welcome to the first lecture on chronic sections in the next series of lectures will be discussing the properties and equations of many different conic sanctions like circles parabolas ellipses hyperbolas
 so in this lecture we will start with circles
 so let us see what the circle means
 so let us say that we have a fixed point o as can be seen here
 so circle is said to be the set of all points that are at the same distance from a certain fixed point
 so let us say that this fixed distance is r
 so can we find one point let us say let us try to find the some point which is at a distance r from this fixed point o
 so again when we talk of distance we are talking of the two dimensional plane plane
 so we are talking of a surface like this
 so to find a point which is at a distance r from this fixed point let us first draw a vertical line and a horizontal line which intersect at this fixed point o now to get another point which is at a distance of lets say some fixed distance r from this point o one way is that we consider any straight line passing from this point o
 so let us say let us draw any straight line
 so for example lets consider this straight line which passes from o obviously this straight line is at some angle theta with respect to this horizontal line and then it is not very difficult to get a point r
 so we can just think of because if we if we start at this point o and if we just walk along this line
 so we can either go in this direction on this direction if we go in this direction
 so for example suppose if we move from here to here then we have covered some distance
 so we can keep on increasing this distance till we reach a certain point let us say p which is at that fixed distance r that we initially wanted to have
 so let us say that
 so starting from o we go in this direction on this green straight line and when we as we move further away from o the distance between where we are and this fixed point o will increase until we reach a point p such that this distance is r now instead of going in this direction we could have gone in the other direction and if we go in the other direction then obviously we will get some other point lets say this point q
 so this distance is also r
 so now we have two points p and q which are at a distance of r from this fixed point o we find more points like this of course we can find its not very difficult because we can draw another straight line which again passes through but at some other angle instead of theta it could be some other angle right
 so for example we have this straight line and it is it is at some other angle with respect to the horizontal line again on this straight line there will definitely exist some point i can call it
 so i will call this p one q one for the first line and then starting from o if i go in this direction on the second line then there will definitely exist some point let us say p two such that o p two is equal to r and similarly on this side also will get a point q two such that o q two is also r
 so now we have four points p one p two q one q two which are at the same distance r from this fixed point o and it turns out that since we can make infinitely many straight lines because a straight line in this case
 so we are only considering those straight lines which are passing through this

fixed point o but there are infinitely many because we just need to change the angle theta between the horizontal blue line and the straight line that we are drawing as theta changes continuously it takes all real values from 0 to 360 degrees and therefore corresponding to any possible angle theta between zero and three sixty corresponding to any such angle that we choose there will exist a straight line for example if we take theta equal to zero degree then we have this line itself this this horizontal line itself and on the horizontal line let us say we have this point p such that o p is equal to r and this point q such that oq is also r if we take theta equal to ninety degrees then we are on this vertical line and then we get let us say this point here p three q three
 so this is also r this is also r and then we can take theta equal to one twenty any possible angle
 so since there are infinitely many angles between zero and three sixty what it means is that we have infinitely many points which are at the same distance r from this fixed point o and now if i join if i join all these infinitely many points which i am showing with this dotted blue line over here now let us say these are those infinitely many points right
 so if i just plot all those points and if i connect them we get our circle
 so in short a circle is a collection of points which are at a fixed distance from a given fixed point in this case the fixed point is o the fixed distance is r as an example as an example the fixed point o here could be the origin in our coordinate system this point o could be zero comma zero and r could be in this case let us say three and a half units also this fixed point o is called the is called the center of the circle
 so the fixed point o is the center of the circle and this fixed distance is called the radius of the circle now if we take any two points on the circle
 so let us say this point here and any other point let us say this one here or let us say not that lets say we take this point and this point
 so we take let us say p 4 n q4 and let us say
 so these two points are they belong to the circle they are on the circle and we connect them by a line segment then such a line segment is called a chord is called a horde
 so chord is any line segment joining any two
 so the chord is a line segment joining any two points on the circle
 so we do have a special chord where suppose if we for example if we take r case
 so suppose we take p one and q one
 so we say p one and q one both of them they are on the circle and therefore this line segment between q one and p one is a horde but its a special horde is a special card because it passes through the center of the circle therefore such a and such a horde is given a special name called as diameter is called a diameter
 so in this case p one q one is a diameter similarly p two q two is a diameter also and as you can see there are infinitely many diameters now what is the length of this ah diameter this line segment if you see any diameter then it is composed of two radius for example in the case of p one q one because a diameter passes through the center this length p one q one is p one o plus oq one because p one q one is a straight line and we know that both p one o n o q one are equal to r then therefore p one q one is two times r
 so from that we see that diameter is always twice the radius
 so so far we have seen how to precisely define or what we basically mean by a circle but in maths we have to always express things formally in terms of equations just as we define the equation of a straight line
 so if we go back to equation of a straight line
 so suppose this is our these are our coordinate axis this point is the origin

