

hello everyone

so this is the first lecture on conic sections

so in this chapter conic sections will study about parabolas ellipses and hyperbola

so let us start with we will discuss parabola ellipse in the hyperbola and a special case of ellipse ellipses circle which you have already studied about

so let us start with first circle let me recall what a circle is

so a circle is a set of points in a plane which are equidistant from a fixed point in the plane

so there is a fixed point let us call c in the plane and then to determine a circle we need all the points which are at a fixed distance from this fixed point

so let us say the fixed distance is r then if we look at set of all points which are at distance r from this fixed point c in this plane we get what is called a circle and this fixed point the fixed point is called the center of the circle and the fixed distance of points on the circle from the center is called the radius of the circle

so you would have also seen how to find equation of a circle given the centre and radius

so let me just briefly recall

so suppose the center c is at a point h comma k and the radius is r now if i take a general point p whose coordinates are x comma y then let the center be c whose coordinates are h comma k and radius r this is some positive real number then to find the equation of this circle we use the definition

so we know that the distance pc is equal to r and then by distance formula we have the distance between the point p and c is x minus h square plus y minus k square square root this is equal to r and therefore x minus h square plus y minus k square is equal to r square this gives the equation of the circle right

so any point on the circle satisfies this equation conversely if x comma y satisfies the above equation then distance the distance between x comma y and the center h comma k is square root of x minus h square plus y minus k square which by this equation is equal to square root of r square which is r thus this equation thus the above equation let me call this equation star is the equation of this circle whose center is at h comma k and radius is r

so now let us discuss some other curve

so we will discuss what is called a parabola

so the definition is a parabola is the set of all points in a plane that are equidistant from a fixed line let us call this line l and a fixed point and we require this point not to lie on the line not on the line l in the plane

so let me describe this geometrically

so suppose i have a line l and there is a fixed point let us call that point f so this point f and the line both lies in this plane x y plane and we are looking for all the points in this plane such that the distance of the point from this line that means the perpendicular distance of the point to this line is equal to the distance of the point p to this point f

so if we draw this

so you can see one point is clearly if i draw this perpendicular line here and if i take the midpoint here then the distance of this point to the line is equal to distance of this point to this point f similarly if you see if you draw this curve you will get a figure which looks like this

so if i take any general point p on this curve then the distance of this point p to the line is equal to the distance of p to f

so if this distance is d then this is also equal to d

so we look at all the points which satisfies this and this will give me a curve

which is called parabola

so this line l the line l is called the directrix of the parabola and the point f is called the focus of the parabola we have directrix and focus also the line passing through the focus and perpendicular to the directrix is called the axis of the parabola

so if i draw again this line and then i have this parabola i have this focus f this is line l

so axis is the line which is perpendicular to the directrix and it passes through this focus

so this is called the axis of the parabola now this axis will intersect the parabola at some point which is called the vertex of the parabola

so the point of intersection of the axis with the parabola is called the vertex of the parabola

so we have axis this is focus this v is the vertex this line is called the directrix and this line which is perpendicular to the directrix and passes through focus is called the axis of the parabola

so now we will try to derive the equation of parabola

so so let me first discuss some standard equations of the parabola

so right now we will discuss the parabola whose

so we will discuss parabolas whose vertex is at the origin and the directrix is parallel to one of the axis one of the coordinate axis let's see

so let me draw the x y axis this is x axis y axis what we want is that the vertex should be at the origin

so this is my origin o and suppose the directrix is parallel to the y axis

so let us take this to be directrix line l then where is focus

so focus will be on the x axis in this case

so let us say the coordinate of this focus is a comma 0 then what is the equation of this line l if you see the distance of this point vertex which is on the parabola to focus this distance is a this distance is a

so the distance of the vertex to the line this should also be equal to a and therefore the equation of this line is x equal to minus a

so this is the case where my focus is f a comma 0 directrix is the line x equal to minus a and vertex in this case is the origin 0 0 now if you draw the curves

so we want any point x y whose distance

so if this is general point p x comma y the distance of this point to the focus should be equal to the distance of the point from this line l

so let us say this is point m

so let p x comma y be any point on the parabola then the distance of p from the focus f which is a comma 0 is equal to the perpendicular distance of p from the line from the directrix whose equation is x equal to minus a that is in the figure we have pm is equal to pf let me draw this figure again this is the directrix x equal to minus a and focus is at a comma zero now any point p now what is the perpendicular distance pm this is nothing but the distance of this point x comma y from the line x equal to minus a which is equal to this distance if you see this is equal to x and this is

