

welcome students welcome to this series of lectures on matrices and determinants matrices is one of the concepts in mathematics which is highly useful in many places well let us look at an example suppose there are two companies a and b and each company produces say three items suppose three items let us give it a name one two and three suppose the company a produces 70 80 and 90 kgs of items one two and three respectively similarly the company

so in fact let us suppose that the company b produces 90 50 and 100 kgs of items one two and three respectively right we have the following data what we have is that we have two companies a and b and not only that you have and each company both of them produces item one two and three

so item one is produced seventy kilo by company a and 90 kilos by company b 80 kilo of item 2 is produced by company a and 50 kilo of item 2 is produced by company b similarly ninety of item three is produced by ninety kg of item three is produced by company a and hundred kilo of item three is produced by company b let us write it in some form let me ha let us put it in the form of a table i have a company a and company b and on the other hand i have item one item two and item three

so a produces seventy eighty and 90 and similarly b produces 90 50 and 100.

so this is one of the typical examples for what is known as the concept of a matrix

so now let us write down what a matrix is let us put it formally in the form of a definition a matrix is a rectangular array rectangular array

so the elements of the array array are called as the entries of the corresponding are the underlying matrix entries of the underlying matrix

so how do we in general denote a rectangular array this is the way you denote right

so a one one a 1 2 a 1 3 up to a 1 n a 2 1 a 2 2 a 2 3 up to a 2 n proceeding this way you will have a m 1 a m 2 a m 3 up to am n

so there are totally m and elements are entries in this matrix

so given a matrix and when you write it in the form of a rectangular array

so the total number of entries that you have

so that is the totality of the whole array

so so well let us notice that there are m rows and n columns in a matrix well this m and n depends on the chosen problem and finally the order of a matrix with m rows and n columns is m cross n right this m cross n denotes the order of the matrix now let us do some examples let us begin with the first example that we started with in this test first example we had an array

so the matrix that we had is a which is seventy eighty ninety ninety fifty and hundred this is the matrix that we had initially and if you look at this matrix

so a has got two rows and three columns therefore the order of the matrix a is the order of the matrix a is two by three right and you can also notice that it has got 2 into 3 that is 6 elements let us do another example a which is given by 1 2 3 4 five six seven eight and nine this is the matrix

so a has got three rows and three columns therefore order of a is three byte now let us do a simple problem to calculate the entries of a matrix suppose that a is a real matrix

so what does it mean by a real matrix by a real matrix we mean a matrix whose entries are just real numbers real matrix whose order is three by two find the entries if if the entries are given by the formula a i j equal to modulus of i minus j whole upon 2 let us try to solve this

so given a matrix a an n by n matrix a the ijth entry will be denoted as a i j given a is a three by two matrix therefore a has three rows and two columns

so the ijth entry is given by modulus of i minus j whole upon two

so therefore the matrix  $a$  is given as  $a_{11} a_{12} a_{13} a_{21} a_{22}$  and  $a_{31}$  let us apply the formula and get the entries  $a_{11}$  which is modulus of  $1 - 1$  upon  $2 - 1$  minus  $1$  is  $0$

so the first entry is  $0$ .

second one  $a_{12}$  it is given by  $1 - 2$  modulus of that is  $1$  whole upon  $2$  so you have half a one three

so one minus three it is minus two and modulus of that is two two upon two it is one second one two minus one it is going to give you two minus one is one

so one upon two it is again going to be half two minus two zero again two minus three it is minus one and

so you will have modulus of that is one

so one by two thus we have found the entries

so now let us look at various different kinds of matrices the first one is what is known as a row matrix row matrix what is a row matrix a matrix with just one row is called a row matrix a row matrix will have the order one by  $n$  right where  $n$  is the number of entries in that row

so let us look at an example first one just look at this one one two three

so this is an example for a row matrix right and its order is one by three second one let us look at one more one by two root two three

so this is again a row matrix and its order is again one by three

so in this case  $n$  is just three a column matrix

so what is a column matrix it is just the similar to the row matrix just the matrix with just one column a matrix with just one column is called a column matrix

