welcome students welcome back to the series of lectures on matrices in this lecture we will try to do some problems on matrices first problem let p be an invertible matrix with i plus p plus p square up to p power n equal to zero that is zero that is the zero matrix So what is given is that when you add i p p square up to p n what you get is the zero matrix find p inverse what you are given the data that is given to you is that p is an invertible matrix and it also satisfies the following that is when you add i p p square up to p power n what will end up is a 0 matrix let us try to find the solution to this in fact i should say find p inverse in terms of
So what is given is that i plus p plus p square up to p power n this is zero now given that p is invertible that means p inverse exists
So let us multiply
So multiply this equation by p inverse then what we will have is that p inverse plus p sorry p inverse plus i plus p plus up to p power n minus one is going to give me zero
So this will imply p inverse is minus i minus p e minus up to minus p power n minus one right
So this is the inverse of the matrix
So which is equal to minus of summation zero to n minus one p power i lets do the second one next problem if a equal to one two one five two six minus two minus one minus three find a cube
So we are given that a is one two one five two six minus two minus one minus three right now let us try to find a square a square is one two one five two six minus two minus one minus three times one two one five two six minus two minus one minus three which is equal to one into one plus two into five which is ten plus sorry minus two two plus four minus one one plus twelve minus three five plus ten minus twelve ten plus four minus six five plus twelve minus eighteen minus two minus five plus six minus four minus 2 minus 4 minus 2 plus 3 minus 2 minus 6 plus 9 minus 2 minus 6 plus 9 which will give me 11 minus 2 which is 9 6 minus 1 which is 5 13 minus three which is ten fifteen minus twelve which is three sixteen minus six which is ten seventeen minus eight which is eighteen minus minus one minus seven plus six which is minus one minus six plus three which is minus three minus eight plus nine which is one
So let us try to find out a cube which is a square into a which is going to give me a square is nine five ten three ten minus one minus one minus three one into the given matrix a which is one two one five two six and minus two minus one minus three which is equal to nine plus twenty five minus twenty eighteen plus ten minus twenty nine plus 30 minus 30 3 plus 50 3 plus 50 plus 2. six plus twenty plus one three plus sixty plus three one sorry i will have minus one minus fifteen minus two minus two minus six minus one minus one plus eighteen minus three thirty six minus twenty which is sixteen thirty eight sorry twenty eight minus twenty which is eight thirty gets cancelled you have nine fifty five twenty seven sixty six minus eighteen minus nine fourteen
So this is a cube next problem the set of natural numbers that is n is partitioned partitioned into arrays of rows and columns in the form of matrices as m one equal to just the first one single m two s two three four five m three six seven eight nine ten eleven twelve thirteen fourteen and
So on right the first matrix consists of just one element one the second one consists of four elements the two by two matrix two three four five wherever we are left with the next one is the first element of the next matrix
So m three is a three by three matrix right six seven eight nine ten eleven twelve thirteen fourteen because five is the last element for m two
So the next one is six it starts with in general m n is a n by n matrix whose first element is the next element corresponding to the last element of the a n minus 1 n minus 1 element of the m n minus 1 matrix fine now the question is this find the trace of mn solution now let us write down the first elements of each one right
So these are all the first first elements of each matrix the first element in the first matrix is 1 2 6 15 it goes and then well one can notice that ah it goes well ah will come back to that one can notice that the difference is going to be just a square that will notice
So let me denote by t n the first element general first element
So 0
So let well let s n equal to 1 plus 2 plus 6 plus 15 up to t n therefore zero equal to s n minus s n
So let me write s n as follows one plus two plus six plus fifteen up to t n well you have

the same thing one plus two plus six plus fifteen up to t n which will give me now let me write this one separate 2 minus 1 plus 6 minus 2 plus 15 minus x up to t n minus 1 minus t n minus 2 plus t n minus t n minus 1 plus t n
So i will have a minus t n over here right therefore
So which is equal to let me write down which is equal to one plus one square plus 2 square plus 3 square up to t n minus t n minus 1 minus t n therefore
So this is my zero your right hand side left hand side is zero and you have this as the right hand side therefore t n let me bring this t into the other end t n is one plus one square plus two square plus three square up to t n minus t n minus one which is equal to one plus well one can notice that the difference is always a square one to n minus one i square i equal to one to n minus one i square which is given by one plus we know the formula right which we must have done in when we do this principle of mathematical induction one can verify that this is going to be n into n minus one into two n minus one whole upon six