so this will be equal to x plus a in modulus and the distance pf is equal to the distance of the point x comma y to a comma 0 which is square root of x minus a square plus y square

so we get square root of x minus a square plus y square equal to mod of x plus a squaring we will get x minus a square plus y square is equal to x plus a square which implies x square minus two a x plus a square plus y square equal to x square plus two a x plus a square

so we see that x square and a square cancels and then we get the equation is y square equal to four ax this implies y square equal to four a x and we assume

that a is a positive real number in this case

so this is the equation for the parabola whose focus is on the positive x axis and vertex is at the origin

so this is the equation $y^2 = 4ax$

so here vertex is at the origin and the focus lies on the positive x axis

so this is one of the standard form of parabola this is a parabola which opens to the right also what do we see in this parabola this is symmetric about the x axis and x axis is actually the axis this is the axis of the parabola in this case x axis which is the axis of the parabola

so this is symmetric means that if I have any point (x, y) on the parabola then we also have the point $(x, -y)$ this also lies on the parabola this can be easily seen from the equation also because if (x, y) lies on this equation we have $y^2 = 4ax$ and then if I put $(x, -y)$ y^2 is same as y^2

so it also satisfies

so we will see some more form of the parabola

so it might happen that we have again we are taking vertex at the origin and suppose my focus is on the negative x axis

so f is $(-a, 0)$

so now we have now consider the parabola whose vertex is at the origin and the focus lies on the negative x axis say f has coordinate $(-a, 0)$ where a is positive

so in this case what will be the directrix

so if we see the distance of the vertex from the focus this is equal to here a is the distance

so the directrix will be perpendicular to this axis axis is the line joining the vertex to the focus

so the directrix will be line $x = -a$ and in this case this is the line $x = -a$ and this distance we are taking a positive

so this distance is also a

so in this case the parabola let me draw in red passes through this vertex $(0, 0)$ and this will open to the left

so we get a graph like this

so in this case if I take any point $P(x, y)$ be any point on the parabola then again we have the distance of this point if I take P here to this line this is the point this is $(-a, y)$

so this is $x = -a$ and y

so the distance PM is equal to PF that is $|x + a|$ this is equal to the distance of P to F will be $\sqrt{x^2 + y^2}$ again by squaring we get $(x + a)^2 = x^2 + y^2$ and this will give $y^2 = 4ax$

so we can see in the previous one where the focus was on the positive x axis we have $y^2 = 4ax$ a is positive

so in this case a can never be negative because we have $y^2 = 4ax$

so x is equal to $y^2 / 4a$

so x is always non negative in this case whereas in this case we have $y^2 = -4ax$ because a is positive x can never be positive in this case so this lies completely in the left half plane and this lies in the right half plane

so so this is the equation of this parabola where the focus is on the negative x axis and vertex is at the origin similarly we can have focus to lie on the y axis

so focus on the y axis and vertex at the origin

so suppose focus f is on the positive y axis sorry $(0, a)$ and vertex is

at the origin then what is the directrix if we see the distance of this vertex to the focus this is equal to a

so the directrix will be the line y equal to minus a now if you take any point so this parabola will look like this which opens up words

so any point $p(x, y)$ on the parabola should satisfy that distance of p to f is equal to distance of p to this line

so in this case pf is equal to square root of $x^2 + y^2$ minus a square and pm is this point m is $x, -a$

so the distance is $y + a$ in mod

so $pf = pm$ is equivalent to writing $x^2 + y^2 - a^2 = (y + a)^2$ which gives $x^2 = y^2 + 2ay + a^2 - y^2 - 2ay - a^2$ that is $x^2 = -4ay$ similarly if focus f is at $(0, -a)$ then the equation of the parabola is $x^2 = 4ay$

so these are four standard forms we had this is $y^2 = 4ax$ then we have $y^2 = -4ax$ this is $x^2 = 4ay$ and the other one will be sorry this graph should pass through the vertex o

so this this is the parabola $x^2 = 4ay$ and if my focus is on the negative x axis then we get parabola facing downwards this equation is $x^2 = -4ay$

so in subsequent lectures we will also discuss about the more general form of parabola where the vertex need not be at the origin also the directrix need not be parallel to any of the coordinate axis but right now let us discuss some other term which is called the latus rectum of the parabola

so let us take say this parabola this is my parabola $x^2 = 4ay$ and here the focus is at the point $(0, a)$ and the vertex is at the origin now the latus rectum is the line segment which is perpendicular to the axis of the parabola

so this is the axis of the parabola here and it passes through the focus

so this line segment here let me call this ab

so latus rectum is the line segment ab which passes through the focus f and is perpendicular to the axis of the parabola and has end points on the parabola

