

welcome today we are going to discuss about the distance of a point from a line

So this is lecture number four

So distance of a point from a line say $ax + by + c = 0$ is a line and (x_1, y_1) is a point this line $ax + by + c = 0$ cut axis at point a and b

So $ax + by + c = 0$ we can reduce this equation in intercept form then we can write it as $x \cos \alpha + y \sin \alpha = p$

So $a = -c/b$ means x intercept equal to $-c/b$ and y intercept means $b = -c/a$

So the coordinate of this point a is $(-c/a, 0)$ and coordinate of this point b is $(0, -c/b)$ the distance of av is equal to $\sqrt{(-c/a)^2 + (-c/b)^2}$

So this is $\frac{c^2}{a^2} + \frac{c^2}{b^2}$ is equal to $\frac{c^2}{a^2 + b^2}$ square root of $a^2 + b^2$

So $v = \frac{c}{\sqrt{a^2 + b^2}}$ into square root of $a^2 + b^2$ now in this figure draw pn perpendicular to given line that is pn perpendicular to ab and say the length of this perpendicular $pn = d$

So let the length of this pn equal to d

So area of this triangle pab area of this triangle pab is equal to $\frac{1}{2} ab \sin \theta$

So area of triangle pab is equal to $\frac{1}{2} ab \sin \theta$ it means $\frac{1}{2} ab \sin \theta = \frac{1}{2} ab \frac{c}{\sqrt{a^2 + b^2}}$ area triangle pab can be find in this way and we can also find area of triangle by using vertices of this triangle pav

when you use this vertices of triangle pav

So again we have area of triangle pab is equal to $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ and $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = \frac{1}{2} |(-c/a)(0 - (-c/b)) + (-c/b)(-c/a - (-c/a)) + (-c/a)(-c/b - (-c/a))|$ and this is zero when we simplify it will get area of triangle pab is equal to $\frac{1}{2} |(-c/a)(c/b) + (-c/b)(-c/a + c/a) + (-c/a)(-c/b + c/a)|$

So from one and two from one and two half into $\frac{1}{2} ab \frac{c}{\sqrt{a^2 + b^2}} = \frac{1}{2} |(-c/a)(c/b) + (-c/b)(-c/a + c/a) + (-c/a)(-c/b + c/a)|$

So half half cancel c/a b/c y/v conceal

So $d = \frac{c}{\sqrt{a^2 + b^2}}$

So in this way we can find distance of any point from the line distance between two parallel lines

So here we have two line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$

So these two line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ are parallel line because when you find slope of these two line you will get slope of first line is $-a/b$ and slope of second line is also $-a/b$

So slope of these two lines are equal

So these two lines are parallel lines

So this is line one and this is line two

So put $x = 0$

So we will get $by + c_1 = 0$ implies $y = -c_1/b$ it means this point p

So we have a point $p(0, -c_1/b)$ on the line one it means this line intersect the y axis at this point now find the distance of this point p from the second line

So distance of p on one from line two

So say this distance is d the distance of this point p from the line 2 is t

So $d = \frac{|a(0) + b(-c_1/b) + c_2|}{\sqrt{a^2 + b^2}} = \frac{|-c_1 + c_2|}{\sqrt{a^2 + b^2}}$

So $d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$ yes you can write it as $\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$ and this d is nothing but the distance between these two parallel line

So we can find distance between any two parallel line by using this formula now let us take some example based on these two concept