and suppose that we have this straight line

so this straight line is essentially defined by all those

so how do we how did we formally define the straight line

so straight line is nothing but the collection of all the points that lie on this straight line and then we said that lets say we have any arbitrary point here whose x coordinate will lets say x and whose y coordinate is let us say y

so we have a point p here now then we went to find out then we went on finding out what property this x and y should satisfy

so that it will lie on this straight line now if you see here this straight line passes through this point x zero y one which is this point further this angle here is forty five degrees that is because the slope of this line is one and that is very easy to see because we see that this straight line also passes through this other point here whose coordinates are x one y two then therefore the slope of the line is two minus one

so y two minus y one divided by x two minus x one which is one slope equal to one means that the angle between this straight line and the horizontal axis is 45 degrees now since the slope is equal to 1 it now becomes very easy because if i try to if we have this point x y and if it is said that it lies on the straight line the same straight line then if i calculate the slope of this line segment between p and zero one then the slope of this line segment should also be one itself

so it will be one if and only if this point x y belongs on the straight line

so the slope of this line segment is y minus one divided by x minus zero and the slope should be one because we calculated that the slope of this straight line is equal to one

so we that is how we get this equation and then if we simplify it we get y equal to x plus one then we say that this straight line is a collection of all those points for which the y coordinate one more than the x coordinate for example if somebody give me a point gives me a point six comma eight x is six y is eight i can immediately check whether this point belongs to this straight line or not because and one can see that it does not belong because y is eight and x plus one is seven and eight is not seven

so i just put ah y equal to eight and x equal to six and i see that this equation is not satisfied by this point and therefore this point does not lie on this straight line and then its very easy to write down ah this

so essentially what we have is a characterization of the entire straight line

so now for the case of circle or our objective is to get a similar characterization or basically some kind of a rule like this which the coordinates of any point on the circle must satisfy let us go back and lets say that this is our con these are the coordinate axes next axis y axis suppose we have a circle whose center has coordinates h comma k

so the x coordinate of the center is h the y coordinate is k and let us say that the radius is r now suppose if we say that there is a point p whose coordinates are x and y and let us say that p lies on this circle having center s k n radius r can we now form a rule for which this x and y must satisfy

so that given any point just like in the case of the straight line even any point if we are knowing its coordinates if we know that this rule then we should be able to check and tell whether that point is going to lie on the circle or not

so let us first connect the center with the point p now since the radius is r the length of this line segment o p will be r let us draw a horizontal line passing through the center row

so obviously the x axis and this line are parallel now and we make a vertical line passing through point p obviously this vertical line and the y axis are

