

on inverse trigonometric functions

so in the previous four lectures we have covered and defined the inverse trigonometric functions we have also defined identities and relations between them and in the last lecture we had started solving some of the problems

so we will continue to do

so so this this lecture is primarily going to be problem solving for inverse trigonometric functions

so this is the first problem

so we have to prove that this left hand side is equal to square root of $x^2 + 1$

so immediately if we see this it appears a little daunting because there appears to be a composition of trigonometric and inverse trigonometric functions and there is a nesting of the composition is up to the third level

so the idea is to actually start with the innermost level

so that is $\cot^{-1} x$ and we say that let $\cot^{-1} x$ be equal to θ and then because the range set of \cot^{-1} is the open interval $(0, \pi)$ it is clear that θ has to belong to the open interval $(0, \pi)$

so the this then becomes $\sin \theta$ and we know that when θ belongs to $(0, \pi)$ from the graph of the sine function it is clear that $\sin \theta$ is non negative

so because θ belongs to this we know that $\sin \theta$ is non negative and therefore we can write $\sin \theta$ as square root

so here this is the positive square root of $\sin^2 \theta$ which is equal to the positive square root of $1 - \cos^2 \theta$ now we need to express this right hand side in terms of x and therefore we will use this relation here

so from this relation if we take if we apply the \cot function on both the sides what we get is that x equals $\cot \theta$ and therefore and this then implies that $\tan \theta$ is $1/x$ then this can be written as $1 - \tan^2 \theta$ which can then be also written as square root of $1 - \tan^2 \theta$ we know the identity that $\sec^2 \theta$ is $1 + \tan^2 \theta$

so we are going to use that identity over here which can be further simplified to square root of $\tan^2 \theta$ over $1 + \tan^2 \theta$ and then in this expression we plan to use this value of $\tan \theta$ which is $1/x$

so we replace $\tan \theta$ by $1/x$ everywhere in this expression here

so then what we get is that $\sin \theta$ becomes equal to the positive square root of $1/x^2$ over $1 + 1/x^2$ which is equal to square root of 1 over $1 + x^2$ and this is the positive square root and that value we essentially put back in this equation over here

so what we then get is that we need to evaluate $\cos \theta$ but $\sin \theta$ is this value which is $1/\sqrt{1+x^2}$

so this is what we are supposed to evaluate and then again we what we say is that let us say that this particular value is equal to ϕ

so let $\tan^{-1} \frac{1}{\sqrt{1+x^2}} = \phi$ now since this is the positive square root this entire value here this value here is non negative and therefore ϕ must belong to the interval $(0, \pi/2)$

so essentially what we have to find out is $\cos \phi$ of course from here it can also be concluded that $\tan \phi$ equals $1/\sqrt{1+x^2}$ now this is $\cos \phi$

so this is what we have to compute now

so $\cos \phi$ now since ϕ belongs to the interval $(0, \pi/2)$ from the graph of the \cos function it is clear that $\cos \phi$ has to be non negative and therefore we can write $\cos \phi$ as the positive square root of $\cos^2 \phi$ which can then be written as $1/\sqrt{1+x^2}$ and

then again using the identity that $\sec^2 \phi$ is equal to $1 + \tan^2 \phi$ we get this to be equal to $1 / \sqrt{1 + \tan^2 \phi}$ and then of course ah in this equation we substitute $\tan \phi$ with $1 / \sqrt{1 + x^2}$ and then that when we do this substitution we finally get

so we do the substitution $\tan \phi$ equal to $1 / \sqrt{1 + x^2}$ and then we get $\cos \phi$ equal to $1 / \sqrt{1 + \tan^2 \phi}$

so this is the positive square root $1 / \sqrt{1 + \tan^2 \phi}$ which is $1 / \sqrt{1 + 1 / (1 + x^2)}$

so $\tan^2 \phi$ is $1 / (1 + x^2)$ and then this comes out to be equal to $1 + x^2 / \sqrt{1 + x^2}$ and this is exactly what was asked to prove in the question

so that finishes the proof of ah this first problem

so here is the second problem

so it states that suppose there is a function f whose domain is 0 to 4π and whose range is the closed interval 0 to π and the function f is defined as $f(\theta) = \cos^{-1}(\cos \theta)$ then the question asks the number of points θ belonging to the domain of the f function which satisfy the equation $f(\theta) = 10 - \theta$

