

okay friends today we are going to discuss about some miscellaneous problem on straight lines we have already discussed various concept on a straight line now we shall discuss some miscellaneous problem

so first problem is find the equation of line which passes through the point  $(-4, 3)$  and the portion of the line intercepted between the axis is divided internally in the ratio five is to three by the this point

so here we have figure of this problem we have to find the equation of this line  $AB$  which passing through this point  $P(-4, 3)$  and this point  $P$  divides this  $AB$  in the ratio five is to three

so first of all we find the point  $B$  and  $A$  by using section formula we know that if this point  $P$  divide this  $AB$  in internally then  $\frac{-4 - a}{3 - b} = \frac{5}{5 + 3}$

so this is  $3 - a = \frac{5}{8}(b + 4)$

so this implies  $3 - a = \frac{5}{8}(b + 4)$  this implies  $a = 3 - \frac{5}{8}(b + 4)$

so this implies  $5b = 24 - 4a$  this implies  $b = \frac{24 - 4a}{5}$  we get the value of  $a$  or we get the coordinate of this two point  $A$  and  $B$

so  $a = -32/3$  and  $b = 24/5$ .

so equation of line equation of line  $AB$   $x \cdot \frac{-32}{3} + y \cdot \frac{24}{5} = 1$

so this implies  $3x - 32y + 24x = 5$  this implies  $27x - 32y = 5$

so this implies  $9x - 20y = 5/3$  this implies  $9x - 20y + 15 = 0$  is a required equation of line

so in this way we can find any line which passing through or which make intercept  $x$  intercept and  $y$  intercept and this line having some point given then we can use section formula and find the equation of line now we have another problem that is find the values of  $k$  for which the line  $kx - 3x - 4y + k^2 - 7k + 6 = 0$  is first parallel to  $x$  axis second parallel to  $y$  axis and third passes through origin when line parallel to  $x$  axis

so parallel to  $x$  axis means what the coefficient of  $y$  equal to zero

so parallel to  $x$  axis first case parallel to  $x$  axis implies coefficient of  $y$  equal to  $0$  this implies  $k^2 - 7k + 6 = 0$  this implies  $k = 6$  or  $k = 1$  question is  $y$  coefficient of  $x$  equal to  $0$  because when line parallel to  $x$  axis then is slope equal to zero that's why coefficient of  $x$  equal to zero now second parallel to  $y$  axis parallel to  $y$  axis what does it mean parallel to axis means slope of that line equal to infinity or you can say not defined it means the coefficient of  $x$  must be equal to zero

so this implies coefficient of  $x$  equal to zero

so coefficient of  $x$  equal to zero means  $k - 3 = 0$  this implies  $k = 3$

so this implies  $k = 3$  or  $k = 6$  third passes through origin passes through or is it

so any line passes through origin that line must be like this means its  $c = 0$  means when you take  $y$  is equal to  $m \cdot x + c$

so if line passes through origin

so then  $c$  must be equal to zero it means

so  $c = 0$  implies  $k^2 - 7k + 6 = 0$  this implies  $k = 6$  or  $k = 1$

square minus six k minus k plus six equal to zero

so this implies  $k - 6 = 0$

so this imply  $k - 1 = k - 6 = 0$  this implies  $k = 1$  or  $6$

so for these two value line is passing through origin

so in this way we can discuss various condition now we have another problem that is find the equation of one of the sides of an isosceles right triangle which hypotenuse is given by  $3x + 4y - 4 = 0$  and the opposite vertex of the hypotenuse is  $(2, 2)$  we have to find equation of one of this side

so what is given we have we have given equation of hypotenuse and its coordinate of opposite vertices and the side makes  $45^\circ$  because right as well as right angle

so this is  $45^\circ$  slope of a b slope of a b equal to  $-\frac{3}{4}$  slope of a b equal to  $-\frac{3}{4}$  let slope of ac equal to  $m$  now this line ac and a b max angle  $45^\circ$

so  $\tan 45^\circ = \left| \frac{m - (-\frac{3}{4})}{1 + m(-\frac{3}{4})} \right| = 1$

