way to come up with a determinant when there is some magic involved in how we define the joint how we come up how someone came up with the idea of coming up with an adjoint ah defining the joint and

so on but ultimately the method is such that it matches what we expect

so therefore we should have confidence after having understood this example that yes defining the inverse by the ah taking the adjoint and dividing it by the determinant that should work ok

so let us look at one more example of this and ah just to solidify or concretify our understanding of how we can calculate matrix determinants

so in this let us look at a two by two example and let us look at a numerical example here

so let us look at an example a which is two 1 3 2

so maybe let us have different parts over here we can say calculate a inverse ok

so looking at this matrix now this is an example of having a matrix um just by inspection it is hard to come up with an inverse of course for a general two by two matrix we can you know develop a formula and come up with the inverse that is also fine but let us do it in the structured way for this the first thing always before doing the calculation of the inverse is we should check whether it is existing or not

so first question to ask is does the inverse exist does inverse exist how will we check that well we will first look at the determinant what is the determinant of a is 2 2 4 minus 3 4 minus 3 is 1 um

so the inverse exists as determinant is not zero all right um okay then next

so we know the inverse exists next how do we calculate it well we can calculate it by defining that joint in fact here the inverse is equal to the joint y because the determinant is one

so what is that joint

so then what is a inverse

so a inverse would be the joint of a which is that as determinant of a is one what is that joint sort joint is again the

so let me write it here perhaps that is a transpose of a matrix obtained by replacing each element with the cofactor

so cofactor of two is two cofactor of three is minus one cofactor of 1 minus 3 minus is coming because it is in the second row first column

so minus 1 power 3 and then if you delete this column and this row we get 3 same here

so this is 2 minus three minus one and two and now we can check

so we have two three one two and two minus three minus one two determinant is one

so this is what the what we have studied in this lecture is the inverse of the matrix um is it the case

so it's always good to check the two

so let us check let us at least check whether a times a inverse is the identity

so let's check
so what is a times a inverse
so this times this
so the first element is 2 times 2 plus 3 times minus 1
so that is 1 second is 2 times minus 3 plus 3 times 2
so minus 6 plus 6 that 0 the this entry is 1 times 2 plus 2 times minus 1 is 0
and the last entry is 1 2 times minus 3 times 2
so 4 minus 3 again that's one
so which is exactly like the identity all right
so um this is ah completes what we want to do is that we want to calculate the
a inverse ah or something which not directly we can say is the inverse
so what is the inverse but maybe for two by two you can come up with formulas
in general it is hard but this is the way we can calculate the inverse for a
general matrix um one of the things also is important to mention is that there
are different ways to calculate the inverse in some sense the determinant plays
an important role in most of them ah but there are other ways
so for example ah how do we calculate the inverse
so calculating inverses right
so what are the ways
so the ones that we have seen is by inspection and that happens in some cases
so by inspection my inspection for example for some cases ah then there is ah
using this definition of the adjoint and the determinant ah which has a plus
point that it also gives you whether we should calculate that joint or not just
by checking that the determinant is zero it gives you condition for the
existence of the inverse
so by using determinant and adjoined
so the bonus here is that bonus is that provides a condition provides a
condition to check for existence of an inverse there are perhaps other ways also
and just to complete this subject just wanted to present a simple example for
this and that is that if you have a matrix a which satisfies a polynomial
equation ah in some cases that can also be used to calculate the inverse ah
so in particularly for the case the example that we just presented ah for the
example that we just presented for previous example if you continue the previous
example the matrix a which is defined to be 2 3 1 2 that can also be shown to
satisfy the equation satisfies a square minus 4 a plus i equal to 0.

so you could check this you can say a times a plus i whether it is equal to
minus 4a or not
so this can be checked can we checked ah how can this be used to calculate the
inverse well consider multiplying how to use this to calculate a inverse well we
can multiply by a inverse then we will get an equation like a inverse times a a
so i have replaced a square with a times a minus 4 a inverse a
so i just taken this 4 outside plus a inverse times i
so this is 0.

so one of these can be combined a inverse times a is identity
so this is identity times a this is again identity
so minus 4 i plus a inverse because identity times any matrix is the matrix
itself or this is a inverse is equal to 4 i minus a
so let's see whether this gives you the inverse
so 4 i will just have 4 0 0 4 minus a and a is written here
so this will be 2 minus 3 minus 1 and 2 and this we can check from what we were
doing before that this 2 minus 3 minus 1 2 is the same as what we have got here
so this is not just one way to calculate the inverse this is also a inverse

so another way to calculate a universe right

so just for completeness i show this because ah the reason i showed this was just because there are different ways to come up with the inverse one is that you have ah by inspection by calculating that joint and the determinant and this is a way in which there may be other ways

so just um determinants are important but of course they are not the only way to calculate the inverse ah and while we are at it i mean one question which might arise is ok where is this kind of equation coming from um surprisingly or perhaps not surprisingly because determinants are very important these equations come from looking at the determinants of some special kind of matrices

so this one you can check this equation is the determinant of a matrix obtained by doing the following

so lambda i minus a or determinant of lambda i

so lambda minus 2

so lambda here is a variable equation is lambda minus 2 minus 3 minus 1 lambda minus 2

so and if you put lambda equal to a then you will get ah this equation

so this can be checked this is not directly related to ah the idea of how determinants help in getting the matrices but what we can check in state in general is that if we have any square matrix a and construct the matrix lambda i minus a take the determinant and then replace lambda with a and whatever one is there you can replace for identity then we will find that that equation that is satisfied by the matrix

so if you replace lambda then that equation obtained in this fashion which also includes a determinant can be ah solvable by the by the matrix and then that fact is also used in some other equations

so that is a more of an advanced topic which is to ah see ah some other properties of matrices but the my main intention of presenting it over here is that ok there may be other ways but there also these determinants are important

so ah the deep determinant is an important tool an important number associated with a square matrix many interesting properties ah there are some geometric ideas some algebraic ideas ah the very interesting properties of the determinant itself ah some we have presented here ah for example the product of the determinant is the determinant of the product that idea it has many applications one of which we have seen here is in terms of finding out the inverse of a matrix and specifically what we have the statement that we have made is that the determinants help in checking the invertibility of matrices and in computing the inverse and this was the significance of the theorem that we presented all right

so with that i end this lecture and i thank you for your attention thank you you