welcome to the third lecture on circles in the last lecture we concluded with the method of deciding whether a point is inside a circle or not in this lecture we will be moving ahead and will see how we can find out if a given line and a given circle intersect and whether they intersect at two points or one point or they do not intersect similarly we will see how to find out the intercept of a circle on both the axis followed by some problems and if we have time in this lecture then we will also cover the method of finding the equation of the tangent and normal to a circle at a given point

so just little bit of recap on what we concluded with in the last lecture

so suppose if we have a circle c whose equation is x square plus y square plus two g x plus two f y plus c equals zero

so we know that this defines a circle let us say over here with center at minus g minus f and radius r equal to square root of g square plus f square minus c now suppose we are given a point p whose coordinates are a and b the question now is how do we check whether this point lies in this inside the this circle or does it lie outside or does it lie exactly on the circumference of this circle

so geometrically if we see this figure here one can easily see that if the point p lies outside the circle then the distance between this point p and the center of the circle

so this distance has to be greater than the radius of the circle on the other hand if the point lies inside then this distance between this point and the center is going to be less than the radius of the circle and if the distance between this point and the center is exactly equal to the radius of the circle then obviously it is this point p is on the circumference of the circle

so this is how we can find out

so essentially what then we have to do is find this distance o p

so o p is equal to square root of minus g minus a whole square plus minus f minus b whole square which equals

so now we say that if o p is less than r then it is it follows that p lies inside this circle see if is greater than r then it follows that p lies outside the circle and if o p is equal to r then p lies on the circle now using the expression of o p from the previous slide

so this is o p and we are checking to find if it is less than the radius r of the circle which is d square plus f square minus c

so if you simplify this condition by squaring both the sides then what we will get at the end is that this condition is equivalent to a square plus b square plus two a g plus two f b plus c is negative and note that this left hand side is nothing but this quadratic form with x and y replaced by a and b respectively

so we just have to put the coordinates of this point p into this quadratic form and then check if what we get is less than zero or greater than zero or equal to zero if it is less than zero then the point p lies inside the circle if equal to zero then its on the circle and if this is greater than zero then its outside the circle for example suppose if we are given this circle c which is whose equation is x square plus y square plus six x minus eight y plus four equals zero

so this circle

so two g is six

so g is three

so the center is the x coordinate is of the center the x coordinate of the center is minus three similarly the y coordinate of the centre will be four because f is minus four and the y coordinate is minus f which is equal to four and the radius will be square root of g square plus f square

so that is going to be 25 minus c which is four

so the radius is square root of twenty one and suppose now we are given a point

two comma minus one and we are asked to find if this point lies inside the circle or outside the circle or on the circle

so we basically have to find this distance o p which will be equal to square root of minus three minus two which is minus five whole square plus four minus minus one which is five whole square

so this is square root of 50 which is greater than the radius and therefore it is not clear that this point p lies outside this circle next suppose that we are given a line and a circle

so suppose we are given this circle having this equation and we are given a straight line l whose equation is y is equal to m x plus d

so now the very first question that comes to mind is if we see geometrically then suppose if we have a circle

so this is a circle here and then a straight line could either be like this

so this is one case where the line does not intersect any point on the circle another case could be where the straight line cuts through the circle

so it intersects the circle at exactly two points and the third case could be where the straight line is actually a tangent to the circle

so this is the third case

so in this case the straight line only touches exactly one point on the circle

so how do we know if we are only given these two equations how do we find this out whether which of these cases is true of course one easy way is that we plot the circle and the straight line but that might be time consuming and of course it is prone to errors the other way is to actually try to solve this system of equations

so since we are given that y is equal to m x plus d let us use this fact in the first equation because suppose that lets say that the circle and the straight line intersect at some point

so let the circle c and the straight line l intersect at some point p those coordinates are a and b

so what it means is that this point lies both on the circle and it also lies on the straight line and therefore the coordinates of this point must satisfy both the equations

so what this implies is that since this point should lie on the circle this equation should be satisfied with x equal to a and y equal to b

so this equation should be satisfied now since this point is also on this straight line this equation should also be satisfied with x equal to a and y equal to b and hence we also have this equation and now what we do is we substitute b equal to m a plus d into this first equation

so in this first equation wherever we have b we replace it with m a plus d we then get the following equation and if we simplify it we get we just have to open up this expression and this here

so if we see this equation it is a second degree equation in a its a quadratic equation and therefore they there can be at most two real roots which are two distinct real roots

so if we get two distinct real roots for a

so if we get two if you get two distinct real roots in this equation then what does that imply that implies that there are two different real values of a