so this implies  $1 = \left| \frac{m + \frac{3}{4}}{1 - \frac{3m}{4}} \right|$

so we can write it as  $1 = \frac{4m + 3}{4 - 3m}$

so  $4 - 3m = 4m + 3$  and  $4 - 3m = -4m - 3$

so this implies  $4m + 3 = 4 - 3m$  or  $4m + 3 = -4m - 3$

so this implies  $4m + 3 = 4 - 3m$  or  $4m + 3 = -4m - 3$

so this implies  $7m = 1$  or  $m = \frac{1}{7}$

so this implies  $m = \frac{1}{7}$  or  $m = -\frac{7}{1}$  we have two equation which passing through this point  $(2, 2)$

so equation of line with slope  $m = \frac{1}{7}$   $y - 2 = \frac{1}{7}(x - 2)$

so this implies  $7y - 14 = x - 2$

so  $x - 7y + 12 = 0$  again equation of line with slope  $m = -\frac{7}{1}$

so  $y - 2 = -7(x - 2)$  this implies  $y - 2 = -7x + 14$

so  $7x + y - 16 = 0$

so these are the two equation of line that is we have a c and a b we have if one diagonal of a square is along the line  $8x - 15y = 0$  and one of its vertex is at  $(1, 2)$  then find the equation of sides of square passing this vertex

so this problem is just like this problem we have discussed

so here this line this is hypotenuse with opposite vertex a is given

so we can find let us solve this problem

so slope of diagonal equal to  $\frac{8}{15}$  and let the slope of side a b equal to  $m$

so  $\tan 45^\circ = \left| \frac{m - \frac{8}{15}}{1 + m \frac{8}{15}} \right| = 1$  because you know that each angle of square is  $90^\circ$  and diagonal is angle bisector

so this is  $45^\circ$

so  $\tan 45^\circ = \left| \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}} \right| = 1$  say this is  $m_1$

so this implies  $1 = \left| \frac{m - \frac{8}{15}}{1 + \frac{8m}{15}} \right|$

so this implies  $1 = \frac{15m - 8}{15 + 8m}$  or  $15m - 8 = 15 + 8m$

so  $15m - 8 = 15 + 8m$  or  $15m - 8 = 15 + 8m$

so this implies  $7m = 23$  or  $m = \frac{23}{7}$

minus 15 minus 8 m

so this implies  $7m$  is equal to 23 or twenty three  $m$  is equal to minus seven  
so  $m$  is equal to twenty three by seven or  $m$  is equal to minus seven by twenty three  
equation of side of a square with slope  $m$  is equal to twenty three by 7  
and passing through a one two  $y - 2$  is equal to twenty three by seven  $x - 1$

so this implies  $7y - 14$  is equal to  $23x - 23$

so  $23x - 7y - 9$  equal to zero again equation of side with slope  $m$  equal to minus seven by twenty three and passing through a one two is  $y - 2$  equal to minus seven by twenty three and  $x - 1$  this implies  $23y - 46$  equal to  $-7x + 7$ .

this implies  $7x + 23y - 53$  equal to 0

so in this way we can find the equation of side a b and a d now another problem and this is most interesting problem find the image of point three eight with respect to line  $x + 3y = 7$  assuming the line to be a plane mirror equation of line  $x + 3y = 7$  this line we assume this line as a mirror and here this point is given  $p = (3, 8)$  we have to find the image of this point  $p$  say the image of this point  $p(3, 8)$  is  $q(\alpha, \beta)$  and let this  $p, q$  intersect this line at  $n$  and this point  $n$  is called foot of perpendicular

so we can solve this problem in various way but let us try to find the this point  $n$

so slope of  $l$  slope of  $l$  means given line is minus one by three

so slope of  $p, q$  which is perpendicular to  $l$  equal to minus minus one by three is equal to three because we know that the slope of perpendicular line related like  $m_1 m_2 = -1$

so slope of perpendicular line is negative reciprocal now we have two information for this line  $p, q$  slope is three and passing through three eight

so equation of line  $p, q$  with slope three and passing through  $p(3, 8)$   $y - 8$  is equal to  $3(x - 3)$