so the order of a column matrix will be  $n$  by one right where  $n$  denotes the number of elements in that column

so let us do some examples first one you have root two root three and root five

so this is an example a typical example for a column matrix and its order is three by one let us look at one more example zero zero this is again a column matrix and its order is two by one third one is what is known as a square matrix a square matrix is a matrix in which the number of rows is equal to the number of columns whenever you find a matrix in which the number of rows is equal to the number of columns then you say that such a matrix is a square matrix let us do some examples let us look at the first example that we had seventy eighty ninety ninety fifty hundred

so what does it mean by a square

so square matrix number of rows is equal to the number of columns which means the if you say that it has got  $n$  rows which means it should also have  $n$  column that means the order should be  $n$  by  $n$  right and we know that its order the order of this matrix is two by three therefore this is not a square matrix let us do one more example well let us write down this way half one by four one by eight one by three one by nine one by twenty seven one by four one by sixteen and one by sixty four

so number of rows is equal to three and similarly number of columns is also same as three therefore this is a square matrix third one let us look at this one one two three four here again in this case number of rows is equal to number of columns equal to two therefore this is a square matrix

so let us make a small remark first one a row matrix of order one by  $n$  is a square matrix if and only if  $n$  equal to one right if you have a row matrix  $i$  have order one by  $n$  right

so when can it become a square matrix this is possible only if  $n$  is one

so if  $n$  is one then it is a square matrix you know that it is a row matrix of order one by  $n$  and if you want that to be a square matrix which means the number of rows should be same as number of columns and you know that it has got just

one row therefore it should have only one column and you know that it has got  $n$  columns therefore the only possibility is that  $n$  should be 1.

similarly second one a column matrix of order  $n$  by one is a square matrix if and only if  $n$  equal to one right let us look at one more variety one more type of a matrix diagonal matrix a square matrix is called a diagonal matrix if all the entries  $x$  of the diagonal entries except the diagonal entries are zero

so if you have a square matrix  $a_{11} a_{12} a_{13} \dots a_{1n}$   
 $a_{21} a_{22} a_{23} \dots a_{2n}$   
and it goes up to  $a_{n1} a_{n2} a_{n3} \dots a_{nn}$  right the entries these entries

so the  $a_{ii}$  entries are called the diagonal entries these are called as the diagonal entries right are the entries in the  $i$ th position entries in the  $i$ th position are called the diagonal entries let us look at an example

so the diagonal entries are zero and zero of course the other entries are also zero

so this is an example for a diagonal matrix let us look at one more example two zero zero three this is again an example for a diagonal matrix let us look at one more example if you look at this one the diagonal entries are zero while the other entries right the entries in one tooth position and the entry in entry in the  $a_{11}$  position not zero and therefore

so this is not an example this is not a square matrix sorry this is not a diagonal matrix let us do one more example

so all the entries or zero you have the diagonal entries these three things but if you look at this entry this is a non-zero entry which is not a diagonal entry right

so one is in is a nonzero entry in the two one position therefore this is not a diagonal matrix let us look at one more variety of matrix of what is known as a scalar matrix a diagonal matrix is called a scalar matrix if all the diagonal entries are given by by multiplication by a unique scalar or obtained by multiplying a unique scalar to one right if all the diagonal entries are obtained by multiplying a unique scalar to one

so let us look at an example two zero zero two

so this is an example for a scalar matrix second one two zero zero two zero zero

so this is not even a square matrix and hence not as not even a square matrix and hence cannot be a scalar matrix well let us look at one more thing there is something called what is known as an identity matrix

so what is an identity matrix an identity matrix of order  $n$  is given it is usually denoted as  $I$  right

so you have one zero at other places you have one in the two tooth position and then zero at other places

so that means what  $a_{ij}$  the  $ij$ th entry is going to be  $i$  can write it as  $0$  if  $i$  not equal to  $j$  and is going to be  $1$  if  $i$  equal to  $j$  this matrix is what is known as the identity matrix let us write to try to write down the 2 by 2 identity matrix let me write it as  $I_{2 \times 2}$  it is going to be given as one zero zero one