So this is the first element
So t n is the this is the first element for the m nth matrix right now therefore
So let a one one denote the first element of m n then a one one is given by one plus n into n minus one into two n minus one whole upon six now the next element is just plus one plus one plus one
So what we wanted as all a i i right for one less than or equal to i less than or equal to n this is what we wanted but one can notice that the next element the difference between the next element and the or for example the difference between a two two and a one one is just n plus one that is the difference right the difference is just n plus one
So i eighth element is just n one plus n into n minus one into two n minus one upon six plus i times sorry i minus one times n plus one right i will just write it as one less than or equal to i less than equal to l right if it is one you will get the same element right other things you will get it as an right because the difference between any two elements is just n plus one any two consecutive terms that is a one one and a two two the difference is just n plus one or more generally the difference between a i i and a i plus one i plus one is exactly n plus one and therefore this is the general a i i once you know a one one
So what we wanted is the trace of m n therefore trace of m n equal to summation one to n a i i which is given by summation one to n one plus n into n minus one into two n minus one whole upon six plus i minus one this is i running from one to ah i minus one into n plus one right
So let me just well ah split this term sum into two terms one to n one plus n into n minus one into two n minus one whole upon six plus summation i equal to one to n i minus one into n plus one this is just a constant term which means summation i equal to one to n just one which means i will end up with n times one plus n into n minus one into two n minus one whole upon six this is just a constant n plus one i will just pull it out plus summation i equal to one to n i minus one
So i minus one which is equal to n into one into n into n minus one into two n minus one whole upon 6 plus n plus 1 into well this is same as i equal to 1 to n i sorry n minus 1 i because i can just pull out i equal to i replace i by i minus one
So that i will have i s
So the one gets replace i by i plus one
So that it turns out to be from zero to n but then the 0
So here you will have 0 to n i but then the term correspond to 0 is just 0
So i will have y 1 to n minus 1 i which is equal to n into 1 plus n into n minus 1 into 2 n minus 1 whole upon 6 plus n plus 1 into
So we know that summation one to n i is n into n plus one it is only f to n minus one
So n minus one into n upon two n into n plus one upon two therefore replace n by n minus one you have n minus one into n upon two which is going to be let me just pull out n by six throughout i will have six plus i have two n square
So i will have two n cube minus three n
So minus three n square plus n again i have a plus n minus one into n plus one which is n square minus one
So three n square minus three n this is what i have n by six into the first term is just two n cube three n square three n square gets cancelled minus two n plus six minus oh i have pulled out an n outside
So i have an n outside

So i haven't pulled out an n outside
So what will i have two n cube minus three n square sorry ah three n square remains
So this three n square gets cancelled and then terms regarding n i will have only just plus 2 a plus n 6 minus 3 is just 3 plus 3
So this is the final solution therefore trace of m n is n by six into two n cube plus n plus three next problem if a equal to minus 1 plus i root 3 upon 2 i minus 1 minus phi root 3 upon two i one plus i root three upon two i and one minus i root three upon two y and f x equal to x square plus one then find f of a
So what is this f of a wherever there is x just replace x by e thats what this f of a means solution
So what is f x
So given that f x is x square plus one and therefore f of a is a square plus identity right one is replaced by the identity matrix
So now let us try to calculate a square minus one plus i root three by two i minus one minus i root three by two i one plus i root three upon two i one minus i root three upon two i into minus one plus i root three upon two i minus one minus i root three upon two i one plus i root three upon two i one minus i root three upon two y if you look at one by two i is a common thing
So let us pull it out in both the things
So what you will finally end up here is minus one by four because i square is minus one times minus 1 plus i root 3 minus 1 minus i root 3 1 plus i root 3 1 minus i root 3 into same thing minus 1 plus i root 3 minus 1 minus i root 3 1 plus i root 3 and 1 minus i root this is what we have you have minus one by four into let us try to multiply
So you have minus one plus i root three whole square minus one plus i root three whole square similarly this into this you have a sq a plus b into a minus b is a square minus b square but you have an i
So a is
So one plus three this into this and similarly here a plus b into a minus b which is a square plus b square plus one plus three again this into this again you have a plus b into a minus b
So the negative
So you will have a square minus b square right
So a square is