so we have to essentially find how many θ s are there in this domain of the f function such that this value $f(\theta)$ is equal to $10 - \theta$

so essentially we need to solve for all the θ belong to the interval 0 to 4π such that $\cos^{-1}(\cos \theta) = 10 - \theta$

so we need to find out how many solutions to this equation exist in this interval 0 to 4π

so clearly this is because this is a trigonometric well we need to first simplify this left hand side over here and then try to equate and see how many such values of θ are there now it is said that θ belongs to 0 to 4π

so clearly when

so let us let us divide this region 0 to 4π into 4 regions

so the first region is 0 to π the second region will be π to 2π then the third region will be 2π to 3π and the last which is the fourth region will be the closed interval between 3π and 4π

so the first case is when θ belongs to 0 to π

so when θ belongs to 0 to π what is $\cos^{-1}(\cos \theta)$ now when θ belongs to 0 to π it is clear that the this particular value well for any θ $\cos^{-1}(\cos \theta) = \theta$ let us say it is equal to some angle ϕ now from the range of the \cos^{-1} function it is clear that for any θ ϕ has to belong to the closed interval 0 to π also by applying the \cos function on both the sides what we get is $\cos \theta = \cos \phi$ now we know that in this interval 0 to π just like these the graph of the sine function between $-\pi/2$ to $\pi/2$ if we look at the graph of the cosine function

so let me quickly draw it over here

so if we see the graph of $\cos x$ versus x between let us say 0 to π

so let us say this is $\pi/2$ let us say this is 1 and let's say this is -1 and the graph is something like this and as you can see the graph the curve the function $\cos x$ when limited to the interval 0 to π is monotonically decreasing with increasing x and therefore if we have two angles θ belonging to 0 to π and ϕ also belonging to the the interval 0 to π

so both θ and ϕ belong to the same interval and $\cos \theta = \cos \phi$ but since the cosine function is monotonically decreasing the only way

this is true is that θ is equal to ϕ and therefore we get that

so this ϕ is basically equal to θ

so therefore we get that for the first case when θ belongs to 0 to π $\cos^{-1}(\cos \theta)$ is equal to θ which is θ

so $\cos^{-1}(\cos \theta)$ is θ itself now when θ belongs to the second region which is between π and 2π it is clear that for this case $\cos^{-1}(\cos \theta)$ cannot be equal to θ because this value $\cos^{-1}(\cos \theta)$ has to belong to the range set of \cos^{-1} which is 0 to π and this θ is belonging to the interval π to 2π

so therefore

so to be precise what we can do is that we can define this second region to be π to 2π but it will be open from this left side

so the value of π does not belong to the second region because and this is because we had defined the first region to be the closed interval

so π belongs to the first region

so now it is clear that for the second case when θ belongs to π to 2π it is clear that $\cos^{-1}(\cos \theta)$ will not be equal to θ

so then how do we find the value of $\cos^{-1}(\cos \theta)$ now as usual let us say that this is equal to some x then it is clear that this x has to belong to 0 to π which is the range set of the \cos^{-1} function and then if we apply the cosine function on both the sides here we end up getting $\cos \theta$ is equal to $\cos x$ we ah realize one thing over here which is that since the \cos function is a periodic trigonometric function $\cos(2\pi - \theta)$ is also equal to $\cos \theta$ and the reason why we took $2\pi - \theta$ was because if θ belongs to π to 2π then that implies that $2\pi - \theta$ would belong to the interval 0 to π

so therefore what we see is that if we ah from essentially what we see is that if θ belongs to π to 2π then $2\pi - \theta$ belongs to 0 to π and this interval is actually a subset of the range set of the \cos^{-1} function

so therefore what we have here is that we have an angle $2\pi - \theta$

so let us say this is ϕ

so we have $\cos \phi$ is equal to this value $\cos \theta$ and we know that this ϕ belongs to the range set we know that this ϕ which is $2\pi - \theta$ belongs to the range set of \cos^{-1}

so we know that ϕ belongs to the range set of the \cos^{-1} function and therefore since $\cos \phi$ is equal to $\cos \theta$ it is immediately clear that ϕ must be equal to $\cos^{-1}(\cos \theta)$ hence ah in this case where θ belongs to π to 2π it is clear that $\cos^{-1}(\cos \theta)$ will be equal to ϕ where ϕ is $2\pi - \theta$

