welcome to the first lecture on inverse trigonometric functions in the previous seven lectures we discussed trigonometric functions like sine x cosine x cot x tan x cosec x and seck x from this lecture onwards we are going to define their inverses and we will also discuss the identities ah between these inverse functions let us take the case of sine x

so as we know from our previous lectures the sign x function has a domain which is the set of all real numbers and the range is all numbers between minus one and plus one what this means is that if we take any real value x then sign of x will definitely belong to the closed interval minus one to plus one the next question is can we define the inverse function of the sine function

so by inverse function what i mean to say is that if we take any y belonging to minus one to plus one the question is when there exist a unique will there exist a unique x belonging to r such that sine of x is equal to this y

so this is the basic question that we are asking

so lets obviously because sine x as we know is a periodic function this is not true

so here on this slide i have plotted the graph of sin x on the vertical axis x is on the horizontal axis and lets say that we have y equal to half and we are trying to find out all those x such that sin x is equal to half

so what we do is we draw a horizontal line here which is parallel to the x axis and which is at a displacement of half on the positive side on the y axis and then what we see is that there are many values of x for example this value and this value here and this value this value of x

so all these values of x are such that sign of x is equal to half and there are infinitely many actually

so this therefore from this we know that we cannot find a unique x such that sine x is equal to half and therefore in general as of now it appears that it will be difficult to define the inverse of the sine function however we can still define its inverse for that we need to take up a smaller ah simpler example

so on this slide we have a plot of the function f x equal to x square

so y equal to x square is plotted on the vertical axis x is on the horizontal axis and this is the the blue dotted curve is the curve for y equal to x square

so the function f is defined from the set of all real numbers to the set of all non negative real numbers which i denote by r plus now as you can see here for any value of y in the range of this function f

so let us take for example this is y equal to one

so for y equal to one we see that there are two values of x one is x equal to plus one and then there is another value of x which is x equal to minus one

so for both x equal to minus one and x equal to plus one we get y equal to one and therefore again there is no unique x such that f x equal to x square is one because for both these x equal to minus one and plus one we get y equal to one and therefore there is no unique x which means that again to define the inverse of this function f x equal to x square appears difficult

so one usual ah trick is that

so we try to restrict and define a new function f which is the same as the old function

so we still have we will still have f x equal to x square but then we will restrict the domain of this function f from the set of all real numbers to a subset of the set of all real numbers

so let us say that we restrict this function f in this case to this following subset let us say that we restrict it to the set of all

so so we take this set which is

so so from 0 to all to infinity

so we say that we define a new function f prime which is from r plus
so r plus is essentially this interval from zero to infinity
so we define a new function f prime from r plus to r plus and f prime of x is again x square right
so f prime is again f prime x is x square which is the same as what we had before its just that the domain now has been restricted from r to r plus but the advantage of doing this restriction is first of all that by restricting the domain from r to r plus we still still see that the entire range of the function is still the same
so earlier the range of this function the previous function f was r plus because f x equal to x square and since x square can only be non negative the range of the previous function was r plus even now when we restrict the domain of this function f from r to r plus even now the domain will still be r plus because all all non negative numbers will be in the range of this new function f prime
so what we see is that we have restricted the domain in such a way that the range of the function is still the same
so we are not changing the range of the function but the important thing that we have done here is that by doing this restriction of the domain now what we see is that
so i am plotting the graph of this function f prime in red
so i essentially only consider the the graph of f x equal to x square only on the positive side of the horizontal axis
so this curve in red is the graph for the new function f prime and now if we take any value of lets say we take
so this is y equal to one two three lets say four here
so if we take y equal to four and what we see is that with the restricted domain there is only one unique value of x which is x equal to 2 for which f prime x is equal to 4 in fact even for y equal to 1 for the restricted function the for the function f prime with the restricted domain there will be only one unique solution which is x equal to plus one and this happens because we have restricted the domain of the function from r to r plus and therefore now there is uniqueness because we can take any other y we can take this y for example let us say this is some value of y
so even for this wire this is the unique x this is the unique x such that f prime of x which is x square is equal to this y and therefore for this restricted function f prime it is now possible to define the inverse meaningfully for for this f prime function the inverse will be let us say g which is from r plus to r plus and for any y ah belonging to r plus the inverse will be g of y will be equal to square root of y where this is the positive square root of y
so this is how we define g
so many times you will write g also as it is many times written as f prime inverse
so we use this superscript on the ah on the name of the original function to whose inverse we are defining
so f prime inverse or many times people do not even write like this they just write f prime inverse is from r plus 2 r plus and f prime inverse of y for any y belonging to the domain of f prime f sorry forgot a prime here f prime inverse of y is equal to the positive square root of y
so now we are going to use the same method of restricting the domain of the function such that the range does not get altered but then the the inverse should be meaningfully defined
so so the whole trick here is to restrict the domain of the function for