let us write down the three by three identity matrix it is given as one zero zero zero one  $0$  and  $0$   $0$   $1$  this is the matrix that we have next one is what is known as an upper triangular matrix variety or a type a square matrix in which all the entries that are below the diagonal or zero which all the entries that are below the diagonal are zero is called an upper triangular matrix us look at an example first one first example one two three zero four five zero zero six

so let's first draw the line right

so you have this

so one four and six these corresponds to the diagonal entries and the entries below that right you have all the entries below that or zero therefore this is

an upper triangular matrix let us look at one more example again

so this is the these two correspond to the diagonal entries and that below this over this is the identity matrix

so this is a upper triangular matrix let us look at one more example in fact this example is a generalization of the previous one the identity matrix every scalar matrix is an upper triangular matrix right

so the next one strictly upper triangular matrix a triangular matrix or i will write it as an upper triangular matrix is called strictly upper triangular if even the diagonal entries are zero

so let us look at some simple examples zero one zero zero

so these are the diagonal entries the diagonal both the diagonal entries are zero and the one even below that is zero

so this is an example for a strictly upper triangular matrix this is an example for a strictly upper triangular matrix

so let us look at the second example  $\begin{pmatrix} 0 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 0 \end{pmatrix}$

so let us first mark the diagonal entries second example

so you have the diagonal entries here fine

so all the entries below the diagonal they are zero

so therefore this is a this is an upper triangular matrix first thing and now let us verify whether this is a strictly whether this is strictly a upper triangular or not

so the second diagonal entry which is four its a non zero number right four is in the two tooth position and hence this matrix is not strictly upper triangular

so having said about the types of matrices now let us try to do something on operations some kind of operations on matrices first one is what is known as addition of matrices two matrices can be added if they are of same order right two matrices of same order only then you can add them if you have it as of same order then you can add it and the  $ij$ th entry of the resulting matrix matrix is obtained by adding the  $ij$ th entry of the given two matrices

so lets do an example in fact lets start with a simple example let us choose a as one two three four let a equal to this and b as five six seven and eight let us try to calculate a plus b we want to add these two matrices

so let us calculate a plus b which is given by

so the first entry or the one month entry is given by adding the one month rays of the corresponding given matrices a and b

so the one month entry of a is one and the one month entry of b is five therefore one plus five it is six similarly the one tooth entry of a is two and the one tooth entry of b is six therefore two plus six it is eight the two one entry of a is three and two one entry of b is seven therefore three plus seven it is going to give me ten two tooth entry of a is four and two tooth entry of b is eight eight plus four it is going to give me twelve

so let us do second example let us do it for a three by three matrix let a equal to this  $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$  b equal to  $\begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$  let us try to calculate a plus b notice that both a and b are square matrices in fact both are of order three by three and therefore one can calculate a plus b

so let us calculate entry way

so one plus nine ten two plus eight ten three plus seven ten in fact you can notice that all the entries are going to be just ten now let us do some properties of addition before we proceed further let us discuss a remark saying that the above said details about real matrices holds without any change to complex matrices that means what just like what you do for real matrices same thing can be done even for complex matrices you can add two complex matrices

so simsing whatever we did for real also can be done for complex matrices

so what is a complex matrix a complex matrix is a matrix in which all the entries are complex numbers example ah look at this matrix a which is given by  $i$   $2 + i$   $3 + i$   $1 + 2i$   $2 + 3i$   $3 + 4i$   $\sqrt{2} + \sqrt{3}i$   $\sqrt{3} + \sqrt{5}i$   $\sqrt{5} + \sqrt{7}i$  what we have are complex entries in this matrix

so this is one of the examples for a complex matrix and just like how you add real matrices one can also add complex matrices

so just like you will have to add entry wise same thing now before we proceed further with the properties of matrices let us write down a note saying that two matrices  $a$  and  $b$  of same order are said to be equal if each entry of  $a$  equals  $z$  corresponding entry in  $b$

so for example if i write  $a$  as the matrix with entries  $a_{ij}$  and the matrix  $b$  as  $b_{ij}$  then  $a$  equal to  $b$  if  $a_{ij}$  equal to  $b_{ij}$  for all  $i$  and  $j$  right