So find the distance of the point $(-2, -3)$ from the line $12x - 5y = 2$

So $(-2, -3)$

So this is point $(-2, -3)$ and $12x - 5y = 2$ we have to find the

distance of this point distance is say this line is twelve x minus five y equal to two
 So this line will be if you try to draw this line roughly
 So put x equal to 0 will give y equal to minus two by five
 So say this point is like minus two by five
 So this point is here minus two by five
 So this way point is somewhere here and put y equal to zero implies x equal to six
 So x equal to one by six one by six
 So this point is one by six means say this point is something here
 So point line is passing like this
 So we have to find distance of this point from this line means this d
 So use formula d is equal to a x 1 plus b y one plus c by square root of a square plus b
 s square here a is equal to here a is equal to 12 and b is equal to minus 5 and c is
 equal to minus 2 and x 1 y 1 x 1 is equal to minus 2 and y 1 is equal to 3
 So now put this value on the equation on the formula d is equal to mod 12 into minus 2
 plus minus 5 into 3 and minus 2 by square root of 12 square plus minus 5 square
 So this is 20 minus 24 minus 15 and minus 2 by 144 plus 25 means 169
 So this is 24 and 39 41 this is 41 by 13 units
 So this is a distance of this point minus two three from this line twelve x minus five y
 equal to two in this way we can use this formula now we have another example find the
 distance between the line three x plus four y equal to nine and six x plus eight y equal
 to fifteen
 So given line three x plus four y equal to nine and
 So 3 x plus 4 y minus 9 equal to 0 another line is 6 x plus 2y equal to 15 we can write
 this line as take 2 common 3x plus 4y equal to 15.
 So three x plus four y minus fifteen by two equal to zero this minus nine this is c one
 and this minus fifteen by two is c two
 So distance between two parallel these two lines are parallel line because three x plus
 four by three x plus four y
 So its slope is equal
 So these two lines are parallel coefficient of x and y in both equation are equal then
 that two line will be parallel line
 So here c one equal to minus sign and c two equal to minus fifteen by two
 So we know that distance between two parallel line is mod c two minus c one you can write
 mod c one minus c two not no problem root under a square plus b square
 So this is minus fifteen by two and minus nine minus minus plus nine modulus by square
 root of a square a square means three a square three a square you can write it again mod
 minus 15 by 2 plus 9 by square root of three s square plus four s square
 So d is equal to mod minus fifteen plus eighteen by 2 and this is square root of 25
 So this is 3 by 2 by 5 means three by ten units
 So this is a distance between two parallel in this way we can find distance between any
 two parallel lines now another problem good problem if the equation of the base of an
 equilateral triangle is x plus y minus six equal to zero say this equation x plus y minus
 six equal to zero this is the equation of base b c and the opposite vertex is the point
 minus one one minus one minus one
 So a is upward opposite vertex of this base b then find the area of triangle means a b c
 since this is equilateral triangle
 So this angle is every angle is 60 degree draw a perpendicular from this a to this base b
 say this is this a n and the length of this altitude or perpendicular is d and this point
 is and see the side of this equilateral triangle is a find the relation between these two
 d and a in triangle a b n a b n angle b and a is equal to ninety degree
 So sine sixty degree is equal to p by h means sine 60 d equal to d by k and sine 60
 degree equal to root 3 by 2 equal to d by a this implies d is equal to root three by two
 a
 So this is a relation between this a and d we have to find the area of this triangle
 So to find the area of this triangle
 So find the equation of this a n equation of this a n
 So slope of slope of b c is equal to minus one slope of a n is equal to one because a n
 perpendicular to b c
 So equation of a n equation of n means equation of this a n whose slope is 1 and passing
 through minus 1 minus 1 is y plus 1 equal to 1 x plus 1
 So is equal to x minus y equal to zero x minus y equal to zero

So this is equation $x - y = 0$ say this is equation two and $d = \sqrt{3}$ by $2a$ say this is 1 and solve this equation 2 with given equation $x + y - 6 = 0$ this is equation three

So from second and third

So $y = x$

So $x + x = 6$ this implies $x = 3$. $x = 3$ and

So $y = 3$ also

So distance between two points

So $a = \sqrt{3^2 + 1^2} = \sqrt{10}$ now in this triangle d is equal to what d is equal to $4\sqrt{2}$ $d = \sqrt{3}$ by $2a$ from 1 implies $4\sqrt{2} = \sqrt{3}$ by $2a$ this implies $a = \frac{\sqrt{3}}{2\sqrt{2}}$

So area of triangle abc and abc equilateral triangle

So $\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} (\frac{\sqrt{3}}{2\sqrt{2}})^2 = \frac{\sqrt{3}}{4} \cdot \frac{3}{8} = \frac{3\sqrt{3}}{32}$ square unit