example even in this case if we if we lets say restrict the domain instead to be instead of r plus instead of r plus if we restrict the domain to let us say some interval minus 2 to plus 2 then this will not work because if we restrict the domain from minus two to plus two then the range of this function will only be from zero to

so let us say that we restrict it to minus two to plus two

so we restricted to minus two to plus two then the range of the function will be only from zero to four right whereas the range of the original function was the set of all non negative numbers and that is not the same as this range zero to 4 right

so this is not a meaningful restriction because we should try to restrict the domain of the function in such a way that the range does not get altered the range is still the same

so we will now use this technique to restrict the domain of trigonometric functions

so that we can define their inverse meaningfully

so we go back again to the y equal to sine x function and now we we know that the range of the sine x function is between minus one and plus one and if we see here if we restrict let us say the domain of the ah sine x function let us say we restrict it from minus pi by 2 to plus pi by 2.

so we restrict the domain of the sine x function to the interval to the closed interval minus pi by two to plus pi by two

so by this by doing this restriction we see that the curve for the restricted function will be the solid red curve which i am going to draw now

so it will be this

so this is the curve for the restricted sine x function where the domain has been restricted to minus pi by 2 to plus pi by 2 and as we can see here even after this restriction all the ah the possible values are there because if we try to follow this curve then for example here it is minus one and then as we go in this direction the values of sin x go from minus one to zero continuously and then further if we go further towards ah further from zero to plus one and therefore even after this restriction the range the range of the sine x function is still minus one to plus one additionally what we also see is that if we take any value of y in the range minus one to plus one

so let us say we take this y here

so y equal to minus half

so if we take y equal to minus half then we see that there is only a unique value of x in this restricted domain such that sign of x is equal to minus half because as you can see in this restricted domain this sine x function when restricted to this domain is strictly monotonically increasing function of x and therefore this for any y that we take in this range minus one to plus one there will be a unique value of x a unique value of x in this interval such that sign of that x will be equal to this y

so for example we see here that for y equal to minus half this value of x is equal to

so this value of x is equal to minus 30 degrees or minus pi over 6 and therefore we say that sine

so we have sine of minus pi by 6 is equal to minus half and from here then we write that sine inverse of minus half is minus pi by six

so now that we have restricted the domain of the sine function to the close interval between minus pi by two and plus pi by two we are in a position to define its inverse uniquely and meaningfully