so for example we could have these values a naught and a one and then corresponding to a naught since b is equal to m a plus d

so when a is a naught b will b will be equal to m a naught plus d

so then we have this as one possible point of intersection

so this could be one point of intersection

so these two are the distinct roots of ah this equation here the other point of intersection will be corresponding to a one

so when the x coordinate is a one the y coordinate will be m a one plus d
so then when we have two distinct real roots then we will have two different points where the straight line is going to intersect the circle
so that will correspond to this green through this case where we have the green line cutting this circle at two different points two distinct points another case could be where in this equation we have equal real roots we have equal real roots
so basically what it means is that we just have one root which is repeated
so let the the root of this equation for this case b denoted by a naught therefore in this case where both the roots are equal to a naught we will only have one point of intersection and that point will be a naught comma m a naught plus d and this case is basically the case where the straight line is such that it is tangent to the circle at some point
so this is the case where we only have this is the case where we have equal roots to this equation and of course the third possibility is that both routes are complex
so there is no real route since we have no real route it basically what it means is that there is no real value of a for which this thing is equal to zero what that means is that the circle and the straight line never intersect in this figure that this thing corresponds to the case of this red line as you can see the red line never intersects this circle and further it is very easy to also see the cases where just by looking at this coefficients and in fact just by looking at the discriminant of this quadratic equation one can figure out which one of these three cases is applicable let us take a little example here to just to illustrate this particular idea let us consider the circle x square plus y square plus two x minus four y minus four equals zero
so as you can see the center of the circle is at minus one comma two the radius is three units let these be the coordinate axis and we have the origin here
so the center is at minus one two
so this is the center here
so this point is minus one and two and the radius is three units
so so the circle is approximately something like this and now suppose if we are given a straight line x plus y equal to minus five
so this straight line is going to intersect the x axis at this point minus 5 0 and it is also going to pass through this point on the y axis
so the straight line is something like this
so in this case obviously we can geometrically by drawing it we can see that there is no point of intersection between the circle and the line but how do we see that mathematically well we can just proceed exactly as we did now if if there is a point having coordinates x and y which lies both on this circle and this line then both this equation should be satisfied by the coordinates of that point x y
so suppose we have a point x y now from here we know that is equal to since this point als lies on the straight line x plus y equal to minus 5 the y coordinate should be equal to minus of x plus 5 where x is the x coordinate of this point using this fact in this equation here because this point of intersection will also satisfy this equation of the circle
so using y equal to this using y equal to minus of x plus five over here we get and if we simplify this we get
so this is what we get and the roots of this equation are sixteen square is two fifty six minus eight times forty eight times forty one is three twenty eight
so as we can see both the solutions are complex valued because the discriminant is negative and since both the roots are complex both roots are complex it follows that there is no point of intersection and that is what we exactly saw

in this figure here

so similarly the student should also try to take some example of some other line which is actually passing through this circle

so its you know some some line which touches both let us say two points on the circle and then try to show it using the analysis which we covered to try to get the second degree equation and then check that there are actually two distinct real solutions to that second degree equation

so the next topic is how to find out the intercepts and by a circle on the coordinate axis

so let us see what we mean by intercept suppose this this is the x axis this is the y axis we have the origin here now suppose that we have a circle whose centre is at one comma one and has a radius of three units

so circle is something like this

so as we can see this circle is cutting or intersecting with the x axis at these two points and it is intersecting with the y axis at these two points

so this distance or this length here

so this length between the two points where the circle intersects with the x axis

so this length is called the intercept made by the circle on the x axis

so this is the x intercept or or the intercept made by the circle on the x axis

so that is this length and similarly the length of this part of the y axis between the two points where the circle intercept with the weather circle intersects with the y axis

so the distance between those two points is basically the intercept made by the circle on the y axis or in short by intercept now the question is if we are given the equation of a circle how do we find out the value of these intercepts

so next we will discuss a method where if we are given the equation of the circle then we should be able to find out expression of the intercept made by this circle on the x and y axis

so suppose this is your x this is our x axis and

so this is the x axis this is the y axis we have the origin over here and suppose if

so suppose this is a circle over here with let us say this being the center

so to find the x intercept to find the x intercept will have to first find the coordinates of these two points now this this point where the circle is intersecting the x axis will obviously have its y coordinate to be equal to zero

so these two points are of the form a comma zero

so we have a point of this type which is on the x axis then this point is also on the circle as we can see

so it must the coordinates of this point must satisfy the equation of the circle that is a square plus zero square is zero plus two g a plus c equals zero as we can see again this is a quadratic equation and the two roots are given by minus g plus minus square root of g square minus c