so in the figure ab is the latus rectum of the parabola $y^2 = 4ax$

so we would like to find the length of this later section

so what is the length of

so what we can see from this figure what are the coordinates of this point a and b we have focus is the has the coordinates $(a, 0)$

so this line is $x = a$

so the what are the coordinates of this

so let us say this is $(a, -y)$ and this is (a, y) the x coordinate is a and y coordinate we are taking y here then this will be minus y

so because we have equation of the parabola is $y^2 = 4ax$ putting $x = a$ gives $y^2 = 4a^2$ which means y is plus or minus $2a$

so this point is $(a, -2a)$ and this is $(a, 2a)$ this is the focus $(a, 0)$

so the length of the latus rectum l is equal to $4a$ this length is $2a$ and this is also $2a$

so $4a$ is the length of the later section let me discuss one problem find the equation of the parabola which is symmetric about the y axis and passes through the point $(2, -3)$

so lets look at how this will look like

so if i have x axis and y axis what is given to you is that the parabola is

symmetric about the y axis

so because it is symmetric about the y axis that means that this will be y axis is the axis of the parabola this is the axis of the parabola since y axis since the parabola is symmetric about the y axis y axis is this axis of the parabola and the vertex is at origin

so we have the vertex is here and this is the axis

so therefore this will be either facing upward like this or it will be like this

so we have to determine which one is this but what we know is also is that the parabola passes through the point two comma minus three whereas two comma minus three this point is in the fourth quadrant

so this point two comma minus three lies somewhere here two comma minus three

so therefore the parabola facing upward is not what we are looking for we are looking for this parabola which faces downward

so we have to find the equation of this parabola which passes through two comma minus three

so we know the general equation of this parabola is $x^2 = -4a y$ right this

so we do not know what a is we need to find this a

so the equation is $x^2 = -4a y$ and then putting since two comma minus three lies on the parabola we have $2^2 = -4a(-3)$ and this gives $a = 1/3$

so therefore the equation of the required parabola is $x^2 = -4 \times \frac{1}{3} y$ that is $x^2 = -\frac{4}{3} y$ this parabola satisfies the given conditions that it is symmetric about the y axis and it passes through the point two comma minus three now let me give you some idea about finding the equation of parabola when the vertex is not at the origin or the directrix is not parallel to the coordinate axis

so in general how do we do that given the focus and the directrix

so let us say we have this line l is the directrix and the focus is at some point f is some (α, β) then how do we get the parabola

so this line l let us say this says equation $ax + by + c = 0$

so let the focus be $f(\alpha, \beta)$ and the equation of the directrix $ax + by + c = 0$ this is the general form of any straight line

so now if you see the vertex will be midpoint on the line segment which is perpendicular to this l and passes through focus and the axis is this will be the axis of the parabola and the vertex is this point say v and if you see in this case the parabola will be symmetric about this axis

so it will look something like this now how to find the equation

so if we take any general point $p(x, y)$ let $p(x, y)$ be an arbitrary point on the parabola then we have distance pf should be equal to the perpendicular distance pm

so then pf we know is $\sqrt{(x - \alpha)^2 + (y - \beta)^2}$ and pm is the perpendicular distance of the point $p(x, y)$ from the line $ax + by + c = 0$ this again you must have seen this formula we will get $pm = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

so we have $pf = pm$ this is equivalent to writing $\sqrt{(x - \alpha)^2 + (y - \beta)^2} = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$ if we square both sides this is equal to $(x - \alpha)^2 + (y - \beta)^2 = \frac{(ax + by + c)^2}{a^2 + b^2}$ this can be written as $(a^2 + b^2)(x^2 + y^2 - 2\alpha x - 2\beta y + \alpha^2 + \beta^2) = (ax + by + c)^2$

so this is $a^2 x^2 + b^2 y^2 + 2abxy + 2a^2 \alpha x + 2b^2 \beta y + c^2 = (ax + by + c)^2$ and this you can simplify further and write it in

this form

so this can be simplified and it can be written as $b^2 x^2 + a^2 y^2 - 2abxy - 2\alpha a^2 + b^2 + acx - 2\beta a^2 + b^2 + bcy + a^2 + b^2 = 0$

so you do not need to remember this formula you just need to remember what is the definition of a parabola

so if you are given equation of any line which is the directrix and you are given focus then you simply use the distance of the point to the focus is equal to the perpendicular distance to the line and then you obtain the equation of the parabola

so we will stop here in the next class we will learn about ellipse thank you