so in this graph we are going to plot the sine inverse function whose domain

will be the interval the closed interval between minus one and plus one and the range will obviously be the closed interval between minus pi by two and plus pi by two here is a table of ah sine x x versus sine x values which will help us to plot this graph
so lets start with ah minus one
so we know that
so this is known to us that sine of minus pi by two is equal to minus one
so from here we can say that sine inverse of minus one is equal to minus pi over two
so sine inverse of minus one
so sine inverse of y equal to minus one
so y equal to minus one is here and sine inverse of minus one is minus pi by two minus pi by two is over here
so we essentially have this point on the
so this is minus 1 and this is minus pi by 2
so sine inverse of minus 1 is minus pi by 2
so we basically have this point on the graph and then moving ah further ahead we have sine of minus pi by 3
so sine of minus pi by 3 which is sine of minus sixty degrees is equal to minus square root of three over two and from there we can write that sine inverse of minus root three over two is minus pi over 3
so sine inverse of y equal to minus root 3 over 2
so this is 4 units and minus root 3 over 2 is around 0 minus zero point eight six
so that will be approximately somewhere here and then minus pi by three
so this is minus pi by two
so two thirds of this will be minus pi by three since this is six units it will be somewhere here and therefore this is the point on the graph that we get
so similarly from this graph we can also show that sine inverse of minus ah sorry sine inverse of ah minus one over square root of two will be equal to sine inverse of sine inverse of minus one over square root of two will be minus pi by four minus one over square root of two is minus zero point seven which is somewhere here and minus pi by four is half of this which is somewhere there
so this and this is going to meet over here and then sine of minus pi by 6 which is minus 30 degrees is minus half and minus half is here minus pi by six is one third of this thing which is over here
so we get another point like this and of course ah sine inverse of zero will be zero which is here will have a symmetrically similar thing on the ah pos for positive values of y
so will have this point for ah because sine inverse of minus sine inverse of one is equal to pi by two and then for plus root 3 by 2 we have sine inverse of root 3 by 2 equal to pi by 3 which is over here
so this and then this will get us this point and similarly for pi over four we are going to get this point here and then for pi over six we have this point here
so if we join all these all these dots we get the
so this dotted red curve is the graph for sine inverse of y similarly for cos y also ah we see that if we do not restrict the domain of cos function then again the reason is that because it is is a periodic function what will happen is that with the domain as the whole real line we cannot define its inverse meaningfully because any value of y that we take in the range of this cos function there will be many different values of x for which cos x will be equal to that y but we see that if we lets say restrict the ah just like the sine function if we restrict the cosine function the domain of the cosine function to the interval from from

0 to pi

so we only consider from 0 to pi

so so 0 to pi the closed interval between 0 to pi if we restrict the cos function to this ah to this subset of the real line then the graph of the cos function will be like this and again what we see is that just like for the case of the sine function this the entire range is covered even if we restrict the domain to 0 to pi the range of the cosine function when restricted to this 0 to pi interval is still between 1 and minus 1 all values between 1 and minus 1 because this red curve is continuous between 1 and minus 1.

so we can start from one here and as we go along the curve we get we reach a value of zero at pi by two and then further down we go all the way till minus one when x equal to pi

so the even by restricting the domain of the cos function to 0 to pi the entire range is still the same further what we see is that if we take any value of y let us say we take y equal to let us say half over here

so so for y equal to half if we look at the original ah cos function whose domain was the entire real line then we will have many different values of x here and then another value here and another value here similarly another value here and here also and

so infinitely many values of x for which cos x is equal to half but when we restrict the domain to zero to pi what we will see is that there is only one value of x one unique value of x which is in this case in this example it is pi by three or sixty degrees because cos of sixty degrees is equal to half and therefore what we see is

so it is not just true for y equal to half we take any other y in the interval ah minus one to plus one which is the range of the cost function what we will see that there will always be a unique x in this in this restricted domain such that cos of that x will be equal to this y and of course it is also visibly clear because as we can see this function cos function when restricted to this domain when restricted to 0 to 0 to pi the graph is monotonically decreasing and therefore for any value of y that we take there will always be a unique x in this interval and therefore the inverse of the cos function can now be uniquely i mean meaningfully defined

so just like for the sine function we have the inverse of the cos function from minus one to plus one being the domain and the range is the closed interval 0 to pi and again here we have a table of values

so just as we did for the sine function we start with

so we have cos of pi equal to minus one and therefore cos inverse of minus one will be equal to y

so cos inverse of y equal to minus one is here will be equal to pi pi is here

so we have this point on the graph ok and then let us take here

so here we have cos of three pi over four is equal to cos of three pi over four is equal to minus one upon square root of two and from here what we get is that cos inverse of minus one over square root of two will be equal to three pi over four

so minus one

so y equal to minus one over square root of two is ah somewhere here and three pi by four is basically three quarters of

so this is nine units

so three quarters will be somewhere around six and three quarters of units three four five six and somewhere there

so the point that we have is this point on the graph and similarly ah when we go further ahead we see that for ah cos x equal to minus half when x is equal to

