

so welcome to the second session on problem solving for trigonometric and inverse trigonometric functions

so just like the previous session will solve some challenging problems involving the identities that we have learnt and discussed ah for both inverse trigonometric and trigonometric functions

so this is going to be the last lecture on ah trigonometric and inverse trigonometric functions

so this is the first problem

so is

so what we have here is that we have angle theta which must lie between minus pi by 6 and minus pi by 12 and it says that suppose alpha 1 and beta 1 are the roots of this quadratic equation and alpha two and beta two are the roots of the second quadratic equation which is this one and it said that if alpha one is greater than beta one

so so alpha one is the larger of the two roots of this quadratic equation and alpha two is the larger of the two roots of ah this second quadratic equation

so it is asking us to find the value of alpha one plus beta two

so we start off with the first quadratic equation which is $x^2 - 2x \sec \theta + 1 = 0$

so the two roots ah the two roots are alpha one and beta one

so the two roots are

so we get two roots one with the plus sign the other one is the with the minus sign here and some simplification will give us $\sec \theta \pm \sqrt{\sec^2 \theta - 1}$ and then of course we use the identity that for any theta $\sec^2 \theta = 1 + \tan^2 \theta$

so the two roots are $\sec \theta \pm \tan \theta$ and then because secant theta is one over cos theta we can also write this as $\frac{1 \pm \sin \theta}{\cos \theta}$ now alpha one

so we have been asked to find the value of alpha one plus beta two and it said that ah alpha one is the larger of the two roots of the first quadratic equation

so we need to find out which one of the two is the larger one out of the two roots here now since theta we know that theta belongs to the interval minus pi by six

so it is actually an open interval minus pi by six to minus pi over twelve and when theta is in this range we know that sine theta is negative and therefore out of the two roots here the larger root is the one with the minus sign and hence alpha one which is the larger root is equal to $\frac{1 - \sin \theta}{\cos \theta}$ and of course the other fact that we have used here is that cos theta is positive

so the because when theta belongs to this interval cos theta is positive and therefore the denominator here is positive but since sin theta is negative the larger root out of these two which is alpha one is going to be $\frac{1 - \sin \theta}{\cos \theta}$ and then we take the second ah equation

so the second equation was $x^2 + 2x \tan \theta - 1 = 0$

so the two roots of this equation are alpha two and beta two which are equal to $\frac{-2 \tan \theta \pm \sqrt{4 \tan^2 \theta + 4}}{2}$ which is equal to $-\tan \theta \pm \sqrt{1 + \tan^2 \theta}$ and then of course here we use the identity that $1 + \tan^2 \theta = \sec^2 \theta$

so we are going to use this identity here and then it follows that the two roots of the second quadratic equation are $-\tan \theta \pm \sec \theta$ which is $\frac{-\sin \theta \pm 1}{\cos \theta}$ now again since

theta belongs to the open interval $-\frac{\pi}{6}$ to $-\frac{\pi}{12}$ it follows that $\sin \theta$ is negative and $\cos \theta$ is positive now the two roots here are so the first root is $\frac{1 - \sin \theta}{\cos \theta}$ and the other root is $\frac{-1 - \sin \theta}{\cos \theta}$

so we know that $\cos \theta$ is positive and we have a $-\sin \theta$ in in in for both the roots but then for this root we have a minus one here and then we have a plus one for the other root and therefore it is clear that ah for θ belonging to this interval since $\cos \theta$ is positive this root is greater than the other root and therefore since ah it was said that out of α_2 and β_2 it was said that α_2 is the greater root

so the greater root is denoted by α_2 and the smaller root is β_2 and if you also see what we have been asked for is to calculate the value of $\alpha_1 + \beta_2$

so we are interested in finding the expression for the for β_2 which is the smaller of the two roots of this quadratic equation

so since this is the smaller root it is clear that this is equal to β_2 and then we just need to add α_1 and β_2 .