so  $3x - y - 1 = 0$   $3x - y - 1 = 0$

so say this is first and given equation of line  $x + 3y = 7$   
so this implies  $x = 7 - 3y$  put  $x = 7 - 3y$  in one

so  $3(7 - 3y) - y - 1 = 0$

so this implies  $21 - 9y - y - 1 = 0$

so  $-10y + 20 = 0$  this implies  $y = 2$

so  $x = 7 - 3(2) = 1$   $7 - 6 = 1$  means  $7 - 6 = 1$

so the coordinate of foot of perpendicular  $n$  is one two

so the coordinate of this foot of perpendicular is one two now this  $n$  is midpoint of this  $p, q$  this any midpoint of  $p, q$  it means this is  $p$  and this is  $q$   $q$  is  $(\alpha, \beta)$  and  $p$  is  $(3, 8)$  and this point  $n$  is  $(1, 2)$  which is midpoint of  $p, q$  since  $n$  is the midpoint midpoint of  $p, q$

so  $\frac{\alpha + 3}{2} = 1$  this implies  $\alpha + 3 = 2$  this implies  $\alpha = -1$  and  $\frac{\beta + 8}{2} = 2$  is equal to two this implies  $\beta + 8 = 4$

so this implies  $\beta = -4$

so image of point  $p(3, 8)$  with respect to a line  $x + 3y = 7$  equal to seven is  $q(-1, -4)$  in another way we can also find this image of point say this  $p, q$  the this line intersect at midpoint

so find the value of this  $n$  in terms of  $\alpha$  and  $\beta$  and put that value in this equation you will get the value of  $\alpha$  and  $\beta$  now another problem if

the area of the triangle formed by a line with coordinate axis is  $54\sqrt{3}$  square unit and the perpendicular drawn from the origin to the line makes an angle of  $60^\circ$  with x axis find the equation of the line we have to find the equation of this line a b this a b may intercept with x axis and y axis say this is a and b and this line form an angle say this is origin o a b and the area of this triangle o a b is given as  $54\sqrt{3}$  given area of triangle o a b is equal to  $54\sqrt{3}$  let the equation of line be  $x/a + y/b = 1$

so the coordinate of this a is a zero and coordinate of this b is zero b now we have two information one formation is given that perpendicular from origin makes angle  $60^\circ$  with the x axis it means  $\alpha$  is given

so we can use this line equation of line in normal form  $x \cos \alpha + y \sin \alpha = p$  p is length of perpendicular from origin or you can say distance of this line a b from origin

so this implies  $x \cos 60^\circ + y \sin 60^\circ = p$  this implies  $x \cdot \frac{1}{2} + y \cdot \frac{\sqrt{3}}{2} = p$  this implies  $x + \sqrt{3}y = 2p$  this is second

so compare on comparing one and two will have a equal to  $2p$  and b equal to  $2p\sqrt{3}$  given area of triangle o a b equal to  $54\sqrt{3}$  square unit

so for right hand angle triangle you know that  $\frac{1}{2} \times \text{base} \times \text{height}$

so  $\frac{1}{2} \times a \times b = 54\sqrt{3}$

so this implies  $\frac{1}{2} \times 2p \times 2p\sqrt{3} = 54\sqrt{3}$  this implies  $2p^2 = 54$  this implies  $p^2 = 27$  this implies  $p = \pm 3\sqrt{3}$  question is we have to find the equation of line

so now for this equation we have length of perpendicular is known and this  $\alpha$  is known

so equation of line equation of line with  $\alpha = 60^\circ$  and  $p = 3\sqrt{3}$  is  $x \cos \alpha + y \sin \alpha = p$  this implies  $x \cos 60^\circ + y \sin 60^\circ = 3\sqrt{3}$  or  $x \cos 60^\circ + y \sin 60^\circ = 3\sqrt{3}$

so this implies  $x \cdot \frac{1}{2} + y \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$  and  $x + \sqrt{3}y = 6\sqrt{3}$