two pi by three and therefore cos inverse of minus half
so cos inverse of y equal to minus half
so y equal to minus half is here
so cos inverse of minus half will be equal to two pi by three which is two thirds of ah this whole range which is equal to six ah little square
so that comes over here
so so far we have gotten this part of the cos inverse curve which i am showing with a dotted line and then further we know that cos of pi by two is zero and therefore cos inverse of zero will be pi by two
so cos inverse of zero will be pi by two is half of this entire range which will be somewhere here and then will have a similar ah symmetric you know curve about this line on the for positive values of y
so for example for we know that cos of 0 is 1 and therefore cos inverse of 1 will be 0
so which is this point over here and let us take one more point
so pi by three we know that cos of sixty degrees which is pi by three is equal to half right
so therefore cos inverse of half will be pi by three
so cos inverse of y equal to half is here
so cos inverse of y equal to half is pi by 3 pi by 3 is here
so we have this point
so so the curve will look something like this something like that
so this is the curve for the cos inverse function
so similarly we have the other trigonometric functions and everywhere we have the same problem that if we just look at the entire domain as we had defined in the previous lectures it will be difficult to define the inverse meaningfully and therefore we have to restrict the domain for example for the tan function the domain is all real numbers except odd multiples of pi by 2 the reason is that at odd multiples of pi by 2 the tan function becomes unbounded
so this is the graph of the tan function on the x axis we have x the vertical axis is tan of x and the blue dotted curve is the tan x function and as we can see this is again a periodic function it repeats with pi and therefore what we can do is that if we restrict the domain of the tan function to the interval between minus pi by two to plus pi by two
so here we have to take the closed interval
so this will take sorry the open interval because
so this is the open interval the open intervals are denoted with a rounded bracket and the closed for the closed interval we take a square bracket
so if we restrict the domain to this open interval the reason for for taking the open interval is because the tan function will become unbounded at minus pi by two and plus pi by two
so we have to avoid those two points
so all other points in between minus pi by two and plus pi by two will be the restricted domain of the tan function and when we restrict the tan function to that domain the curve for the restricted tan function becomes i am showing it with the with the red curve
so the red curve as we can see now first of all the red curve is going to cover the entire range of the tan function from minus infinity to plus infinity
so therefore even by restricting the domain of the time function to this open interval between minus pi by 2 and plus pi by 2 what we see is that the the range still remains the same as before which is the set of all real numbers the interesting thing however that happens is that if we take because we see that this red curve is a monotonically increasing curve right and therefore it is clear that if we take any value of y in the any value of y in the range of the

tan function
so any real value of y then what we will see is that there will be always a unique value of x in this interval minus pi by two to plus pi by two there will be unique value of x such that tan of x will be equal to that y
so therefore again for the tan function it is clear that by restricting the domain to the open interval between minus pi by 2 to plus pi by 2 the inverse of the tan function can be uniquely defined
so on this slide we again just like the sine and the cosine function we plot the graph of the inverse of the tan function
so as we can see the the domain of the tan inverse function is the set of all real numbers the range of course is the open interval between minus pi by two and plus pi by two as shown here and let us try to plot the graph now
so as we can see from this table of values here that first of all tan of 0 is 0
so we know that tan of 0 is 0 and from that it follows that tan inverse of zero will be equal to zero
so tan inverse of y equal to zero
so tan inverse of y equal to zero is equal to zero
so in this graph we have y is plotted on the horizontal axis and tan inverse of y is plotted on the vertical axis
so y equal to zero is this point and tan in tan inverse of zero is also zero
so therefore we have this point on the graph of tan inverse then we see that ah tan of pi by six is one over root three and therefore tan inverse of one over root three will be equal to pi by six
so tan inverse of one over root three
so one over root three is ah should be somewhere over here
so this is y equal to 1 over root 3 and the value of the tan inverse function and y equal to 1 over root 3 will be equal to pi by 6 which is one third of this entire range which will be somewhere here will be somewhere there and then of course we know that tan of pi by 4 is 1 and since tan off since tan of tan of pi by 4 equals 1 it follows that tan inverse of 1 will be equal to pi by 4
so tan inverse of y equal to one will be pi by four pi
so y equal to one is here and pi by four is here
so we have this point on the on the graph right and similarly we can plot the other points also
so what we will end up getting is some curve like this but note that the value of the tan inverse will as y goes to as the value of y goes to infinity as it becomes larger and larger the value of tan inverse on the positive side for the positive values of y as y goes to infinity tan inverse y will go to pi by two but will never be equal to pi by two it will converge as you can see this graph is going towards ah
so this is the line corresponding to the value of vertical axis being the value on the vertical axis equal to pi by two
so this dotted curve will never go beyond this ah solid red line and similarly on the negative side also using this table we can draw the negative side the the curve for negative values of y and even here the curve that will get is something like this
so even
so here we have this line which is which corresponds to tan inverse y which corresponds to minus pi by 2 on the vertical axis and as y goes to minus infinity
so if as we go in this direction what happens is that the graph will go as you can see from here the graph will go nearer and nearer to this value of minus pi by 2 but will never exceed that in fact tan inverse y will never be equal to minus pi by 2.