so if you remember α_1 was this value

so from the previous slide we have α_1 equal to $\frac{1 - \sin \theta}{\cos \theta}$ which is ah from the first slide and therefore if we add α_1 to β_2 what we get is

so this gets added to this

so the one over $\cos \theta$ is going to get cancelled then at the end what we get is $-\frac{2 \tan \theta}{\cos \theta}$

so the final the final answer is that $\alpha_1 + \beta_2$ equals $-\frac{2 \tan \theta}{\cos \theta}$ you take up the second problem now

so this is the second problem we are asked to find the number of possible values of θ such that θ lies in the open interval 0 to π for which this system of equations has a solution but if you see here we have θ as one of the variables then the other variables are x , y and z and the question is asking us that this system of equations

so there are three equations has a solution x naught y naught z naught with y naught z naught not equal to zero

so with y naught times z naught not equal to zero the first equation was $y + z \cos 3\theta = xyz \sin 3\theta$ and this and then in the second equation we multiply both the left and right hand side with y times z because y times z is we know that it is not

so we are looking for a solution where y times z is not equal to zero and that is why if we will multiply y times z on both the sides of this equation and when we do that what we get is $xyz \sin 3\theta = 2yz \cos 3\theta + 2y \sin 3\theta$ and then of course we have the last equation which is $xyz \sin 3\theta = y + 2z \cos 3\theta + y \sin 3\theta$

so in all these three equations what we see is that

so this is common to all three equations

so essentially what we have is two equations and

so the two from

so the two equations are as follows

so the first equation is $y + z \cos 3\theta = xyz \sin 3\theta$

so this obviously has to be equal to this which is $2z \cos 3\theta + 2y \sin 3\theta$ and then the second equation that we have is that this quantity here should all be also equal to $y + z \cos 3\theta$

so so $y + z \cos 3\theta$ is also equal to $y + 2z \cos 3\theta$

three theta plus y sine three theta

so if we see for a given theta what we have is essentially a system of two equations and two unknowns y and z the solution set should be such that the y z neither should y be equal to zero nor should z be equal to zero what we see from both these equations is that ultimately these right hand sides have to be equal and the right hand side here can be written as $y \cos 3\theta$ and then plus $2z \cos 3\theta$ plus $y \sin 3\theta$

so if we see $2z \cos 3\theta$ is also here

so since this entire right hand side has to be also equal to the right hand side here because both of them are equal to the same quantity which is $y \cos 3\theta + 2z \cos 3\theta + y \sin 3\theta$

so eventually what we get is that $2z \cos 3\theta + 2y \sin 3\theta$ equals $y \cos 3\theta + 2z \cos 3\theta + y \sin 3\theta$ of course this and this term get cancelled and then what remains is that $y \sin 3\theta$ is equal to $y \cos 3\theta$ which can be written as $y \sin 3\theta - y \cos 3\theta = 0$ now since we are looking for a solution to this

so we are looking for such a solution says that neither should y be equal to zero nor should z be equal to zero because it was mentioned in the question that the solution should be such that the product of y and z is not zero

so if the product of two numbers is not zero it basically means that neither of the two numbers is zero

so since y cannot be equal to zero from this statement it follows that $\sin 3\theta - \cos 3\theta$ has to be equal to zero

so that has to be satisfied since y is not equal to zero from this equation it follows that $\sin 3\theta - \cos 3\theta$ should be equal to zero or essentially that $\sin 3\theta$ should be equal to $\cos 3\theta$

so now if we go back and if we use this fact in the first equation here what we get is that $y \cos 3\theta + 2z \cos 3\theta + y \sin 3\theta = 2z \cos 3\theta + 2y \cos 3\theta$ because $\sin 3\theta$ and $\cos 3\theta$ are the same

