

welcome to iit palm mathematics problem solving session this is lecture number three in today's lecture i will solve some more problems related with matrices and determinant

so let's start with problem number one question find the total number of distant  $x$  belonging to  $\mathbb{R}$  that is the real numbers for which  $x^2 + x^4 + x^6 + x^8 + x^{10} = 10$

so we need to find  $x$  which satisfies this determinant equation

so let's solve when  $x$  has to be real number okay

so let's simplify the let's break this determinant in two parts the first part is  $x^2 + x^4 + x^6 + x^8 + x^{10}$  this is the first determinant the second determinant is  $x^2 + x^4 + x^6 + x^8 + x^{10}$  okay

so now let's uh let's solve this these determinants simplify these determinants okay

so what i will do is i'll take  $x$  common from the from the first determinant i will take  $x$  column from the column 1 and  $x^2$  from the column 2

so we get  $1 \ 2 \ 3 \ 1 \ 4 \ 9 \ 1 \ 1 \ 1$  plus i'll take  $x$  from column 1  $x^2$  from column 2 to  $x^3$  from column 3  $x^4$  into  $x^2$  into  $x^6$  and we get  $1 \ 2 \ 3 \ 1 \ 4 \ 9 \ 1 \ 8 \ 27$  this is equals to 10 okay

so this implies this is  $x^3 - 1$  into  $4 - 9$  minus  $1$  into  $2 - 3$  plus  $1$  into  $18 - 12$  okay plus this is  $x$  to the power 6 and now here i will take 2 common from the row 2 and 3 from the row 3.

so 2 into 3 we get  $1 \ 1 \ 1 \ 1 \ 2 \ 4 \ 1 \ 3 \ 9$  this is equal to 10 okay this implies what what do we get there it's a  $x^3 - 2$  minus  $5$  plus  $1$  plus  $6$  plus  $6x$  to the power 6 and this is nothing but  $1$  into  $18 - 12$  minus  $1$  to  $9 - 4$   $1$  into  $3 - 2$  this is equals to 10 okay

so this implies that this is  $2x^3 + 6x^6$  and this is  $6$  and minus  $5$  plus  $1$  is equals to 10 all right

so we get  $12x^6$  right

so we have  $2x^3 + 12x^6$  equals to 10 this implies  $6x^6 + x^3 - 5$  is equals to 0

so let's factorize it

so we have get  $6x^6 + 6x^3 - 5x^3 - 5$  is equals to 0 this implies we get  $6x^6 + 1 - 5x^3 + 1$  equals to 0

so this implies  $x^3 + 1$  into  $6x^3 - 5$  is equals to 0

so this is 0 either  $x^3 + 1$  is equals to 0 or  $6x^3 - 5$  is equals to zero or both ok

so the solution of  $x^3 + 1$  is equals to zero or  $6x^3 - 5$  is equals to 0 okay

so let's solve let's solve this let's find the find the roots of these these two equations

so this i can further simplify it

so this is nothing but  $5/6$  to the power sorry equals to 0

so here what i will do is i will i will just take some general form see for example i will calculate the root of  $x^3 + a$  what is that this is nothing but  $x + a^{1/3}$  minus  $a^{1/3}$  plus  $a^{1/3}$

so this is equals to zero

so this implies  $x$  is equals to minus  $a^{1/3}$  this is the this is the first food and  $x$  is equals to for this quality equation we get  $x + a^{1/3} - a^{1/3}$  square minus  $4$  square divided by 2 this is nothing but a plus minus  $a^{1/3}$  divided by 2 okay

so these things okay

so let's okay

so  $x$  is equal to  $-a$  and  $x$  is equal to  $a \pm \sqrt[3]{\frac{a^3 + 27b^2}{4}}$  are the roots of  $x^3 + ax + b = 0$ .