similarly for the cotangent function also that we have here the cotangent function the cortex function is nothing but 1 over tan x
so which is an or we can even write it as cos x over sin x sin x
so cotangent x is equal to cos x over sine of x and therefore what you will see is that whenever x is a multiple an integer multiple of pi whenever x is an integer multiple of pi sine of x is 0 and therefore cotangent of x for x being integer multiples of pi is not well defined and that is clear from this graph where we have plotted x on the horizontal axis and cot of x on the vertical axis we can see that at integer multiples of pi
so for example x equal to zero or x equal to pi or two pi we see that the graph goes to either plus infinity or minus infinity
so it becomes unbounded right and that is why when the domain of the cot function is defined it is defined like this
so we can for the domain we consider all real values except this set and this set is precisely what we mean by all integer multiples of pi
so this is the domain set of the chord function and of course the range will be the set of all real numbers because the chord function will take all ah real values between minus infinity and plus infinity and of course it is also periodic with pi
so cot x is a periodic function ah with pi
so cot x is equal to cot of pi plus x i mean if you just consider the same domain which is defined over here it will be defined difficult to define the inverse of the chord function any value of y that we take here let us say we take this y
so this corresponds to y equal to three by two
so there will be many different values of x many different values of x for which cot of x will be equal to this value three by 2 and therefore there is no unique x and hence with this domain the inverse of the cot function cannot be defined meaningfully and hence we again have to restrict the domain of the chord function
so what we do is we restrict the domain to this interval from 0 to pi
so we take this interval 0 to pi but we cannot include the point 0 and pi because the chord function is unbounded it is not well defined at these two end points
so therefore we take take the open interval between 0 to pi and restrict the domain of the chord function to this open interval when we restrict it what we see is that the curve of the chord function restricted to this open interval 0 to pi now becomes what i am drawing with the red solid red curve here and it is very easy to see now that even after restricting the domain of the cot function to the open interval 0 to pi what we see is that the range is still from minus infinity to plus infinity
so the range of the chord function is still the set of all real numbers even after restricting the domain from this to the open interval 0 to pi and secondly it is very easy to also see that the solid curve here is monotonically decreasing and therefore if we take any y in the range of the chord function
so if we take any y let us take a y here
so we take this y
so this y is ah minus three by two
so if we take any y there will be a unique x which is here which will lie in this restricted domain of the chord function such that cot of x will be equal to this y
so this will be true for any value of y that you take and that is very much clear also because this chord function is a continuous function in this it is a continuous function in this restricted domain and additionally it is

monotonically decreasing and therefore by restricting the chord function to this domain we should be able to define its inverse meaningfully
so we again try to plot the graph for the cot inverse function whose domain will be the set of all real numbers and the range will be now the restricted domain of the chord function which is the open interval between zero to pi now we know that let us start from here we know that cot of pi by two is equal to zero and therefore cot inverse of zero will be equal to pi by two
so cot inverse of y equal to zero
so y equal to zero is here will be equal to pi by two
so pi by two is here
so so we get this point on the on the graph of cot inverse y
so here in the graph we have y is plotted on the horizontal axis and the corresponding cot inverse of y will be plotted on the vertical axis similarly we have cot of pi over three equals one over square root of three and therefore cot inverse of one over square root of three will be equal to pi over three
so cot inverse of one over ah square root of three will be somewhere there and pi over three is somewhere here
so it will be somewhere there and then as we
so will get this point here
so this is
so this was pi over two this is equal to pi over three and then as we increase y to more positive values
so we are going in this direction here
so so for example cot inverse of square root of ah 3 will be pi by 6
so as we increase y what we will see is that cot inverse will decrease and go towards zero
so cot inverse
so from here from the table what we see is that cot inverse of square root of three is equal to pi by six
so square root of three will be somewhere here and pi by 6 is somewhere
so pi this is this is pi by 6.