parallel and because the x and y axis are at ninety degrees it is easy to see that see that this angle is also ninety degree let us denote this point of intersection of these two lines which i constructed by q
 so now what we have is a right angle triangle here
 so we have a right angle triangle opq now from the pythagoras theorem then follows that o p square is equal to o k square o p square is equal to o q square plus p q square but o p square is r square how much is o q square if we see because the x coordinate of this point p it is x this horizontal distance is x the x coordinate of the centre o is h therefore this distance is as and therefore it is easy to see that o q square will be x minus h the whole square and similarly this vertical distance is y which is the y coordinate of this point p this vertical distance is k which is the y coordinate of the center o and therefore p q square is y minus k whole square
 so this is the rule that we were talking about
 so here just as for the case of straight lines even in the case of circles we get a general rule which the coordinates of any point on the circles the which the coordinate of any point on the circle must satisfy for example let us say that we have a circle whose center is the point one comma two and radius is five units
 so let us say we have a point x y which is equal to minus one comma three and now we have to check whether this point minus one comma three lies on this circle
 so the circle is specified by the center one comma two and the radius five
 so it is not very difficult to check
 so in this case this is r this is h and this is k this is x this is y
 so first we just try to put these values here and check if we get an equality the left hand side of course is five square which is twenty five on the right hand side we have x minus h
 so minus one minus one which is minus two
 so minus two square is four y minus k y minus k three minus two
 so one three minus two whole square is one
 so on the right hand side we have five on the left hand side we have twenty five and they are not equal hence this point minus one comma three does not lie on the circle having center at one comma two and radius five this particular ah form of equation of a circle is called the center radius form and it is quite clear why we have that name center radius because just by looking at the this expression we see that the center is h comma k and the radius is r which is on the left hand side for example if ah if i give if i tell that
 so let us consider this equation of some circle in the
 so this equation is an equation of a circle in the center radius form and just by looking at it we can very easily say that the center of the circle is three comma seven and the radius is
 so this is r square
 so r is 8 radius is it but this is not the only way of specifying the equation of a circle there are some other ways
 so one such way is known as the parametric form
 so what we have in the parametric form is that let us again draw the coordinate axis this of course is the origin and suppose we again have suppose we again have a circle with center at h comma k and radius r
 so let us say this is the center s comma k radius is r now we need to go back to that construction of all the points of on a circle based on drawing a straight line drawing any straight line passing through the center now if we draw a straight line let us say this line here
 so this line which of course passes through the center of the circle h comma k

and let this straight line have make an angle of theta with the x axis
 so i draw a dotted line here parallel to the x axis and this straight line makes an angle of theta
 so always measure theta in the anti clockwise direction
 so if i go like this i have positive theta if i go like this i have negative theta let us say that here have a point p whose x and y coordinates we have to find or let us say if you remember the way we found this point p given this radius r was that we started moving on the straight line in this direction and we moved until we reached this point p such that o p is equal to r
 so that is how we started finding points on the circle if we go back to the first or second slide of today's lecture
 so this is r
 so if this is r and if i also draw a line parallel to the y axis then it is clear that these two lines constructed by me will meet at some point q over here at an angle of ninety degrees and then i have this little right angle triangle here o p q where we know that o p is equal to r but because o p is r from our knowledge on the trigonometric ratios we know that oq will be equal to r cos theta which is here
 so this is r cos theta and q p will be r sin theta which is this now from this we should be able to find out the two coordinates of this point here which lies on the circle
 so now what we are going to do is we are going to express the x and y coordinates of this point p which lies on the circle in terms of r and this angle theta and that is not very difficult because the x coordinate of this point p is equal to this distance
 so let us say that we have p x y
 so the x coordinate is this much this is x but this x is nothing but
 so in fact this is also x but this x is nothing but this distance plus this therefore x is this distance is equal to the x coordinate of the center which is h plus and this distance oq we have already found it to be r cos theta similarly the y coordinate of this point p which is this distance here this is y is nothing but this plus q p but this is nothing but equal to the y coordinate of the center o which is k
 so y is equal to this much which is k plus q p but q p is already r sin theta
 so indeed we now have expression for the x and y coordinate of this point p in terms of the coordinates of the centre of the circle the radius r of the circle and the angle theta between the straight line and the horizontal x axis but can this be formed into some kind of a rule just as we had in the center radius formulation and it is not very difficult we can define the entire circle to be all those points x and y where x is h plus r cos theta y is k plus r sin theta indeed if we go back we can see that x minus h whole square is
 so x minus h is r cos theta square of that is r square cos square theta y minus k is r sin theta square of that is r square sin square theta now we add these two then what we get is r square cos square theta plus r square sin sin square theta which is nothing but r square and which is nothing but the rule which we just saw now in this when we were discussing the center radius form
 so if we add these two we end up getting this which means that any point p on the circle can firstly be written the coordinates of any point p on the circle can always be written like this where theta is some angle between zero and two pi further if for any theta we choose between zero and to power two pi zero to two pi if we form a point having x and y coordinates as h plus r cos theta n the y coordinate has k plus i sin theta then such a point will obviously lie on the circle and that is what we proved here
 so with both these arguments what we can conclude is that the circle is nothing

but the set of all points x and y where the x and the y coordinate are essentially h plus r cos theta n k plus r sin theta theta belonging to the interval zero to two pi