so here if you see we have only one real root which is  $x = -a$  ok

so  $x^3 + ax + b = 0$  has only one real root which is sorry  $x = -a$  similarly  $x^3 - ax + b = 0$  has only one real root given by  $x = a$  okay other two are complex one okay

so we can do the industry in the same way

so this means what this implies  $x^3 + 1 = 0$  has one real root  $x = -1$  and  $x^3 - 5 = 0$  has one real root  $x = \sqrt[3]{5}$  okay

so this means this implies we only get 2 the real root for one for this equation  $x^3 + 2x + 1 = 0$  and they are given by  $x = -1$  and  $x = \frac{1 \pm \sqrt{1 - 12}}{2}$  okay

so so

so this is the this is the final answer let's solve another problem okay

so let  $A = \begin{pmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha^2 \\ \alpha^2 & \alpha & 1 \end{pmatrix}$  such that  $\det A = -64\alpha$  and then what will be the value of  $\alpha$  okay

so we need to first find a determinant in terms of  $\alpha$  and then we try to solve this equation okay

so so let's let's do that

so determinant of  $A$  is nothing but let's simplify those terms

so it is nothing but  $1 + 2\alpha + \alpha^3 + 4\alpha + 4\alpha^3 + 4\alpha^3 + 6\alpha + 9\alpha^3$  okay then the second row  $4 + 4\alpha + \alpha^3$  okay and this is  $4 + 8\alpha + 4\alpha^3$  this is  $4 + 6\alpha$  no it's a 12 sorry this is  $4 + 12\alpha + 9\alpha^3$  okay

so let me just write this again sorry for that okay

so what was that this is  $4 + 12\alpha + 9\alpha^3$  and  $5\alpha^3$  okay the third row is  $9 + 6\alpha + \alpha^3$  okay then  $9 + 12\alpha + 4\alpha^3$  square all right and then third entry is  $9 + 18\alpha + 9\alpha^3 - 5\alpha^3$  okay

so this is nine okay

so so now now this is the determinant  $A$

so let's try to simplify it okay

so we'll we'll do some we'll make some elementary row operation

so what i will do is i will apply this operation  $r_2 = r_2 - r_1$

so that means we will subtract the row 1 with the row 2 and row 3 also will subtract row 1 with the row 3

so  $r_3$  goes to  $r_3 - r_1$  okay

so so as we know that with these elementary row operation the determinant doesn't change

so this is nothing but okay

so what is that

so this is uh for there is no change in the first row

so we'll just write it which is  $1 + 2\alpha + \alpha^3 + 4\alpha + 4\alpha^3 + 4\alpha^3 + 6\alpha + 9\alpha^3$  and  $1 + 6\alpha + 9\alpha^3 - 5\alpha^3$  then  $r_2$  is minus

r1

so what we get we get 3 plus 2 alpha here we get 3 plus 4 alpha here we get 3 plus 6 alpha okay and uh third row is r three minus r one

so we get eight plus four alpha okay and then eight plus eight alpha okay and we get 8 plus 12 alpha okay

so this is what this is what we get now we again uh make some changes uh we again do some elementary row operations

so for example if i multiply uh the second row by two and uh subtract it with the third row

so this is the operation i will do  $r_3 - 2r_2$  all right okay

so let's see let's see what we get

so here there is no change in the first and second row determinant a is equals to okay there is no change in the first two rows

so one plus 2 alpha plus n phi square 1 plus 4 alpha plus 4 l phi square 1 plus 6 alpha plus nine alpha square okay the second row is three plus two alpha 3 plus 4 alpha and 3 plus 6 alpha then the operation was  $r_3 - 2r_2$

so  $r_3 - 2r_2$  is we get

so alpha term will be knocked out

so we get here 2 and 2 and this is also 2.

okay

so let me just verify one more time okay

so all right let me just delete this all right

so can we can we further simplify this

so yes we can do now we'll do some column operation for example i will make this operation  $c_2$  goes to  $c_2 - c_1$  and  $c_3$  we will apply  $c_3 - c_1$  okay then let's see what we get

so we know that determinant value doesn't change

so there is no change in the first column

so 1 plus 2 alpha plus 1 phi square 3 plus 2 alpha 2 and this is nothing but 2 alpha plus 3 alpha square and this is 4 alpha plus 8 l phi square this is 2 alpha and this is 4 alpha all right and uh this one enter will be zero this is also zero okay

so now now we can we can just open the determinant

so this is nothing but we'll just uh open it across the third row

so this is nothing but 2 times 4 alpha into 2 n plus 3 l pi square minus 2 n point 2 4 alpha plus 8 alpha square and this gives me this gives me 2 times 8 alpha square i del y square plus 16 alpha cube okay

so this is nothing but 8 l phi square plus 12 alpha q minus 8 alpha square minus 16 alpha cube okay and this is equals to uh little pi square will be knocked out

so this is equals to minus 8 alpha cube

so the determinant of a is equals to minus 8 alpha cube okay

so in the question it is given that the determinant is equals to minus 648 alpha this implies minus 8 alpha cube is equals to minus 648 alpha