so we get this point on the curve right
so essentially what we have is
so far we have plotted and it will go like this but the value as
so as y goes to infinity cot inverse y will go towards zero but it will never be equal to zero right and then for the negative values of y what we will see is that will have a similar type of curve
so and which we can be drawn by looking at the negative values of cot x
so for example from here what we will see is that ah cot inverse of minus 1 for example will be equal to three pi over four
so minus one is over here and three pi over four will be
so this is six units
so three pi over four will be four and a half units right
so it will be somewhere there somewhere over here
so this is y equal to minus one and this value here is three pi over four and then ah for on for for further even more negative values what we will see is that the value of the cot inverse function will go towards pi
so now we are going in this direction
so as we make the value of
so this is the value of y on in this column as y becomes more and more negative
so from minus one to minus square root of three as y becomes more negative the value of ah this x is going towards pi
so what we will have is

so this is ah
so this is pi
so this curve the dotted curve will go something like this
so we finally have the red dotted curve for cot inverse of y
so as y becomes more and more negative cot inverse y will tend towards pi but will always be less than pi in fact it will never be equal to pi the next function is ah cosecant function
so the cosecant function cosecant of x is as we know defined as 1 over sine of x and it is also a periodic function trigonometric function and because it is 1 over sine of x it will not be meaningfully defined whenever sin x is equal to zero
so therefore whenever x is an integer multiple of pi cosecant of x is not meaningfully defined and therefore the domain of the cosecant function is the set of all real numbers except integer multiples of pi
so that is the domain of cosecant x and for the range what we see is that because cosecant x is 1 over sine x and sine the range of sine x is the all values between minus 1 and plus 1 what will happen is that 1 over sine x will take all values outside the interval minus 1 to plus 1.

so and that is very much clear from this graph where we have x on the horizontal axis and we have cosecant of x on the vertical axis as we as we can see the the the blue dotted curve will never take any value between minus one and plus one and this is because it is one over sine x and sin x takes values between minus one and plus one and therefore the range of the cosecant function is the set of all real numbers except this range minus 1 to plus 1 and therefore now with because it is still periodic as we can see it is still a periodic function cosecant x is a periodic function of x if we have this as the domain we still cannot define the inverse of the cosecant function meaningfully and therefore we need to restrict the domain of the cosecant function as we have done for the other previous trigonometric functions
so in this case one possible choice is that we consider the interval minus pi by 2 to plus pi by 2 except the point 0.

so we are going to restrict the domain of the cosecant function to the closed interval minus pi by two to plus pi by two but except the point zero because at x equal to zero cosecant of x is not meaningfully kind defined because it becomes unbounded
so by now by restricting ah the cosecant function to this set or this subset of the real line the restricted cosecant function the graph of the restricted cosecant function will look something like this
so i am drawing it with the solid red curve here and as we can see even by restricting the domain of the cosecant function to this interval we see that all the the range of the restricted cosecant function is still the same thing which is all real values except minus one to plus one because it cannot take my any value between minus one to plus one anyways but apart from this range of values it does take all other possible values because for for the interval between minus pi by 2 and 0 if we go if we start from here
so this this value over here is this is minus one and if we go like this then we see that the curve is continuous and starting from minus one it it goes all the way till minus infinity and similarly if we if we start from here then this value is plus one and if we go like this the curve is again continuous and it goes from the value goes from one to infinity
so we have values going from one to infinity and then from minus 1 to minus infinity which is basically the same range