so as we and it is very clear that as we vary theta from 0 to 2 pi we are essentially moving

so when theta equal to 0 our point is somewhere here and then we start moving like this and then if you further increase theta will reach somewhere here

so so if we reach here then we are basically having theta equal to 90 degrees because in that case this will be the straight line and this angle will be now 90 degrees and then we can go beyond theta 90 and then complete an entire revolution

so when we express the circle in this form it is in terms of a parameter theta and therefore this type of equation of a circle is known as the parametric point parametric equation of a circle

so essentially what we have said is that we have said two things if a point x comma y belongs to a circle having center h k and radius r then x must be then next must be equal to s plus r cos theta and y must be equal to k plus r sin theta for some theta belonging to zero two pi

so there should exist some theta such that x is this and y is k plus r sin theta

so this is one thing that we have said the other thing that we have said is that for any angle theta belonging to zero to two pi the point s plus r cos theta and k plus r sin theta

so this particular point belongs to the circle having center thats comma k and radius r

so these two things we have shown if we go back to our center radius form and if we recollect it it was of this form but if you open up these squares we get x square minus two h x plus h square plus y square minus two k y plus k square equals r square or x square plus y square minus two h x minus two k y plus plus k square plus s square minus r square equals zero

so starting from the center radius form we eventually get this and our claim is that the equation of a circle can in general be written in this type of form which is x square plus y square

so the coefficient of both x square and y square will be the same then that is because ah that is starting basically starting from here and then we will have a term containing x only multiplied with some coefficient

so here just like we had this term then another term linear in y

so this is another term which is y times some constant plus a constant c in this case in the case of the center radius form c is this 2 f is minus 2 k 2 g is minus two h

so this is the most general form of a circle this is basically a second degree equation in x and y but with this pair with this with ah some special properties that the coefficient of x square and y square are the same and secondly that there is no term which contains the term there is no term here which is some constant times x y because in a general second degree equation as general second degree equation is of this form a x square plus b y square plus c x y plus two d x plus two e y plus f equals zero

so this is the form of a

so this is the form of a general second degree equation but here if we see the coefficient of x one and y square need not be the same in general plus this coefficient of x y need not be zero but if in this second general second degree equation if we have a equal to b and c equal to zero if these two conditions are satisfied then what we get is an equation of a circle because if we if we put a equal to b and c equal to zero here then we will get a x square plus a y square

because b is a and then c x y will disappear because c 0 two d x plus and if we divide everything by a we get and this obviously is of the form x square plus y square plus two g x plus two f y plus c is zero

 so that is why this is the most general form more general form of an equation of a circle now when we have a circle in this form how do we find the center and radius well we can simplify this

 so we bring these two terms together y square and two f y together and then we complete this you put a plus g square and a minus g square you put a plus f square under minus f square

 so what we get is x plus g whole square plus y plus f whole square is g square plus f square minus c and this reminds us of the form of the center radius form which is x minus h whole square plus y minus k whole square equals r square now this obviously represents a circle only if if and only if this right hand is r square r square is always non negative and therefore this represents a circle if and only if this is non negative

 so this condition must always hold if this is negative if we get an equation of and if we calculate d square plus f square minus c to be negative then this is not an equation of a circle but if it is non negative then it is clear that it is an equation of a circle because this is the same as this over here in which case the radius is simply because r square is this therefore the radius is square root of g square plus f square minus c and the center is h comma k but h is minus g because this and this are equal and k is minus f and therefore we can conclude by saying that the general equation of this form is a circle if and only if d square plus f square minus c is non negative um in which case the radius of the circle is r equal to square root of d square plus f square minus c that is from here because this r square and this term have to be the same and the center of the circle is in the center of the circle is minus g comma minus f that again we can see from here because g has to be equal to minus h and therefore h is minus g similarly k has to be minus f hence we conclude this lecture with the the understanding of this general form of the equation of a circle where we saw that this represents a circle if and only if g square plus f square minus c is non negative in this case this is the radius of the circle and minus g comma minus f is the center of the circle

 so essentially if we are given any equation of this form we should be able to find first we should be able to check whether it is a circle or not and then we should be able to find the radius and the center of the circle

 so we will take more on this and other methods other type of forms of equations of a circle in the next class you