so one word of caution is that i have been using a i would say an upper case c to denote a circle and this lower case c is for the constant term on the left hand side of the circles equation

so this is the lower case c

so this is also the lowercase c now we can clearly see from here that if g square is less than this c

so if g square is less than c then there are no real roots of this equation then since there are no real roots of this equation it means that there is no point of the form a comma 0 which lies on this circle and that what it essentially means is that the circle never intersects the x axis

so if g square is less than c then the circle does not intersect the x axis in

which case the intercept is there is no intercept essentially
 so this is the case of no no intercept with x axis the other case is if g square is equal to c
 so this particular case could ah
 so this figure does not correspond to this case because in this figure we see that there are two points where the because in this figure we see that the circle is intersecting the x axis at two points now if g square is equal to c then this is zero and both the roots are equal to minus g
 so if g square is equal to c then we have two equal roots which implies that there is exactly one point where the circle intersects with the x axis but basically there is only there is exactly one point which lies both on the x axis as well as on the circle and that point obviously then
 so therefore if g square is equal to c then the circle touches the x axis at the point
 so the the value of a is minus g
 so the point will be minus g comma zero and in this case since both the roots are equal the value of the intercept will be zero
 so if g square is equal to c the intercept of this circle with the x axis is zero and the third case is when g square is greater than c
 so if d square is greater than c then we have two distinct roots
 so one of the roots
 so we will have two distinct roots of this of this equation
 so the two roots will be minus g plus square root of g square minus c the other root is minus g minus d square minus c
 so for this case where when g square is greater than c this circle intersects the x axis at two distinct points which are
 so this is one of the points and the other point is this and then the value of the intercept made by the circle on the x axis will be the distance between these two points intercept made by the circle on the x axis equal to on the x axis will be equal to
 so the distance between these two points will be two times square root of g square minus c
 so this is the expression for the intercept made by the circle on the x axis and similarly it is left as an exercise for due to show that if f square is greater than c then the intercept made by the circle on the y axis is two times square root of f square minus c if f square is equal to c then the intercept made by the circle on the y axis is zero and if f square is less than c then
 so in this case the circle basically touches the y axis at the point and the coordinates of the and the coordinates of that point will be zero comma minus f and in this third case if f square is less than c then the circle does not intersect with the y axis next let us look at some problems
 so here is one problem where it is said that we have a circle whose equation is this x square plus y square minus two x minus six y plus six equal to zero
 so the center of this circle is at one comma three
 so this central is this circle is drawn here in green color and the radius of this circle is two units and then it is said that one of the diameters of this green circle which is drawn in green
 so lets consider this diameter p q p to q
 so this diameter is actually record to another circle which is drawn partially in red and this red circle is centered at the point two comma one the question is asking us to find out this radius of this red circle
 so it is not very tough because it is given that since this green line segment is a diameter of this green circle
 so let us say that the center of the green circle is o which is here um the

coordinates is one are the coordinates of o r one and three and since this diameter of this green circle is a horde of the red circle it is clear that will have two points p and q which are the end points of the diameter but then this p and q will also lie on the red circle because p q is also a chord of this other red circle let the center of this red circle be this point a whose coordinate is given to be two comma one this coordinate is given to be two comma one and we are asked to find this distance a p which is will be the which will be the radius of this red circle

so now if we connect this point a to this point o then we know that this angle poa is going to be 90 degrees that is because we connect a to q then we see that these two triangles p o a and q o a are congruent because this side from this side

so this side of poa is equal to this side of q a q o a because both these lengths are radius of the red circle and then between these two triangles this is this side is common further because o is the center of the green circle p o of triangle p o a is equal to q o of triangle q a because p ah because this is the center of the diameter and since these two triangles are congruent these two angles will be equal and therefore since p oq is a straight line both these angles will be 90 degrees now if you look at this right angle triangle poa

so if we zoom it and show it here let us say this is a which is two comma one this is o one comma three and this is p

so we are asked to find this distance now p o is the radius of the green circle and from this equation we can see that from this equation we can see that the radius of the green circle is two units and therefore this distance o p is equal to two units this distance o a can be easily calculated because we have the coordinates for both o and a and that is equal to square root of five and then it is very easy to see that if we apply the pythagoras theorem

so this triangle oap will get this length to be units and therefore the radius of the other circle shown in red is 3 units

so with that we will finish this lecture in this in the next lecture we will see some more problems and will also see how to derive the equation of the tangent and normal to a circle at a given point thank you you