so we can write it as $2z \cos 3\theta + 2y \cos 3\theta$

so essentially what we have now is that these two equations are equivalent to these set of equations of course we are using the fact that the solution set is such that y is not equal to zero and neither is z now from the first equation here what we will end up getting is that we can write $\sin 3\theta - \cos 3\theta = 0$ as

so this left hand side can be written as $\sin 3\theta - \cos 3\theta = 0$ and this can then be written as $\sin 3\theta - \cos 3\theta = 0$ because $\cos 3\theta = \sin(\frac{\pi}{2} - 3\theta)$ so $\sin 3\theta - \sin(\frac{\pi}{2} - 3\theta) = 0$ but this is of the form $\sin a - \sin b = 0$ which is $\sin a - \sin b = 0$

so this is $\sin a - \sin b$ which is $\sin 3\theta - \sin(\frac{\pi}{2} - 3\theta) = 0$

so we have used the $\sin a - \sin b$ formula here with a equal to 3θ and b equal to $\frac{\pi}{2} - 3\theta$ and then the solution to this trigonometric equation as we all know is that $3\theta - (\frac{\pi}{2} - 3\theta) = n\pi$ for some integer n which essentially implies that $6\theta - \frac{\pi}{2} = n\pi$ or $6\theta = n\pi + \frac{\pi}{2}$ or $3\theta = \frac{n\pi}{2} + \frac{\pi}{4}$ for some integer n but remember that it was mentioned that theta has to belong to the open interval $(0, \pi)$ and therefore the three possible solutions are $3\theta = \frac{\pi}{4}$ since theta has to belong to $(0, \pi)$ we can only choose n to be 0, 1 and 2.

so with n equal to 0 we get the solution $3\theta = \frac{\pi}{4}$ with n equal to 1 we get the

solution as π by three plus π by twelve which is actually five π over twelve with n equal to two we get two π by three plus π over twelve which is three π over four

so these are the three possible values of θ for which the set of given solutions to this through this equation are such that y is not equal to 0 but we still need to test go back and test this other equation but what we see here is that whenever θ is taking either of these three values we know that $\cos 3\theta$ is not equal to zero

so and the only way then that this equation is satisfied is that $y + z$ is equal to zero because from this equation we can write that

so from this equation we can write that $y + z \cos 3\theta = 0$ but since $\cos 3\theta$ for any of these angles is not zero

so since $\cos 3\theta$ is not zero the only other option is that $y + z = 0$ but in any case we have already answered our question because the answer to this question is that the number of possible values of θ for which the system of equation have has a solution where $y \times z \neq 0$ is 3 because there are 3 solutions $\pi/12$, $5\pi/12$ and $3\pi/4$

so that finishes the solution to the second problem also we now take up another interesting problem and we have been asked to find the number of distinct solutions to this trigonometric equation

so initially when we look at it we might get a little disturbed by looking at the sixth power of sine and cosine and the fourth power but another thing that should be observed and can be seen is that we do have whenever we have sine we also have cosine with the same power

so $\sin^6 x$ and $\cos^6 x$ similarly sine to the power four and then cosine to the power four also

so that suggests that one possible way is to use the fact that $\sin^2 x + \cos^2 x = 1$ and then you know take the cube of this equation and then from that try to find out an expression for $\sin^6 x + \cos^6 x$

so we will first do that since we know that $\sin^2 x + \cos^2 x = 1$ if we take the cube then this is also true and then we use the formula for $(a + b)^3$ which gives us on the left hand side with $a = \sin^2 x$ and $b = \cos^2 x$

so if you recollect $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

so if we use this formula here with $a = \sin^2 x$ and $b = \cos^2 x$ what we get on the left hand side is $\sin^6 x + \cos^6 x$

so these two are the terms which are actually present here and then we get the remaining terms

so we get plus three times $\sin^4 x \cos^2 x$ plus three times $\cos^4 x \sin^2 x$ and this is equal to one and therefore if you take these two terms on the right hand side what we will get is that $\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^4 x - 3\cos^2 x \sin^4 x$

so we have this little identity

so far in a similar manner we can also find an expression for $\sin^4 x + \cos^4 x$

so instead of performing the performing the cube we have to square so $\sin^2 x + \cos^2 x$ has to be squared

so this is also equal to one and then if we use the $(a + b)^2$ formula what we get is $\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x = 1$ and therefore from here it is clear that $\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cos^2 x$ and then of course the other term was five by four times $\cos^2 2x$ but we know that $\cos 2x = \cos^2 x - \sin^2 x$ is equal to