so this  $i$  and  $j$  varies from one to  $n$  and one to  $m$  if you assume that  $a_{ij}$  are of order  $n$  by  $m$  right with this note let us proceed to prove some properties of matrices for any two matrices in fact square matrices  $a$  and  $b$  of same order  $a + b$  equal to  $b + a$

so to

so how to show that  $a + b$  equal to  $b + a$  will have to use the above note that is just look at the  $ij$ th entry of  $a + b$  and similarly the  $ij$ th entry of  $b + a$  one should show that these two match now let us go with the proof of this let  $a$  equal to  $a_{ij}$  and  $b$  equal to  $b_{ij}$  where  $1 \leq i \leq n$  and  $1 \leq j \leq n$  that is we are assuming that  $a$  and  $b$  are of order  $n$  by  $n$  now what we wanted is  $a + b$  that means we are trying to add the matrix with entries  $a_{ij}$  and  $b_{ij}$  respectively if you look at this one this is going to be same as by the definition of matrix addition this coincides with  $a_{ij} + b_{ij}$  but what we know is that complex addition and real addition whatever it be they are commutative right for scalar for complex scalars or real scalars we know that addition is commutative let us use this fact and therefore  $a_{ij} + b_{ij}$  this is same as  $b_{ij} + a_{ij}$  which is same as again by the definition of addition of matrices this is same as  $b_{ij} + a_{ij}$  that is the matrix with entries  $b_{ij}$  plus the matrix with entries  $a_{ij}$  but matrix with entries  $b_{ij}$  is the matrix capital  $b$  and the matrix with the entries  $a_{ij}$  are just  $a$  thus  $a + b$  equal to  $b + a$  in fact the above set property is what is known as commutative property

so this property is called as the commutative property let us prove next property for any three matrices  $a$   $b$  and  $c$  of same order  $a + b + c$  is equal to  $a + b + c$  let us go with the proof the proof of this is more or less same as the one that we gave for the commutative property

so we will assume that  $a$  equal to matrix with entries  $a_{ij}$   $b$  equal to matrix with entries  $b_{ij}$  and  $c$  is the matrix with entry  $c_{ij}$  right where  $1 \leq i \leq n$  and  $1 \leq j \leq n$  because all three matrices are of same order this is possible that means we are assuming that  $a$   $b$  and  $c$  are any three matrices of order  $n$  by  $n$  now what we wanted is  $a + b + c$  which is equal to we are adding matrix  $a$  that is with entry small  $a_{ij}$  plus a matrix  $b_{ij}$  plus a matrix with entries  $c_{ij}$  equal to let us do what is within the parenthesis  $a_{ij} + b_{ij} + c_{ij}$  plus if you look at within the parenthesis by the definition of addition of matrices this is same as  $b_{ij} + c_{ij}$  which is equal to what we are doing is we have two matrices one with entries  $a_{ij}$  and the other one with entries  $b_{ij} + c_{ij}$  and again now let us try to use the definition of the addition of matrices

so what we will end up with is a matrix with entries  $a_{ij} + b_{ij} + c_{ij}$  whether they are real matrix or complex matrix we know that addition is associative and therefore this is same as  $a_{ij} + b_{ij} + c_{ij}$  plus  $a_{ij} + b_{ij}$  plus

$c_{ij}$  now let us again use the definition of the addition of matrices and then split this this is  $a_{ij}$  plus  $b_{ij}$  plus a matrix with entries as  $c_{ij}$  but again if you use the definition of addition of matrices then this is same as matrix with entries  $a_{ij}$  plus matrix with entries  $b_{ij}$  plus the remaining which is matrix with entry  $c_{ij}$  if you expand or if you write on what these are the first one is the matrix with entries  $a_{ij}$  is the matrix  $a$  second one is the matrix  $b$  plus matrix thus  $a + b + c$  is equal to the matrix  $a + b + c$  so this property is called as a associative property so with this let me stop for today's lecture thank you

Prutor@MITK