so this implies alpha q minus 8 81 okay l pi is equals to 0 this implies alpha times alpha minus 9 alpha plus 9 is equal to 0

so this implies alpha values are 0 9 and minus 9

so these are the three values of alpha for which that equation holds

so this is the this is the final answer okay okay

so let's solve another problem question let in and in b2 3 cross 3 matrices such that  $m \cdot n$  is equals to  $n \cdot m$  further if  $m$  not equals to  $n$  square and  $m$  square equals to  $n$  to the power 4

so that the first part is determinant  $m$  square plus  $m \cdot n$  square equals to zero the second part is there is a  $a^3$  cross three non-zero matrix  $u$  such that  $m$

square plus  $m \times n$  square  $u$  is zero matrix okay okay  
 so let's solve this problem answer okay  
 so so this is what is given it is given that  $n$  is equals to  $n \times m$  let's see what it gives us right  
 so if  $i$  multiply uh  
 so  $m \times n$  into  $n$  from and from the right then  $m \times n$  square is equal to  $n \times m \times n$  and now  $i$  can apply the  $m \times n$  is equals to  $n \times m$  here  
 so we get  $m \times n$  square is equals to okay and this is equals to  $n$  square  $m$   
 so we have  $m \times n$  square is equals to  $n$  square okay  
 so now take this this is what is given  $m$  square is equals to  $n$  to the power 4  
 so this implies  $m$  square minus  $n$  to the power 4 is  $\emptyset$  matrix  
 so this  $\emptyset$  represents the  $\emptyset$  matrix  $\emptyset$  matrix is the matrix with all entries are  $\emptyset$  okay  
 so now let's try to do some manipulation in this equation  
 so  $i$  can always write  $m \times s$  square minus  $m \times n$  square plus  $m \times n$  square minus  $n$  to the power 4 is equals to  $\emptyset$  matrix this implies  $m \times i$  can take common here then it is nothing but  $m$  minus  $n$  square here  $i$  will change  $m \times n$  square to the  $n$  square  $m$  from here  
 so  $i$  can write  $n$  square  $m$  minus  $n$  to the power 4 is equals to  $\emptyset$   
 so this implies  $m \times m$  minus  $n$  square plus  $n$  square if  $i$  take from here then we will get  $m$  minus  $n$  square is equals to  $\emptyset$   
 so we have we have  $m$  plus  $n$  square into  $m$  minus  $n$  square is zero  
 so let's call this equation number one okay  
 so now consider two cases case one where determinant of  $m$  plus  $n$  square is zero okay this implies then the first part was that we need to prove the determinant of  $m$  square plus  $m \times n$  square  $i$  mean square to be  $\emptyset$   
 so this determinant  $i$  can write it as a determinant  $m$  into  $m$  plus  $n$  square this is equals to determinant of  $m$  into determinant of  $m$  plus  $n$  square and determinant of  $m$  plus  $n$  square we are considering to be zero  
 so this implies this is zero  
 so in this this zero is the scalar  $i$  know  
 so don't get confused with these zeros all right okay  
 so this determinant is zero  
 so that means the that means the first part is done under this case  
 so there is  
 so but another case is also possible that means case 2 is determinant of  $m$  plus  $n$  square not equals to zero  
 so these are the only two cases possible  
 so if this is the case this implies  $m$  plus  $n$  square is invertible that means this inverse exists ok then then we multiply equation 1 from the left with the  $m$  plus  $n$  inverse okay this implies  $m$  plus  $n$  square inverse into  $m$  plus  $n$  square  $m$  minus  $n$  square to be equals to zero all right okay  
 so here what we did we just multiply them plus  $n$  square inverse to equation one from the left  
 so this gives identity matrix  
 so this implies that  $m$  minus  $n$  square is a  $\emptyset$  matrix all right  
 so let me just come right here this implies that  $m$  is equals to  $n$  square and this is not possible right which is not possible because in the question it is given that  $m$  is not equal to  $n$  square which is not possible ok all right  
 so this implies that case 2 cannot occur because if it occurs then then we reach to the contradiction ok  
 so this implies that determinant of  $m$  square plus  $m \times n$  square is zero as concluded in in the under case one okay  
 so the first part is first part is done okay

so what is the second part

so let's prove the second part in the second part there is we need to show that there exists there exists a non-zero matrix  $u$  all right