So here a square it is going to give me ah it will give me minus over here a square minus b square right a square is one minus of b square which is i root three which will give me fine and here a square will give me one well b square now a square is i root 3
So minus 3 and b is a square minus b square fine i have this and here a square which is minus three minus b square
So minus one plus this into this plus 1 plus 3 and then finally this into this what you will have is minus of 1 plus i root 3 whole square into 1 minus i root 3 whole square right let us try to calculate
So minus one by four into if you try to expand this what you will have is one minus two i root three minus three minus one minus two i root three plus three
So this is going to give me zero again here i will have a zero
So here i will have a plus right this enter this plus i have a plus minus one minus two i root three minus one minus two by root three minus three plus one minus two by root three
So i have minus three just going to give me minus one by four into this one this gets cancelled this and this gets cancelled what i will have is minus four i root three zero zero this one and this one gets cancelled this is going to be plus similarly these two gets cancelled and i will have minus 4 i three which will just give me i root 3 0 0 i root 3 now let us try to calculate f of a therefore f of a equal to a squared plus i what is a square a square is just pi root 3 0 0 i root 3 plus 1 0 0 1
So give me 1 plus i root 3 0 0 1 plus i root this is what i have lets do the next one next problem if a equal to two minus two minus four minus one three four one minus two x is an idem pattern matrix is an idempotent matrix then find the value of x what is an item partner matrix a matrix a says that a square equal to a is called an idempotent matrix or whenever if you have a square matrix a square matrix a which satisfies a square equal to a is called an idempotent matrix now first let us try to find a square which is a into a two minus 2 minus 4 minus 1 3 4 1 minus 2 x times 2 minus 2 minus 4 minus 1 3 4 1 minus 2 x which is equal to four plus one sorry four plus two minus four minus four minus x minus eight minus eight minus eight minus four x minus two minus three plus four

2 plus 9 minus 8 4 plus 12 plus 4 x plus two plus x minus two minus x minus six two x minus 4 minus 8 plus x square which is equal to 2 minus eighteen minus four minus six plus eight right there should be plus eight
So you have minus two minus sixteen minus four x minus five plus four is minus one eleven minus eight just three sixteen plus four x four plus x minus eight minus two x minus 12 plus x square what is given is that a square equal to a right what is given is that a square equal to a
So that means what do we have that is two minus one four plus x minus two three minus eight minus two x minus sixteen minus four x sixteen plus four x minus twelve plus x square this should be equal to two minus 2 minus 4 minus 1 3 4 1 minus 2 x just by comparing what we obtain is that sixteen plus four x should be four or equivalently four plus x when you divide this by four implies four plus x should be one right
So that will give me x equal to minus three that is one minus four which is minus three
So let us do the next problem suppose a matrix a satisfies a square minus five a plus seven i equal to zero if a power five is five a plus b i then find the value of a and solution now we are given that a square minus five a plus seven i is zero and also what is given is that it also satisfies sorry a power five is a times capital a plus b times the identity matrix
So you will have to find the values of a and b now let us try to express
So well before we go to the right
So given a square minus five a plus seven i is zero that will imply that a square is five a minus seven now let us try to write down a cube which is a square into a and we know that a square is five a minus seven i times a which is equal to five a square minus seven a which is equal to five into five a minus seven i minus 7 times a which is equal to you have 25 a here minus 7 minus 7 a which will give me 18 a minus 7 i
So we have expressed a cube in terms of a now let us try to calculate a power five which is a cube into a square a cube which is eighteen a minus seven i into a square which is five a minus seven i eighteen into five which is ninety a eighteen a minus seven i 126 i sorry a sorry this should be a square a and then you have minus 35 a plus 49 i which is equal to a square i know that it is 5 a minus seven i minus one twenty six a minus thirty five a thirty one sixty
So this is one sixty one a plus forty nine i which is equal to four fifty a minus six thirty i minus one sixty one j plus forty nine i which is equal to ten minus one it is nine for fourteen fourteen minus six which is eight
So you have three three minus one minus two a minus ten minus nine just one have thirteen thirteen minus four which is nine five five ninety one ten one sorry this is two
So this should be eight times right but on the other hand what is given to us is that
So given also we are given that a power five is small a times capital a plus b times identity right
So let me call this as one and let me call this as two comparing one and two we get small a equal to two eighty nine and small b equal to minus five eighty one let us do the next one if a equal to alpha 0 1 1 and a square equal to identity then find alpha solution