$\cos^2 x - \sin^2 x$ and therefore $\cos^2 2x$ will be $\cos^4 x + \sin^4 x - 2\sin^2 x \cos^2 x$

so we now are going to use all these three identities that we have derived
so we will replace all these three by the corresponding right hand side

so so now i am going to write an expression for the entire left hand side here in this equation

so the first thing is $5 \cos^2 2x$

so for $\cos^2 2x$ i am going to use this right hand side expression

so it will be $5 \cos^4 x - 10 \sin^2 x \cos^2 x + 5 \sin^4 x$

so we are not going to

so let us not use this right hand side let us just keep it as $\cos^2 2x$ at the moment

so we still keep it as $\cos^2 2x$ and then we have $\cos^4 x - \sin^4 x$

so from the previous slide we know that $\sin^4 x + \cos^4 x$ is this value

so instead of $\sin^4 x + \cos^4 x$ we are going to use the value $1 - 2\sin^2 x \cos^2 x$ and then for $\sin^4 x - \cos^4 x$ we are going to use this right hand side which is $\cos^4 x - 4\sin^2 x \cos^2 x + \sin^4 x$ and it is given in the question that this entire we need to find an x such that this whole left hand side is equal to two

so this two obviously gets cancelled with this one and one here and then what we end up getting is and we can further combine these two terms because

so the left hand side becomes $2\cos^2 2x - 2\sin^2 2x$

so we can take all these three terms on the right hand side

so $2\cos^2 2x - 2\sin^2 2x$ and then these two terms we can take as common $2\cos^2 2x - 2\sin^2 2x$

so this comes from these two terms taken on the right hand side and of course $\sin^2 x + \cos^2 x$ is equal to one

so this gets simplified to $2\cos^2 2x - 2\sin^2 2x$

so we finally end up with $2\cos^2 2x - 2\sin^2 2x = 2$ which is the same as writing that $\cos^2 2x = 2\sin^2 2x$ which is equal to $2\sin^2 2x = \cos^2 2x$ but $2\sin^2 2x$ is the same as $\sin^2 4x$

so this becomes equal to $\sin^2 4x = \cos^2 2x$

so x must satisfy this equation

so we have $\cos^2 2x = \sin^2 2x$ and that can be then be written as $\cos^2 2x - \sin^2 2x = 0$ but then we see ah that this is nothing but $\cos 4x$ because here we use the $\cos 2\theta$ formula

so we know that $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

so with $\theta = 2x$ this is what we get

so from here it follows that $\cos 4x = 0$ and

so if we go back to the question we were asked to find all the values of x or the number of distinct values of x which are in the interval 0 to 2π which are solutions to this trigonometric equation

so the solution to this equation in the general solution is that $4x$ is of the form

so $4x$ has to basically be an odd multiple of $\pi/2$

so i can write it as $(2n+1)\pi/2$ where n is an integer and from here it follows that x has to be of the form

so x is basically an odd multiple of $\pi/8$ and we are we also need to find only those

so this is the general solution

so we can take n to be any integer will get infinitely many different solutions to the trigonometric equation but we are only interested in those solutions which lie in the closed interval zero to two π