so that  $m$  square plus  $m$   $n$  square  $u$  is equals to zero

so in the second part we need to show that  $m$  square plus  $m$   $n$  square  $u$  equals to  $0$  for some non-zero matrix  $u$  okay

so this zero this represents the zero matrix right

so i don't need to mention all the time i think it is easier to follow okay whenever there is a scalar that zero represents the scalar whenever on the other side it is a matrix it represents the matrix okay

so that means we need to identify a non-zero matrix  $u$  for which this is true okay

so now let's uh go back to the equation one what was the equation number one from one we had this  $m$  plus  $n$  square into  $m$  minus  $n$  square is equals to zero

so now you just this implies you just multiply this by  $m$  okay this implies  $mn$  is  $m$  square plus  $m$  and square  $m$  minus  $n$  square is equals to zero okay then define  $u$  that non-zero matrix as a  $m$  minus  $n$  square which is not equal to  $0$  from the from the question from the hypothesis of the question okay this implies  $m$  square plus  $m$  square  $u$  is equals to zero matrix

so this is what hence proved okay yeah

so let's solve another problem

so let  $m$  is  $3$  cross  $3$  matrix given by  $\begin{pmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{pmatrix}$  and it's a joint is given by a joint  $m$  is given by  $\begin{pmatrix} -1 & 1 & -1 \\ -6 & 2 & -5 \\ -3 & 1 & -1 \end{pmatrix}$  okay second we is eight minus six two minus five three minus one okay where  $a$  and  $b$  are real numbers  $a$  and  $b$  are real numbers okay and then show that number one is  $a$  plus  $b$  equals to three second part is a joint  $m$  of inverse plus a joint of  $m$  inverse is equals to minus  $m$  the third part is if  $m$  into  $\alpha$   $\beta$   $\gamma$  which is three cross one vector is equals to  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  then  $\alpha$  minus  $\beta$  plus  $\gamma$  equals to  $3$  okay

so so this is the problem

so let's let's solve it answer okay

so given uh  $m$  is given the joint is also given and  $m$  has to know uh entries which are  $a$  and  $b$

so basically first let's try to compute  $a$  and  $b$

so let's solve the first problem first part

so here  $m$  is  $\begin{pmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{pmatrix}$  all right

so so what is the what is the one one cofactor of  $m$

so this is nothing but the determinant of sub matrix which is obtained by deleting the first row and first column

so which is nothing but  $\begin{vmatrix} 2 & 3 & b \\ 3 & 1 & 1 \end{vmatrix}$  the determinant of the sub matrix

so this determinant is  $2$  minus  $3$   $b$  okay

so given we know the joint matrix or joint of  $m$

so this  $2$  minus  $3$   $b$  this implies  $2$  minus  $3$   $b$  should be equals to the  $1$   $1$  entry upper joint  $m$  right which is nothing but  $-1$

so this implies  $3$   $b$  is equals to  $3$  implies  $b$  is equals to  $1$  okay

so  $b$  we have obtained

so now let's find the three one cofactor of  $m$  which is nothing but the determinant of sub matrix obtained by deleting the third row and first column

so this is nothing but  $\begin{vmatrix} 1 & a \\ 2 & 3 \end{vmatrix}$  determinant of this  $2$  cross  $2$  matrix and this is nothing but  $3$  minus  $2a$  okay

so given  $um$

so as we know that joint  $m$  is uh is a transpose of a cofactor matrix

so this three one cofactor will be stored in at the one three place of a joint  $m$

so this implies that  $3$  minus  $2a$  this will be the  $1$   $3$  entry of a joint  $m$  which

is nothing but minus 1 all right

so this implies  $2a$  is equals to  $4$  implies  $a$  is equals to  $2$

so  $a + b$  is equals to  $2 + 1$  is equals to  $3$

so first part is done okay

so let's go to second one okay

so this says that let me just write the expression and joint  $m$  inverse plus a joint of  $m$  inverse this is equals to minus  $m$

so this is what we need to we need to show okay all right given we know the  $a$  and  $b$  value

so using these values what is  $m$   $m$  will be  $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 3 & 1 \end{pmatrix}$  in place of  $a$   $i$  will put  $\begin{pmatrix} 2 & 1 & 2 \\ 3 & 3 & 1 \end{pmatrix}$

