welcome to the fourth lecture on circles

so in the last lecture we had discussed the relation between a line and a circle we had also seen how to calculate the intercept made by a circle on both the x and the y axis

so in this lecture we are going to take a few ah problems on how to find the intercepts made by a circle on the axis and also the intercept made by a circle on any arbitrary straight line and after that we will start a new sub topic where we will derive the equation of tangent and normal to a circle at a given point and that will be followed by the definition of the power of a point ah with respect to a circle

so let us take this problem here it is said that the

so we have to find the equation of circle which touches the x axis at a distance of 3 units from the origin and further the circle has an intercept on the y axis of length two times square root of seven

so let the next square plus y square plus two g x plus 2 f i plus c equal to 0 with the general equation of the circle and it is said that the circle touches the x axis at a distance of 3 units from the origin

so it is mentioned that the circle touches the x axis

so by touches we mean that if this is the origin this is the x and this is the y axis what we mean to say is that the circle touches the x axis at a point which is three units from the origin

so one situation is where the circle touches

so this is the circle and it touches the x x axis at exactly one point which is three comma zero

so this is one possible case since

so what this basically implies is that the if we take this case of 3 comma 0 the other case could be that the circle touches the x axis at this other point minus three comma zero which is also on the x axis and which is also at a at a distance of three units from the origin here since this and this are both three

so if we for example if we take this case then the only point which lies both on the circle and the x axis is three comma zero now let us see what are those ah what are those points on the x axis which can also lie on this circle

so we can generally denote a point on the x axis by these coordinates the y coordinate is zero and let us see which are these points of this type or for what values of a will such a point lie on this circle

so that basically means is that this equation has to be satisfied with x equal to a y equal to zero

so what we get is this equation and since the circle touches the x axis at exactly one point it means that although this quadratic equation has two roots

so in general there are basically two different values of a which will satisfy this equation but then and those two values will basically correspond to two different points on the x axis which will lie on this circle but in this problem since we know that the circle touches the x axis at exactly one point it is implied that both the roots of this equation must be equal to three and that is only possible if the discriminant of this quadratic equation is zero which implies that g square is equal to c further since a equal to three is the root of this equation this should be satisfied with when we put a equal to three here

so then if we put a equal to three we get nine plus 6 g plus c equals 0 but c is g square

so that is the same as writing 9 plus 6 g plus g square equals zero and this is nothing but

so this is the left hand side is g plus three whole square equal to zero that implies that g is equal to minus three and therefore since c is g square c is equal to nine it is also said that the intercept of the same circle on the y

axis is of length two times root seven units now in the previous lecture we have already seen that the length of the intercept of a circle on the y axis is given by this equation which is equal to two into square root of f square minus nine because we have just seen that c is equal to nine in the previous line and this is given to be two times root seven from here it follows that f is equal to plus minus four

so essentially we get two different circles if we consider the case where the circle touches the x axis at this point three zero and that is very natural also
so the first circle c one corresponds to value of g which was minus three let us say f equal to minus four
so the center of the circle minus g minus f is equal to three comma four which is this point and the if you draw the circle it's going to be something like this
so this is the circle c one and if we take the instead of taking f equal to minus four if we take f equal to plus four then we get the other solution if you take f equal to plus four we get the center minus g minus f as three minus four and that is this and that is this circle in red and similarly if we had started instead of starting with 3 0 as the point on the x axis which touches the circle if we had started with minus three comma zero then we will again get two circles which are one would be here and the other would be like this
so totally there will be four circles four possible circles which are going to satisfy these two conditions in the question
so i will draw the other two circles over here also another one will be
so these two are the other two circles which touch the x axis at a distance of three units from the origin and whose intercept on the y axis is 2 times root 7
so we did not show the intercept but that should not be very difficult we just have to extend this y axis upwards and then we will see that if we if we draw this circle c one completely something like this and this c one is going to intersect the y axis at two points and this length you can check will be equal to two times square root of seven and the intercepts of the other three circles will also be the same in the last lecture we had derived the formula for finding the intercept made by a circle on both the axis but what if we are asked the following question
so we are given a circle and we are also given a straight line and we are asked to find this length which is the
so this length is called the intercept made by this circle on this straight line
so we had not derived any general formula like that but it is not very difficult to do
so and this is what we will illustrate through this next problem
so let us take another problem l one is a straight line passing through the origin and l two is this straight line x plus y equals one further it is said that the intercepts made by this circle is the same on both the straight lines it is of the same length
so the lengths are the lengths l one lengths of the intercepts are equal then then it is asked because the equation of the first line l one is not given
so it is asked as to which of this four equations could possibly be the equation of l one
so let us first find the value of the intercept
so so we have this circle and we have a straight line l two whose equation is x plus y equal to one
so it is something like maybe where to draw then this be the origin and clearly this straight line is the one which have drawn in blue and the circle if we see it has a center at x equal to half and y equal to minus three by two and the