so therefore in this case in the second case $\cos^{-1}(\cos \theta)$ is equal to $2\pi - \theta$

so $\cos^{-1}(\cos \theta)$ is $2\pi - \theta$ similarly for the third case where θ belongs to 2π to 3π again in this case also $\cos^{-1}(\cos \theta)$ will not be equal to θ and in fact we can show that when θ belongs to 2π to 3π $\theta - 2\pi$ will belong to the interval 0 to π and further that $\cos(\theta - 2\pi)$ is equal to $\cos \theta$

so this is equal to $\cos \theta$

so again we have a similar situation here where \cos of this angle is the same as $\cos \theta$ and we know that this angle $\theta - 2\pi$ belongs to the range set because this particular interval here is a subset of the range set of the \cos^{-1} function and therefore we know that this angle belongs to the range set of \cos^{-1} and from therefore it follows that $\theta - 2\pi$ should be equal to $\cos^{-1}(\cos \theta)$ and therefore for ah when θ belongs to

this interval two pi to three pi $\cos^{-1}(\cos \theta)$ will actually be $\theta - 2\pi$ and the same thing is true for the last scenario where θ belongs to three pi to four pi

so in this case we see that $4\pi - \theta$

so if θ belongs to three pi to four pi $4\pi - \theta$ belongs to the interval zero to pi which is again a subset of the range set of \cos^{-1} further $\cos(4\pi - \theta) = \cos \theta$ and this again is because of the periodicity of the cosine function

so $\cos(4\pi - \theta) = \cos \theta$ and $4\pi - \theta$ this particular angle belongs to the

so this one belongs to the range set of \cos^{-1} and therefore again from here it follows that $4\pi - \theta = \cos^{-1}(\cos \theta)$ hence when θ belongs to this fourth interval $\cos^{-1}(\cos \theta)$ is actually equal to $4\pi - \theta$

so if we now we can now summarize these four different cases in this slide here

so this slide tells us the value taken by this function $\cos^{-1}(\cos \theta)$ for all the different ranges of θ

so this precisely defines the $\cos^{-1}(\cos \theta)$ function

so in this in this graph we have plotted both the curves for the function $\cos^{-1}(\cos \theta)$ as well as the function $\frac{10 - \theta}{10}$

so on the horizontal axis we have θ on the vertical axis we plot the values taken by these two functions

so in black we have plotted the graph of the function $\cos^{-1}(\cos \theta)$ in blue we have plotted the graph of the function $\frac{10 - \theta}{10}$ and ah we see that since ah we were asked to find out all those values of θ for which these two functions take the same value or for which $\cos^{-1}(\cos \theta) = \frac{10 - \theta}{10}$

so that is what was being asked and it is very clear from the graph that ah both these curves they intersect at ah first of all over here and then the second time they intersect here and then the third time they intersect here

so there are basically three distinct values of θ for which $\cos^{-1}(\cos \theta) = \frac{10 - \theta}{10}$

so the final answer to this question will be the number of points θ belong to θ belonging to zero to four pi satisfying the equation $\frac{10 - \theta}{10} = \cos^{-1}(\cos \theta)$ is three

so there are only three distinct points in the next problem we are asked to solve ah the following inverse trigonometric equation ah

so we have to solve and find all those values of x which satisfy this equation here

so the same thing can be written as $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1} 3x$ but we know that $\tan^{-1} 1 = \frac{\pi}{4}$ or rather $\tan^{-1} 1 = \frac{\pi}{4}$

so we replace $\frac{\pi}{4}$ by $\tan^{-1} 1$ and therefore the right hand side is equal to this expression here and then we have to use the formula for $\tan^{-1} x - \tan^{-1} y$ or rather we can actually write it as $\tan^{-1} 1 + \tan^{-1} (-3x)$

so we need to use the $\tan^{-1} x + \tan^{-1} y$ type of formula which we have already seen in one of our previous lectures

so here is the expression

so for this case what we have now is that we have $x = 1$ and we have y equal to $-3x$ and we see that the product of x and y is -3 for this case

so one more thing that has to be observed for this problem is that the value of the tan inverse function

so so this particular value and also this value

so both these values add up to pi by four

so now if if x is negative then we know that if x is negative this value will also be negative and therefore and also this value

so therefore if x is negative then this entire thing will be negative but since in this problem we have been asked to find out those values of x for which this is positive it clearly can be it can be basically concluded that only those x will satisfy this equation for which x is greater than zero