so  $b$  is  $\begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

so let's calculate

so determinant of  $m$  is nothing but uh  $1$  minus  $9$

so this is  $1 \times 1 - 9 \times 2 = 1 - 18 = -17$

so this is nothing but  $8$  minus  $10$  is equals to minus  $2$

so determinant of  $m$  is equal to minus  $2$  okay

so as we know that uh determinant of a joint of  $m$  is nothing but the determinant of  $m$  whole square because  $m$  is  $3 \times 3$

so it's a  $4$  which is not equal to  $0$

so this means that a joint  $m$  matrix it is invertible all right ok

so let's let's prove the expression

so we know that at joint  $m$  into a joint  $m$  and joined  $m$  inverse

so this is true for any two matrices right which is nothing but identity matrix all right

so what is our joint  $m$   $i$  joined  $i$  joined  $m$  is the determinant  $m$  into  $m$  inverse joint  $m$  inverse is equals to  $i$

so this implies that we can multiply by  $m$  from both side from the left

so we get adjoint  $m$  inverse is equals to  $m$  upon determinant of  $m$

so as we know that determinant of  $m$  is equals to  $1$  upon determinant of  $m$  inverse

so this is nothing but determinant of  $m$  inverse  $i$  can also write this  $m$  inverse of inverse okay

so this is nothing but a joint of  $m$  inverse okay

so what do we have we have this relation and joint of  $m$  whole inverse is equals to  $m$  upon determinant of  $m$  is equals to a joint  $m$  inverse okay

so this means this means a joint  $m$  inverse plus a joint of  $m$  inverse is equals to  $2m$  upon determinant of  $m$  and determinant of  $m$  is minus  $2$

so this is equals to minus  $m$

so this is what we wanted to prove right

so this implies let me just write the last line which is nothing but a joint  $m$  inverse plus a joint of  $m$  inverse is equal to minus  $m$

so hence okay

so now let's go back to the third part let's solve the third one third one says that  $m$  if  $m$  and for  $\beta$   $\gamma$  is equals to  $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix}$  then and minus  $\beta$  plus  $\gamma$  is equals to  $3$  okay

so given uh determinant of  $m$  minus  $2$  all right

so that means the that means  $m$  is invertible

so what is  $m$  inverse  $m$  inverse is the adjoint  $m$  divided by determinant of  $m$  right

so what is this we know that joint matrix  $i$ 'll just divide that matrix by minus  $2$

so we get  $\frac{1}{-2} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -1 & -\frac{3}{2} \\ -1 & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

so this is 5 by 2 this is minus 3 by 2 and this is 1 by 2  
 so this is the m inverse right okay  
 so ah  
 so now now let's use this relation right  
 so m of alpha beta gamma is equals to 1 2 3  
 so this implies that alpha beta and gamma is equal to m inverse 1 2 3 and if  
 you multiply this column vector with the m inverse we get  
 so let me just just write it down the same inverse again  
 so this is nothing but 1 by 2 minus 1 by 2 1 by 2 minus 4 let me just check  
 again minus 4 3 minus 1 5 by 2 minus 3 by 2 1 by 2 multiply with the 1 2 3 ok  
 so what will be this this is nothing but 1 by 2 minus 2 3 2 5 1 6.  
 uh three minus two three one second four minus two two by two through one and  
 this is minus 1 which minus 4 minus 5 plus 3 divided by 2 and this is 1 okay  
 so this is what we get alpha beta gamma is this  
 so this implies this implies alpha is 1 beta is minus 1 gamma is 1 and this  
 implies alpha minus beta plus gamma is equals to 1 plus 1 2 plus 1 3 this is 3  
 so this is this is what we wanted to prove okay  
 so hence the third party done  
 so done okay okay  
 so let's solve another problem let x belongs to real number and let p is equals  
 to 1 1 1 0 2 2 0 0 3 and q is equal to 2 x x 0 4 0 x x 6 and r is equals to p q  
 p inverse and then  
 so that first part is determinant of r is equals to determinant of matrix 2 x x  
 0 4 0 x x 5 plus 8 second part is for x is equals to 0 if r times 1 a b is  
 equals to 6 times 1 e b then a plus b equals to 5 okay  
 so let's solve this problem okay  
 so what we start  
 so let's start with this r is equals to p q p inverse okay then determinant of  
 r is nothing but determinant of p into determinant of q into determinant of p  
 inverse as we know that determinant of p inverse is one upon determinant p  
 so this is nothing but determinant r is nothing but determinant of q okay  
 so let's take q matrix  
 so let's write down um okay let me just delete this all right  
 so sorry i'm making too much mistake determinant of r is equals to the  
 determinant of q  
 so what is the determinant this matrix what is the q matrix let me write it 2 x  
 x 0 4 0 x x and x 6  
 so i'll just break six energy five plus one okay then i can break the  
 determinant in two determinants which is nothing but determinant of 2 and  
 so here i can write it as a 0 plus 0 all right then i can write it 2x x 0 4 0 x  
 x 5 plus determinant of 2 0 x x 4 x 0 0 1  
 so determinant of this second matrix is what this is just it if you just open  
 through the third column right then this value we said  
 so this is what we wanted to prove  
 so that means determinant of r is equals to determinant of 2 x x 0 4 0 x x 5  
 plus 8  
 so this is what this is what we wanted to do all right okay  
 so let's go to the part okay  
 so part two is that it says that i mean we need to take x is equals to zero  
 so if if x is zero then what will be the what will be the q matrix the q matrix  
 will be 2 0 0 0 4 0 0 0 6 all right  
 so this is the this is the q matrix we have all right  
 so r is now now we need to calculate the r explicitly remember r was r was p q  
 p inverse