radius of the circle is

so as we can also see this circle passes through the origin

so essentially this distance which is square root of 5 by 2 is going to be the radius of this circle

so i have drawn it approximately

so and as we can see this circle intersects the straight line l two at two points and this is the length of the intercept made by this circle on this line l two

so now the question is how do we find this length

so obviously the simple method is to find both these points of intersection now any point on this straight line is going to satisfy this

so suppose if we have a point x comma y then this x and y has to satisfy this equation because this point is on this straight line

so in in general any

so from here we see that y is equal to one minus x

so any any general point on the straight line will be given by the coordinates x n the x coordinate is is x and y coordinate is one minus x

so if we add both the coordinates we will get one

so all points of this type are basically on this straight line and then we are looking for such type of points which also lie on the circle because we are interested in finding these points of intersection

so the points of intersection will be those points which are also on the which are both on the straight line as well as on the circle and therefore any such point which is on the straight line and which is also on the circle must satisfy this equation with y equal to 1 minus x

so we write down that equation we get x square plus one minus x whole square minus x plus three into one minus x equals zero and if you simplify this we get x square minus three x plus two equal to zero which implies that either x is one or two and that basically ah corresponds to the two points of intersection

so so when x is one the y coordinate will be zero

so so one of the points of intersection will be one comma zero

so that corresponds to taking x equal to one

so as

so that we have got by solving this quadratic equation and the other possible value of x is two but then the point will be two n one minus two

so the coordinate of the other point of intersection will be two n minus one

so now that we have both the points of intersection the length of this intercept is very easily seen to be square root of two and then coming to the second part of the question it says that there is another straight line l 1 which passes through the origin

so there is another straight line which passes through the origin

so it could be it could be something like this

so this is the other straight line l one which passes through the origin and it is said that this straight line also makes an intercept on the circle and the length of the intercept should be the same as the intercept length of the intercept made by l two which is square root of two since this line passes through the origin the general equation of this line will be y equal to m x where m is the slope of this line and then as we did for the line l two we will also have to find the points of intersection of this line with the circle where a job is made little easy because we already know that the origin is one of the points of intersection because both the straight line and the circle pass through the origin and then let us try to see what the coordinates of the point of intersection is

so any point on this straight line will be of the type x comma m x because the

y coordinate is m times the x coordinate and for any such point to be also on this circle the equation of the circle must be satisfied with y equal to m x that is if we put y equal to m s we get this equation

so there are two values of x which satisfy this quadratic equation and those two values will basic respond basically correspond to the two points of intersection

so the two roots are x equal to zero and x equal to one minus three m by one plus m square

so the solution x equal to zero corresponds to the point zero comma zero the intersection point of intersection zero comma zero and x equal to one minus three m over one plus m square corresponds to this other point of intersection whose coordinates will be

so now we got the coordinates of both the points of intersection therefore l one which is y equal to m x intersect the circle at the origin and this other point and therefore the length of the intercept will be the length of the intercept made by the line l one on the circle c is going to be the distance between these two points and this should be equal to the intersect made by the line l two on c which is square root of two then hence we have the equation which is square root of two should be equal to the distance between these two points which will be on simplifying this we get

so there are actually two different values of m the intercept of l one will be square root of two

so from here m is either one or minus one by seven

so l one could be the equation of l one could be y equal to x or

so both these straight lines will have the same intercept of square root of two on the circle

so we see from all possible choices then we see that choice b corresponds to this y equal to x one two s b corresponds to the straight line y equal to x which is one of the possibilities that we found and also choice c corresponds to the other possibility that we found