so that is something that we can immediately see from this equation here

so therefore in the subsequent discussion we will only restrict our discussion to positive values of x

so again coming back to the tan inverse x plus tan inverse y formula now we know that since x is positive for our problem here

so what that means is that y is negative because y is minus three x and therefore x times y

so we have x times y is less than zero and therefore out of these three cases our case will basically be this particular case because for us x y is less than zero and of course zero is less than one

so in our case ah in our for for our problem x y is less than one and therefore we have to use this particular formula with x equal to one and y equal to minus three x and when we do that we get this right hand side to be equal to tan inverse of $\frac{1 - 3x}{1 + 3x}$ and then we have

so finally what we get is that tan inverse of two x is equal to tan inverse of $\frac{1 - 3x}{1 + 3x}$ now clearly because ah we know that if tan inverse a is equal to tan inverse b if this is true then it must be true that a is equal to b then it must be true that a is equal to b and therefore i mean that this can be seen by just applying the tan function on both the sides of this equation

so we applied tan on both the sides we get a equal to b and therefore using this fact in this particular equation it follows that this is true if and only if two x is equal to $\frac{1 - 3x}{1 + 3x}$ and then if we ah if we work it out a little bit of course ah then we have a little bit of algebraic manipulation which gives us this and then we have $6x^2 + 5x - 1 = 0$ and the left hand side can be factored as $(6x - 1)(x + 1) = 0$

so there are two solutions now

so x is either one over six or it is minus one but we already know that ah x must be greater than zero and therefore ah this is not minus one is not a value solution

so the only possible solution is x equal to one over six

so this is the final answer

so here is another ah very interesting problem and i think it is one of the je problems

so it says that if x y and z are in arithmetic progression and tan inverse x tan inverse y and tan inverse z are also in arithmetic progression then which of the following is true now since x y and z are in arithmetic progression we have y is equal to $\frac{x + z}{2}$ or in other words y minus x is equal to z minus y now tan inverse x tan inverse y and tan inverse z are also in arithmetic progression and therefore that fact can also be written as tan inverse y minus tan inverse x is equal to tan inverse z minus tan inverse y now let us denote this angle by theta let us say it is non negative now let us make the following observation

so let's say this is $-\pi/2$ and this is $\pi/2$ then for this case it should be true that $\tan^{-1} z$ is greater than $\tan^{-1} y$ and the least is $\tan^{-1} x$

so and this and this are

so these are equal these are these angles are equal in measure and because these two angles are equal in measure and all of these $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ have to lie in the interval $-\pi/2$ to $\pi/2$ it is it follows that the magnitude of this angle

so let us say if θ is positive then

so it follows that this

so this is θ and this is also θ and therefore if we add both these θ s we get 2θ which is this value

so it is clear that this 2θ should be less than the length of this entire interval which is π and therefore it is clear that θ should be less than $\pi/2$ or yeah because the \tan^{-1} function is actually an open interval

so that is why we have strict less than here

so essentially we have θ greater than equal to zero and less than $\pi/2$

so now let us apply the \tan function on both sides of this equality now we know that this angle θ is between 0 and $\pi/2$ and therefore \tan of θ can be written as \tan of $\tan^{-1} y$ minus \tan of $\tan^{-1} x$

so here we are going to use the \tan of a minus b formula

so let us say this is a this is b

so this will be equal to $\frac{\tan a - \tan b}{1 + \tan a \tan b}$

so $\tan a$ is y minus $\tan b$ is x over $1 + xy$ now because this θ is between 0 and $\pi/2$ it follows that this value of $\tan \theta$ because when θ is between 0 and $\pi/2$ the graph of the \tan function the \tan function takes non negative values

so this has to be greater than equal to 0 .