so obviously those solutions are x equal to

so we start with ah

so we cannot take n equal to minus one because then x becomes negative

so we have to start with n equal to zero

so with n equal to zero the first solution is $\pi/8$ and then with n equal to one the second solution is $3\pi/8$ n equal to two we have $5\pi/8$ and then $7\pi/8$ followed by

so all the odd multiples of $\pi/8$ $11\pi/8$ $13\pi/8$ $15\pi/8$ but we can't go beyond because the next is $17\pi/8$ and $17\pi/8$ is more than two π

so this is not allowed and if you see all of these solutions are distinct

so therefore the answer to this question is that there are eight distinct there are eight distinct solutions of this equation in the interval zero to two π

so these are the eight one two three four five six seven and eight

so the next question is here

so it is asking us to prove that the values taken by ah this particular ratio of trigonometric functions does not lie between one by three and three for any real value of x that we might take

so starting off we immediately realize that this is nothing but because we see $\sin x$ and $\cos x$

so $\sin x$ over $\cos x$ is $\tan x$ and then in the denominator we have $\sin 3x$ and $\cos 3x$ in the numerator

so so this entire thing is essentially equal to $\tan x$ by $\tan 3x$ and here we will have to use the formula of $\tan 3x$ in terms of $\tan x$

so if we remember the formula was for any angle x \tan of $3x$ is $3 \tan x$ minus $\tan^3 x$ over $1 - 3 \tan^2 x$

so we use this right hand side over here this ratio becomes equal to $1 - 3 \tan^2 x$ over $3 \tan x - \tan^3 x$ now we know that $ah \tan x$ takes all values between minus infinity and plus infinity and therefore $\tan^2 x$ will be taking all values between zero to infinity

so $\tan^2 x$ will be a non negative ah real number

so then what we will do is we will represent this as $1 - 3a$ over $3a - a^3$ where a is defined as $\tan^2 x$ and of course we know that a is greater than equal to zero

so a could be any depending on x a could take any value between zero to infinity

so so a essentially belongs to zero to infinity

so what the question is asking us to prove is that for a belonging to the interval 0 to infinity we have to show that for a belonging to the interval

so for a belong to the interval 0 to infinity we have to show that this ratio $1 - 3a$ over $3a - a^3$ it never takes any value between

so this value will never belong to the interval one by three to three

so it says it does not lie between one by three and three

so this is essentially what we have to prove now we start off with

so this is what we have and that can be written as $9 - 3a - 8$ over $3a - a^3$ and this $9 - 3a$ is three times the denominator

so this is equal to $3 - 8/3a$ over $3a - a^3$

so at this point we have to divide the we have to treat two different ah set of values that a can take a of course is non negative

so we first consider all those values of a which belong to the interval between 0 to 3 of course we will not because at a equal to 3 this is not defined

so that is why we have an open interval here
so for a greater than equal to zero and strictly less than three
so for when a belongs to this interval it is easy to see that three minus a is
so the denominator here which is three minus a is greater than zero and also
that since a is greater than zero it also follows that three minus a has to be
less than or equal to three
so this is what is true or this could even be written as a minus three is
greater than equal to minus three n is negative that's one and the same thing
and then if we
so now we need to find well this can also be written as three plus eight over a
minus three and then we need to find the range of values for 8 over a minus 3.

so it is clear that since a minus three is greater than minus three it is clear
that eight over a minus three because

so essentially we are going to use ah this particular inequality here and we
know that a minus 3 is negative when a belongs to this range
so obviously 8 over a minus 3 is bounded from below by minus infinity
so so that is obviously correct that it is greater than minus infinity that is
coming from this particular inequality in the on the previous line and then we
also have that this is

so this is less than equal to from this one here minus eight by three
so if we use ah this inequality here what we
so we just have to add three to all the
so to three to everywhere in this inequality and therefore if we add three
everywhere in this inequality

so we add 3 here we add a 3 here and we add a 3 here
so the final inequality after that we get is
so then