so this means we know the  $p$  and we need to find the  $p$  inverse okay  
 so  $p$  is let me just recall here what is  $p$   $p$  is  $\begin{pmatrix} 1 & 1 & 1 & 0 & 2 & 2 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 3 \end{pmatrix}$  into  $q$   $q$  is  
 so we can easily check that the determinant of  $p$  is not equals to zero hence it  
 is inevitable and i will just leave this as exercise the  $p$  inverse you can  
 calculate by finding its a joint and you just divide that joint by  $p$  all right  
 so determinant of uh by determinant  $p$  and determinant  $p$  is six right you can  
 calculate determinant  $p$  is equals to six  
 so i'll just leave this uh as exercise  
 so what is  $p$  inverse is 1 i'll just directly write it minus 1 by 2 0 0 half  
 minus 1 by 3 0 0 and 2 all right  
 so yeah no sorry 0 let me just yeah zero zero one by three all right yeah one  
 minus one by two zero zero one by two minus one by three zero zero one by three  
 yeah  
 so this is  $p$  inverse this you can take it as exercise it's not very difficult  
 right okay okay  
 so now now let's calculate the  $r$  matrix  
 so what is  $r$  then  $r$  is  $r$  is nothing but the  $p$  is  $\begin{pmatrix} 1 & 1 & 1 & 0 & 2 & 2 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 3 \end{pmatrix}$  into  $q$   $q$  is  
 the diagonal matrix  $\begin{pmatrix} 2 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 2 & 3 & 0 & 0 & 1 & 3 \end{pmatrix}$  and  $p$  inverse  $p$  inverse is 1 minus 1 by 2  
 0 0 1 by 2 minus 1 by 3 0 0 1 3 one second one by three okay  
 so let's let's multiply these matrices  
 so this is nothing but one one one 2 2 0 0 3 and what is this this is nothing  
 but 2 and then it's a minus 1 and 0 and then 0 this is will be 0 and then 0 1 by  
 2  
 so it's 2 and then zero minus four by three plus two two minus uh yeah it's a  
 minus four by three okay and then the last one is 0 0 and 2 all right  
 so this is what we get okay now let's multiply this again then we get what do  
 we get we get 2 1 2 by 3 0 4 4 by 3 and this one is 0 the 0 and 6 all right  
 so this is the this is the  $r$  matrix all right okay  
 so what is given is given by given is this follows  $r$  1 a b is equals to 6 times  
 1 a b okay  
 so if you multiply  $r$  with 1 a b you get 2 plus a plus 2 by 3 b is equals to 6  
 okay 4 a plus 4 by 3 b okay is equals to 6 a and this is 6 b is equals to 6 b  
 all right  
 so if you simplify it then you get a plus a plus 2 by 3 b is equals to 4 and 2  
 a minus 4 by 3 b is equals to 0  
 so if you solve this solve this equation this will give you 4 a is equals to 8  
 so a is 2 and b is b is 4 by 3 b is equals to 4  
 so b is 3 okay  
 so i'll just complete it here itself this implies a plus b is equals to 2 plus  
 3 equals to 5  
 so this is what this is what we wanted to show okay okay students  
 so i will stop here now thank you for attending this session in the next  
 session i'll solve some more problems related with matrices and determinant and  
 i will also start a new topic which is on system of linear equations thank you  
 you