so even by restricting the domain to this subset of the real line the range of the cosecant function is still the same it still remains the same and additionally ah what we also see is that if we take any y
so let us take any y in this range set
so let us say that we take this particular value of y here
so this value of y corresponds to two point five
so so this is ah
so this is y equal to two point five
so for this ah what we see is that there is exactly one unique value of x which is here such that cosecant of this x this value is equal to 2.
5 and of course this value lies in the restricted domain if we had not restricted the domain that there would have been many other values for example this value of x or for example this value of x or this value of x
so there would have been infinitely many different other x values of x such that cosecant of x would have been 2.
5 and therefore there is no unique value as such but when we restrict the domain to this interval we see that for any value of y in this range there will always be a unique x there always be a unique x in the restricted domain such that cosecant of that x will be equal to y that value of y and therefore again by doing this restriction we should be able to meaningfully define the inverse of the cosecant function which i have will try to plot it in this slide here and this can be just done just similar to what we have been doing for the other for the other functions and i will quickly show it to you how it looks
so this is how it looks
so again using the table we can plot
so the what we have is we have y on the horizontal axis and cosecant inverse of y on the vertical axis the domain of the cosecant inverse function is will be the range of the cosecant function
so the domain of the cosecant inverse function is all real numbers except the interval minus one to plus one and the range of the cosecant inverse function will be the modified or the restricted domain of the cosecant function that we saw on the previous slide which is the closed interval minus pi by two to plus pi by two except the point zero and then again using this table of values we through the red curve here and here is what is the graph of the cosecant inverse function and as we can see ah with as y becomes more and more positive the value of cosecant inverse will go towards this value of zero
so this curve red curve here will always be above the horizontal axis on the positive side and for negative values of y the red curve will always be below the horizontal axis
so i mean this part of the curve and similarly the secant function will have a similar problem and for the secant function also we need to to define the inverse of the secant function meaningfully just like the other trigonometric functions we have to also restrict its domain because ah secant of x is equal to one over cos of x and we know that cos of x is zero whenever x is a odd multiple of pi by two
so therefore the domain set of the secant function is defined to be all the set of all real numbers except odd multiples of pi by two because at odd multiples of pi by two secant of x is not well defined as can also be seen from this graph where we have x on the horizontal axis and secant of x on the vertical axis and what we see is again the secant function is also periodic and the range as we can see is that because secant of x is one over cos x and cos x is between minus one and plus one
so what happens is that the range of the secant function is the entire real line which is the entire vertical axis here except this interval between minus

one and plus one
so that is this range and just like the other trigonometric functions we again have the same problem that because secant is function is periodic with this domain with this as a domain we cannot define the inverse meaningfully and therefore we need to restrict the domain of the secant function also
so what we do is that we restrict the domain between 0 and pi
so we take the restricted domain to be the closed interval between zero to pi but as we can see we have to we should not have the point x equal to pi by two and the reason is because at x equal to pi by two secant of x is unbounded it is not well defined right
so the modified domain will be 0 to pi interval except the point pi by 2 and the graph of the secant function will then be what i am drawing with the red curve
so this will be the graph of the secant function when the domain is restricted to this interval x at pi by 2 and as we can see the entire the range even after this restriction of the domain is still the same it is still all real values except the interval minus 1 to plus 1 right
so all values are there except minus 1 to plus 1 and also what we see is that for if we take any value of y then we can always
so if we take any value of y there will be a unique value of x such that secant of x is equal to y and that value of x will be in this interval and therefore by this restriction the we can meaningfully define the inverse of the secant function which i am defining here
so on this graph we have y on the horizontal axis and secant inverse is plotted on the vertical axis and as we can see we are again going to make the use of the table values for x versus secant x the secant inverse function is defined from the this will be the domain which is basically the range of the secant function and the range of the secant inverse function will be the restricted domain of the secant function which is 0 to pi except the point pi by 2.
this is what we get
so in this lecture we had we had initially tried to define the inverse of the trigonometric functions but we immediately ran into a problem because the the trigonometric functions are ah most all of them are periodic and therefore with the with their normal definition of their domains it is difficult to define their inverses meaningfully and but then what we saw was that if we restrict the domain of these functions we can define their inverses meaningfully
so in the next class we will take all these will try to define and prove lot of identities and relations between these 6 inverse trigonometric functions that we defined in today's class thank you you