so next let us derive equation of the tangent to a circle at a given point

so suppose we have this circle and we would like to find the equation of the tangent to the circle at a point having coordinates x one y one

so we have this circle here the center as minus g minus f then we have a point p which is on the circle having coordinates x one y one and we would like to find the equation of the straight line which is the tangent of the circle at this point p

so this is the tangent and we want to derive the equation of this tangent now suppose we have a point x y on this tangent then the slope of the tangent is y minus y one divided by x minus x one the slope of the line joining this point p to the center of the circle

so the slope of this line is y one minus minus f divided by x one minus minus g which is equal to y one plus f divided by x one plus g

so this is the slope of the line this slope of o p now we know that the tangent at any point p on the circle makes 90 degrees with the line segment joining that point p to the center of the circle

so essentially this line op and the tangent are perpendicular to each other and therefore the product of the slopes of this line o p line segment o p and the tangent should be minus one

so the slope of the tangent times the slope of the lino p is minus one which implies that then a little bit of simplification gives us but since the point p lies on the circle c the coordinates of this point p must also satisfy the equation of the circle and therefore this must be true or

so essentially we can replace this right hand side in the equation of the

tangent by this quantity and therefore we get the equation of the tangent to be which can be simplified to
 so this is the equation of the tangent to a circle having the general equation x square plus y square plus two g x plus two f y plus c equals zero at the point p which is x one y one
 so just as we had derived the equation of tangent to a circle at a given point p we next find the derive the equation of the normal to a given circle at a point on the circle
 so suppose we have this circle and this is the center and we have a point p on the circumference
 so the normal is given by the line joining the center to this point p which is this line
 so now the objective is that if we are given the equation of the circle and if we are given the coordinates of this point p which is on the circumference of the circle then the objective is to find the equation of this normal
 so suppose that the equation of the circle is this with the center at minus g minus f and suppose there is a point p having coordinates x two y two which is on the which is on the circumference of the circle then our objective is to find the equation of this normal line or equation of the then our objective is to find the equation of the normal to the circle at the given point p now it is clear that the slope of the normal slope of the normal at p equals y two minus minus f divided by x two minus minus g now we have another point where we have any general point on this normal say x y then the slope or
 so suppose we have this point this is any general point on this normal line then this slope must also be equal to the slope of the line segment between any point on the normal line and the center of the circle
 so this slope of the line between the center and the point p must be the same as the slope of the line between any point on this normal line let us say q and the center of the circle because essentially both these lines are the same line they are basically the normal therefore this must be equal to the slope of the line segment o oq which is equal to y minus minus f over x minus minus g and if we simplify it further we get which implies that is
 so this is the equation of the normal to the circle at that point p next we derive the length of the tangent to a given circle from a given point
 so let us say we have a circle here whose equation is this and this is the center of the circle and suppose we are given a point p having coordinates x one y one and then we are asked to find the length of this tangent p t
 so this length p t where p t is a tangent to this circle at this point t
 so clearly this is 90 degrees and we are asked to find this point this we are asked to find this length we know that this length o t is the radius of the given circle which is g square plus which is square root of g square plus f square minus c and this distance can also be calculated it is given by ops square root of this expression which is square root of x square root of x one plus g whole square plus y one plus f whole square we realize that opt is a right angle triangle and therefore from the pythagoras theorem we know that o p square is o t square plus p t square that is p t equals square root of o p square minus o t square which is
 so o p square can be found from this equation
 so p square will be x one plus g whole square plus y one plus f whole square and ot square is the square of the radius which is d square plus f square minus c
 so at the end this is the expression of the length of the tangent p t from a given point p to this circle whose equation is this
 so essentially here we if will be usually given the the circle of the the the

equation of this circle will be given to us

so this coefficients g f and c will be known similarly the coordinates of this point p will be given to us they will also be known and then will be asked to find this length the length of this tangent

so we can readily use this formula where all these

so these x one y one is known and g f and c are also known we in the next lecture we will define what is meant by the power of a point with respect to a circle and we will also discuss some problems solve some problems related to tangent to a circle at a given point and also normal to a circle at a given point on the circle thank you you