so from here it clearly follows that we have $\tan \theta$ equal to this value and θ of course belongs to this interval which is actually a subset of the range set of the \tan^{-1} function and hence it follows from here that θ must be equal to \tan^{-1} of $\frac{y - x}{1 + xy}$ and similarly ah the same thing can be done for this right hand side

so the right hand side here which is $\tan^{-1} z$ minus $\tan^{-1} y$ will come out to be equal to \tan^{-1} of $\frac{z - y}{1 + zy}$ and because they are equal we finally have this equality and this basically implies that now we can we just need to take apply the \tan function on both the sides of this equality and therefore what we get is that this this value here is equal to this value here which we write as $\frac{y - x}{1 + xy} = \frac{z - y}{1 + zy}$ and then further simplification gives us that $y - x$ into $1 + zy$ is $z - y$ into $1 + xy$ and then $y + z$ into $y^2 - x^2$ is $z + x$ into $z^2 - y^2$ now we know that because x , y and z are in arithmetic progression $y - x$ is equal to $z - y$ and therefore these four terms cancel out and then what we get is $2xy = z^2 - y^2$

so we take this term on this side is equal to and then we bring this term on the left hand side y^2 into $x + z$ but then $x + z$ is equal to $2y$ and that is because x , y and z are in arithmetic progression

so this becomes $2y^3$ and that can be written as $2y^3 - xz = 0$

so in the interest of time ah without considering the second case even if we

so even for the second case where we consider this value to be negative and if we just follow a similar derivation we are also going to arrive at this

condition and therefore it follows that this is a necessary and sufficient condition

so this is a necessary and sufficient condition if both x , y , z and $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ both of them have to be in arithmetic progression

so this is a necessary and sufficient condition for that now from here there are only two possibilities

so either $y = 0$ or $y^2 = xz$ now if $y = 0$

so if $y = 0$

so if y is equal to 0 then we have $x + z = 2y = 0$ and we also have we also had that $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ are in arithmetic progression

so well this will be

so this we are taking the case $y = 0$ here but this is zero

so and $x + z = 0$

so so this

so for this case since $z = -x$

so this actually implies that $z = -x$

so then this third $\tan^{-1} z$ becomes equal to $\tan^{-1}(-x)$ which is $-\tan^{-1} x$

so of course this is one possibility that $y = 0$ and therefore we have $x = 0$ and $z = 0$

so this is that $z = -x$ this is in arithmetic progression and also $\tan^{-1} x$, 0 , $-\tan^{-1} x$

so these three are also in arithmetic progression

so this is one possibility and the other possibility is that $y^2 = xz$

so this is the other possibility but this actually implies that x , y and z are in geometric progression and at the same time we have $x + z = 2y$ which basically means that

so this means that they are in arithmetic progression

so because x , y and z are both in arithmetic as well as geometric progression the only possible way is that $x = y = z$

so we only have two possibilities

so the possibility number one is that $y = 0$ and $z = -x$ and the second possibility is that possibility number 2 is that $x = y = z$

so these are the only two possibilities and if we go back to the multiple choice question that was asked

so since $y = 0$ is not mentioned here the only possibility that is correct is this first scenario

so all of the other ones are not correct

so we take up another interesting problem here

so in this problem we are said that it is said that the value of x is between 0 and 1 and then we are asked to find the value of this very lengthy expression here

so as always we always go to the innermost expressions because this is a nested because here we have trigonometric composition of a trigonometric and an inverse trigonometric function

so we start off with defining $\cot^{-1} x = \theta$ now because x is between 0 and 1 it follows that θ must belong to the open interval $(0, \frac{\pi}{2})$

so because it is zero and one it should actually belong to the interval from $\frac{\pi}{4}$ to $\frac{\pi}{2}$

so this follows from the graph of the cot inverse function and then we just replace of course from here it also follows that x is equal to $\cot \theta$ and then we replace this $\cot^{-1} x$ by θ everywhere in this equation here so then whatever is inside this big equation becomes

so this root of one plus x^2 becomes root of one plus $\cot^2 \theta$ which is root of one plus one over $\tan^2 \theta$ which is equal to root of $\frac{1 + \tan^2 \theta}{\tan^2 \theta}$ which is can be further simplified because this numerator here becomes $\sec^2 \theta$ and that $\sec^2 \theta$ is one over $\cos^2 \theta$

so this will finally simplify to one over $\sin \theta$ it should be also noted that for θ in this range $\sin \theta$ is strictly positive

so the the expression

so what we try to do is that we try to move this root of one plus x^2 inside this other square root and then what we get is square root of square root of sorry the square root will not be there anymore because it will be $1 + x^2$ into $x \cos \theta + \sin \theta$ whole square minus one plus x^2

so this is what we get then we can try to move this $1 + x^2$ inside this expression

so that will be square root of $1 + x^2$ times $x \cos \theta + \sin \theta$ whole square minus one plus x^2 but we know that this is square root of one plus x^2 is actually one over $\sin \theta$