so  $x + \sqrt{3}y - 6\sqrt{3} = 0$  or  $x + \sqrt{3}y + 6 = 0$  another problem that is find the distance of the point p (4, 1) from the line  $4x - y = 0$  measured along the line making an angle of  $35^\circ$  with the positive direction of x axis what is given equation of this line is given this l is given  $4x - y = 0$  and we have to find the distance of this line from this point p (4, 1) situated on this line say this line is l one which makes angle  $35^\circ$  in the positive direction of x axis slope of given equation of line given equation of line  $4x - y = 0$  this is line l say this is equation one also given line l one makes angle  $35^\circ$  with x axis in positive direction

so slope of l one is equal to  $\tan 35^\circ$  it means  $m = 1$  now for this line we have two information that is its slope is  $m = 1$  and this line passing through point p (4, 1)

so equation of line l one with slope  $m = 1$  and passing through p (4, 1) is  $y - 1 = 1(x - 4)$  this implies  $y - 1 = x - 4$  or  $x - y + 3 = 0$

so  $x - y + 3 = 0$  say this is line two and line one is what  $4x - y = 0$  is line one

so this implies  $y = 4x$

so put  $y$  equal to  $4x$  into  
 so  $x + 4x - 5 = 0$   
 so this implies  $5x = 5$   
 so this implies  $x = 1$   
 so  $y = 4$  it means we get the point of intersection this point  $q$  as  $(1, 4)$   
 now question is find the distance of point  $p(4, 1)$  from the line  $4x - y = 0$  measured along the line making an angle of  $35^\circ$  with the positive direction of  $x$ -axis means we have to find the distance of this line from along with this line  
 so we just find the distance between this  $p$  and  $q$   
 so distance of line  $l$  from  $p(4, 1)$  along the line  $l$  is  $pq \sin 35^\circ$   
 so  $pq = \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2}$   
 so distance of this line from this point along this line is  $3\sqrt{2} \sin 35^\circ$  units now  
 so if the line  $3x + y - 2 = 0$ ,  $px + 2y - 3 = 0$  and  $2x - y - 3 = 0$  are concurrent find the value of  $p$  since we have three equations of line is given and these three are lines are concurrent concurrent means these three lines passing through one point  
 so different line three or more than three lines passing through same point is called concurrent length this  $c$  line passing through this point then we have to find the value of  
 so if any three lines say  $a_1x + b_1y + c_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ ,  $a_3x + b_3y + c_3 = 0$  are concurrent if these  $c$  lines are concurrent then  $a_1 b_2 c_3 - a_2 b_3 c_1 - a_3 b_1 c_2 = 0$   
 we just try to expand this quantity known as determinant when we have to find the value of this determinant we expand and we follow this sign rule  
 so  $a_1 a_2 a_3$  then  $b_2 c_3 - b_3 c_2 - b_1 c_3 + b_3 c_1 - b_2 c_1 + b_1 c_2$   
 means  $a_2 c_3 - a_3 c_2 - a_1 c_3 + a_3 c_1 - a_2 c_1 + a_1 c_2$   
 so  $a_2 b_3 - a_3 b_2 = 0$  now come to the problem we have given these three equations of line  
 so given equation of line lines are  $3x + y - 2 = 0$ ,  $px + 2y - 3 = 0$  and  $2x - y - 3 = 0$   
 so given since say this is 1, 2 and say this is 3 since given lines one, two and three are concurrent  
 so  $3 \cdot 1 \cdot 2 - p \cdot 2 \cdot 2 - 2 \cdot 2 \cdot 1 - 3 \cdot 2 \cdot 1 - 3 \cdot 1 \cdot 2 + 3 \cdot 2 \cdot p = 0$  this implies  $3 \cdot 2 - 4p - 4 - 6 - 6 + 6p = 0$   
 into  $6 - 4p - 4 - 6 - 6 + 6p = 0$  this implies  $2p - 10 = 0$   
 so  $2p = 10$   
 so  $p = 5$   
 so this implies  $6 - 4(5) - 4 - 6 - 6 + 6(5) = 0$  this implies  $6 - 20 - 4 - 6 - 6 + 30 = 0$   
 so this is  $6 - 20 - 4 - 6 - 6 + 30 = 0$   
 so  $p = 5$   
 so in this way we can find the value of unknown quantity if equations of lines are concurrent by using this condition  
 so ok we will discuss some other problems in next session thank you