so this is exactly this quantity
so at the end what we get is that if a belongs to the interval zero to three
then this particular ratio which is this will take all values which are less
than equal to and if you calculate this this is one over three
so so when a belongs to zero to three then one minus three a over three minus a
belongs to minus infinity to one over three

so this is the set to which one minus three a over three minus a belongs to
so let me summarize that

so what we have shown on the previous slide is that if a belongs to zero to
three then one minus three a over three minus a belongs to minus infinity to one
over three and then of course we take the second case

so if a belongs to three to infinity which means that a is greater than three
it cannot be equal to three

so in that case again what we see is that

so if we remember we had one minus three a over three minus a to be equal to
three plus eight over a minus three and we have that

so from here it follows that since a is greater than 3 this implies that a
minus 3 is greater than 0 and therefore this quantity here is always positive
so this quantity here is always positive and hence this particular value here
is always larger than three

so for this region we therefore what we get is that one minus three over three
minus a belongs to

so will take the values three two infinity and that is because ah this thing is
positive

so from here what if we use this equality then what we get is that 8 by a minus
three is strictly greater than zero and it is also less than infinity and then

if we add three on all the sides we add three everywhere then we will get this thing

so hence what we have shown is that

so by taking these two by considering these two cases together we see that the values taken by this fraction are either low below $1/3$ or they are greater than 3 or they are greater than 3 and hence this shows that this fraction $1 - 3 \sin^2 a$ will never take any value which is between $1/3$ and 3

so that finishes the proof of the fourth question and now we take up the last question of this session

so we are asked to find the value of

so the value of this summation which is thirteen terms is the k th term is $\frac{1}{\sin \frac{\pi}{4} + k \sin \frac{\pi}{6}}$

so this reminds us of the $\sin a \sin b$ formula we know that $2 \sin a \sin b$ is equal to $\cos(a - b) - \cos(a + b)$

so that is what we are going to use here with this entire thing as a and this as b and then if we do that well but we also need a factor of two here

so we multiply both the numerator and the denominator by two and then this denominator is simply equal to $\cos a - \cos b$

so $\cos a - \cos b$ is going to be $\cos \frac{\pi}{6} - \cos a$

so $\cos a$ will be $\frac{\pi}{2} + k \sin \frac{\pi}{6}$ and we know that $\cos(90^\circ + \theta) = -\sin \theta$

so $\cos(90^\circ + \theta)$ is $-\sin \theta$

so we know that for any θ , $\cos(\frac{\pi}{2} + \theta) = -\sin \theta$

so we use this fact here the k th term of this summation is equal to $\frac{2 \cos \frac{\pi}{6}}{\cos \frac{\pi}{6} - \cos(\frac{\pi}{2} + k \sin \frac{\pi}{6})}$ and therefore this entire summation simply becomes $\sum_{k=1}^{13} \frac{2 \cos \frac{\pi}{6}}{\cos \frac{\pi}{6} - \cos(\frac{\pi}{2} + k \sin \frac{\pi}{6})}$ we know that $\cos \frac{\pi}{6}$ is nothing but $\frac{\sqrt{3}}{2}$

so we write that here directly and then plus sign of $2k \sin \frac{\pi}{6}$ over six another thing that we realize by looking at this term is that if we look at the k th term

so this is the k th term let us look at the $k + 6$ th term

so the $k + 6$ th term in the summation is going to be $\frac{2 \cos \frac{\pi}{6}}{\cos \frac{\pi}{6} - \cos(\frac{\pi}{2} + (k + 6) \sin \frac{\pi}{6})}$ instead of k we have to write $k + 6$ into $\frac{\pi}{6}$ which is equal to $\frac{2 \cos \frac{\pi}{6}}{\cos \frac{\pi}{6} - \cos(\frac{\pi}{2} + k \sin \frac{\pi}{6} + 2\pi)}$