so we use this fact now

so this is essentially one over $\sin \theta$ and that is what we use here

so we get this to be equal to square root of $x \cos \theta + \sin \theta$ plus one

so this when divided

so we divide both these by $\sin \theta$

so this becomes one and this becomes $x \cot \theta$

so eventually we have root of $x \cot \theta + 1$ the whole square minus one plus x^2 but we know that $\cot \theta$ is in fact equal to x

so we replace this by x and then we finally get square root of $x^2 + 1$ the whole square minus $1 + x^2$ and this if we simplify further we get this to be equal to square root of $x^2 + 1$

so this finishes the computation of this expression which we have found to be equal to x into square root of one plus x^2 let us take the next problem in this problem we have to find the number of positive solutions which satisfy this following inverse trigonometric equation

so by positive solutions we mean the values of the variable x which are positive

so $x > 0$ which satisfy this inverse trigonometric equation

so as we can also see in this equation on the left hand side we have an inverse we have the sum of the tan inverse of two different values

so this immediately reminds us of the tan inverse a plus tan inverse b type formula

so just for you to recollect we had in one of the previous lectures proved this particular identity that $\tan^{-1} x + \tan^{-1} y$ can be written as the $\tan^{-1} \frac{x + y}{1 - xy}$ if the product xy is less than 1 and if the product xy is greater than 1 then if both x and y are positive then we just need to add plus π if xy is greater than 1 but both x and y are negative then we need to add minus π ah for this current problem at hand what we see is that since we are only interested in finding the number of positive solutions

so $x > 0$

so whenever x is greater than zero it is clear that one by two $x + 1$ is also greater than zero and

so is one by four x plus one

so both of these arguments of the tan inverse function are positive and therefore obviously this particular case is ruled out

so the only two cases which can i now apply are either this case or this case ah but we also see that if you see this value if x is greater than 0 then $2x + 1$ is of course greater than 1 and therefore $1/(2x + 1)$ has to be strictly less than one and of course positive

so this is

so one by two x plus one which is the argument of the first ah tan inverse here is between zero and one and a similar thing can be said about the second ah argument here which is one by four x plus one

so we see that both one by two x plus one and one by four x plus one lie between zero and one and therefore from this it follows that the product one by two x plus one times one by four x plus one has to be also less than one because both of them are less than one and therefore if we come back to our formula we see that the the out of these two cases the case which applies is $x < y$ less than one which is that the product of the two arguments of tan inverse x plus n inverse y

so the arguments are x and y

so the products x and y if it is less than one then this first case is applicable which is true here because the product we have already shown the product to be less than one

so therefore we have tan inverse of one by two x plus one plus tan inverse of one by four x plus one to be equal to

so we will use ah this first formula here tan inverse of this plus this over one minus the product of these two values and which can be simplified too this expression here in the problem it is said that we need to find a positive we need to find positive solutions which means that we need to find positive values of x such that this expression is equal to tan inverse of two by x square

so we want this to be equal to tan inverse of two by x square

so from this equality over here

so this and this are equal and therefore it follows that this argument here should be equal to two by x square that is $\frac{6x + 2}{2x + 1} \times \frac{4x + 1}{4x + 1} - 1 = \frac{2}{x^2}$ and if we simplify this ah expression algebraically what we end up getting is that x must satisfy this polynomial equation and this can be further factored out as x into $3x^2 - 7x - 6 = 0$

so ah this can be further factored out also

so we can write $3x^2 - 7x - 6$ as $(3x + 2)(x - 3)$

so from here it is clear that there are exactly three values of x which satisfy this equation and the values are $x = 0$ from here $x = -2/3$ because of this factor and $x = 3$ from this last factor but if we go back to the problem the problem was asking us to find the number of positive solutions which satisfy this equation and from here it is clear that this is zero this is $-2/3$

so the only positive solution is $x = 3$ and hence the number of positive solutions satisfying this equation is equal to one

so the number of positive solutions satisfying this equation is one

so there is only one solution which is $x = 3$

so that finishes ah this fifth lecture and we will continue to do some more problems in the next lecture which is going to be our last lecture on inverse trigonometric functions and thereafter we plan to start a new topic where we discuss the properties of triangles where i think a lot of this material that we

have covered for the trigonometric functions and the inverse trigonometric functions is going to be extremely useful thank you you

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