So we are given that a square equal to identity
So to use this lets try to calculate a square a square which is equal to alpha 0 1 1 times alpha 0 1 1 which is equal to alpha square zero alpha plus one one
So we are given that a square is identity that is alpha square zero alpha plus one one this is equal to one zero zero one
So now if you compare these two matrices what will have is alpha square equal to zero and alpha plus one is sorry alpha square is one and alpha plus one should be zero
So this will imply that alpha equal to minus one
So alpha equal to minus one is the solution next problem if a equal to alpha zero two three and a square is nine i then find alpha solution again you are given that a square is nine and nine high which means you will have to find a square a square which is equal to alpha 0 2 3 times alpha 0 2 and 3 which is equal to alpha square second term zero two alpha plus six sorry two alpha plus six six and then the last term is nine now we are given that a square is nine times i that is the 2 matrices matrix alpha square 0 2 alpha plus 6 9 equal to 9 0 zero nine
So what we have is alpha square is nine just by comparing the coefficients and similarly this one you will have it as two alpha plus six equal to zero
So these two together will imply that alpha equal to minus three
So alpha equal to minus three is the solution let us do the next problem if x x x times y

y y y y y if this is equal to 1 by 3 times 1 1 1 1 1 1 1 1 then show that x equal to y by nine sorry one by nine y one by nine solution what we are given is that x x x x x x x x when you multiply it with y y y y y y y y y the resulting matrix is just one by three one by three one by three one by three one by three one by three one by three one by three this is what is given now let us try to find out what the left hand side is if you look at the left hand side it is just the multiplication of matrices with entries just equal to x and y respectively right these two are the matrices which is equal to three x y throughout right x y x y x y
So you will have three x y throughout fine if you compare with the given equation with the first one what we obtain is that three x y equal to one by three that means x y equal to one by nine or equivalently x equal to one by nine let us do one more problem if the equations x minus two y plus three z equal to zero minus two x plus three y plus two z equal to zero minus eight x plus lambda y equal to zero half non trivial solution then find lambda solution
So the let us write the augmented matrix or the coefficient matrix because your constant of matrix is just zero
So one minus two three minus two three two minus eight lambda zero this is the constant of matrix now let us try to convert this into its row reduced echelon form let us try to convert this minus two and minus eight into zero r two is replaced by r two plus two times r one r three is replaced by r three plus eight times r one one minus two three because the first row is unaltered second one r two minus two plus two times one which is zero three plus two times minus two which is three minus four which is minus one two plus two times three
So two plus 6 which is 8 minus 8 plus 8 times 1 this is going to give me 0 lambda minus 16 right lambda plus eight times minus two which is lambda minus sixteen zero plus eight times three which is twenty four now let us try to multiply this row into one
So that you will make this into this minus one into one r two is replaced by minus of r two one minus two three zero one minus eight zero lambda minus sixteen twenty four lets try to convert these two elements into zero r one is replaced by r one plus two times r two r three is replaced by r three plus sixteen minus lambda into r two
So the first two columns are just going to look like one zero zero zero one zero fine r one three plus two times minus eight
So three minus sixteen which will give me minus thirteen the next one is remains unaltered next to one twenty four plus sixteen minus lambda into minus eight right this is what i am going to have
So let us write it down 1 0 minus 13 0 1 minus 8 0 0 one twenty one twenty eight
So minus one twenty eight plus twenty four
So you will have eight lambda minus one zero four right eight lambda minus one zero four now i want what is x given is that the system has got a non trivial solution if the system has got a non trivial solution which means that the rank of the coefficient matrix should be less than three
So if you want the rank to be less than three this eight lambda minus one zero four should be zero
So for the existence of a non trivial solution eight lambda minus one zero four should be equal to zero therefore eight lambda minus one zero four zero that will imply lambda is one zero four upon eight which is equal to twelve sorry thirteen which is equal to thirteen thus lambda equal to thirteen is the only value for which the given system will have a non trivial solution
So if lambda is not equal to thirteen if you just look at the previous one this eight lambda minus one zero four is going to be non zero
So that you can divide this by a now because it is non-zero you can divide by that and you will get as 1 and you can make the other things into 0
So that what you will end up with is a trivial solution because what you will have is an identity matrix in that case the only solution that you get in this case is just going to be the 0 0 0 solution which is a trivial solution
So for the existence of