so this over $6 + 2\pi$ which is equal to \sin of two $k \sin \frac{\pi}{6}$ one $\frac{\pi}{6}$ plus 2π but \sin of two π plus some angle is the sign is equal to the sign of the angle itself

so this is equal to therefore \sin of two $k \sin \frac{\pi}{6}$ one $\frac{\pi}{6}$ which is nothing but the k th term

so we realize that the k th term and the $k + 6$ th term are indeed the same and therefore it follows that the first and the seventh term are the same the second and the eighth term are the same

so we can add both of them together the

so what we can do is we can write rewrite this entire summation as $\sum_{k=1}^{13} \frac{2 \cos \frac{\pi}{6}}{\cos \frac{\pi}{6} - \cos(\frac{\pi}{2} + k \sin \frac{\pi}{6})}$ because if you see the first and the seventh term are together by what I mean by together is that they are equal and then the second and the eighth term are equal the third and the ninth term are equal the fourth and the tenth are equal the fifth and the eleventh are equal and the sixth and the twelfth terms are equal and the 13th term has to be treated separately

so essentially if we have to add all these first 12 terms it only suffices to add the first six terms and multiply that multiply

that sum by a factor of two because the seventh term is the same as the first term the eighth is also the same as the second

so adding all the twelve terms is equivalent to just adding the first six terms and multiplying the sum by a factor of two

so then the entire summation of the thirteen terms becomes equal to

so the the double of the kth term is going to be four over root three over two plus sine of two k minus one pi over six and then of course we have a remaining ah 13th term which still remains because since the 13th num term is left out we need to write another term here which is 2 over root 3 over 2 plus sine of

so when we put k equal to 13 we get sine of 25 pi by 6

so we get sine of 25 pi by 6 which is equal to sine of 4 pi plus pi by 6 but sine of 4 pi plus pi by 6 is the same as sine of pi by six because the sign function is periodic ah in with integer multiples of two pi

so so this is our ah the last term

so now we have to basically calculate the sum of these six terms

so that is what our goal is

so we can just write down ah all of these six terms now

so the the kth term is 4 upon root 3 by 2 plus sine of 2 k minus 1 times pi over 6.

so we will write all the four terms now

so all these sorry we will write all the six terms now

so with k equal to one the the first term is four divided by square root of 3 over 2 plus sine of pi by 6 and then the second term is four over square root of three over two plus sine of three pi by six when you put k equal to here you get sine of three pi by six but sine of three pi by six is sine of pi by two which is exactly equal to one k equal to three we get four over square root of three over two plus sine of five pi by six but sine of five pi by six sine of 5 pi by 6 is the same as sine of pi over 6 and sine of pi by 6 is exactly equal to half

so if we want we can even replace this by half here and this is also half and then for k equal to 4 we get 4 over square root of 3 over 2 plus sine of 7 pi by 6 and sine of seven pi by six is equal to minus half

so we will write minus half here k equal to five we get four divided by root three over two and then when we put five here we get nine pi by six sine of nine pi by six is sine of nine minus ah sine of nine pi by six is sine of three pi by two which is minus one and then the last and the sixth term is

so when we put six here we get eleven pi by six

so sine of eleven pi by six is the same as minus half because sine of 11 pi by 6 is equal to minus of sine of pi by 6

so this is minus half

so we just need to add all of this now and a simple algebra will show that and its a little ah little bit of an exercise left for you is that if you add all of these six terms you will get the sum to be zero

so essentially what then happens is that this entire big summation goes to zero and therefore this is the final answer

so the final answer is two over square root of three over two plus half which is four divided by square root of three plus one and if i multiply both the numerator and denominator by square root of three minus one then what i get is 4 into square root of 3 minus 1 and in the denominator i get 2 which gets cancelled in the numerator

so the final answer is 2 times square root of 3 minus 1.

so that finishes ah the last problem also and with that we finish the second problem solving session from the next lecture onwards we are going to start

discussion on properties of triangles thank you you

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