So in this way we can find the area of this triangle abc we can also find the distance of

this or d by using distance formula now let us try by using distance formula

So we have given in this triangle abc $a = 1$ and since this triangle abc is equilateral triangle

So this is 60° an equation of this bc is given $x + y - 6 = 0$ and draw this perpendicular say this is a and the length is perpendicular is d and the side of this equilateral triangle is a

So $d = \frac{a \sin 60^\circ}{\sin 30^\circ} = \frac{1 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

So this is $\frac{\sqrt{3}}{2}$ by $\sqrt{2}$ is equal to $\frac{\sqrt{3}}{2\sqrt{2}}$ or you can write it as $\frac{\sqrt{3}}{2\sqrt{2}}$

So this is a distance of or length of this perpendicular d and in triangle abn because n is 90°

So $\sin 60^\circ = \frac{d}{a}$ we can do like this $\sin 60^\circ = \frac{d}{a}$

So $\sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{d}{2a}$

So this is $\frac{\sqrt{3}}{2} = \frac{d}{2a}$

So $a = \frac{2d}{\sqrt{3}}$

So area of triangle abc is equal to $\frac{1}{2} a d = \frac{1}{2} \cdot \frac{2d}{\sqrt{3}} \cdot d = \frac{d^2}{\sqrt{3}}$ means $8\sqrt{2}$ by $\sqrt{3}$ and d means $\sqrt{3}$ this is $16\sqrt{2}$ by $\sqrt{3}$ square unit

another problem if p, q are the length of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k$ and $x \sin \theta + y \cos \theta = k$ respectively prove that $p^2 + 4q^2 = k^2$

So given equation $x \cos \theta - y \sin \theta = k$

So $x \cos \theta - y \sin \theta - k = 0$

So distance of this line from origin and this is given as p

So $p = \frac{|0 \cos \theta - 0 \sin \theta - k|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = \frac{k}{1} = k$

So $p = k$ is equal to $\frac{0 \cos \theta - 0 \sin \theta - k}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = \frac{-k}{1} = -k$ but distance is positive so $p = k$

So this is $k \cos 2\theta$ again another equation of line is given $x \sec \theta + y \operatorname{cosec} \theta = k$ we can write it as $x \cos \theta + y \sin \theta = k$

So this can be written as $x \sin \theta + y \cos \theta = k$ and this can be written as $x \sin \theta + y \cos \theta = 1$ by $\frac{k}{2} \cdot 2$

So this is $\frac{k}{2} \sin 2\theta$ means we can write it as $x \sin \theta + y \cos \theta - \frac{k}{2} \sin 2\theta = 0$ say this line is two this line two from origin is given as according to question q

So q is the distance of second line from origin

So $q = \frac{|0 \sin \theta + 0 \cos \theta - \frac{k}{2} \sin 2\theta|}{\sqrt{\sin^2 \theta + \cos^2 \theta}} = \frac{\frac{k}{2} \sin 2\theta}{1} = \frac{k}{2} \sin 2\theta$

So we will get $q = \frac{k}{2} \sin 2\theta$ because $\sin^2 \theta + \cos^2 \theta = 1$ now we have to show that $p^2 + 4q^2 = k^2$

So $p^2 + 4q^2 = k^2 + 4 \left(\frac{k}{2} \sin 2\theta\right)^2 = k^2 + k^2 \sin^2 2\theta = k^2 (1 + \sin^2 2\theta)$

So $k^2 \cos^2 2\theta + q^2$ is equal to $k^2 \sin^2 2\theta$
 So $4k^2 \sin^2 2\theta + q^2 = k^2 \cos^2 2\theta + 4k^2 \sin^2 2\theta$
 So $k^2 \cos^2 2\theta + q^2 = k^2 \sin^2 2\theta + 4k^2 \sin^2 2\theta$ because
 $4k^2 \cos^2 2\theta$ cancel
 So this is k^2
 So $p^2 + 4q^2 = k^2$ another problem find the equation of
 line which is equidistant from the parallel line $3x + 2y + 6 = 0$ and
 $9x + 6y - 7 = 0$
 So given lines $3x + 2y + 6 = 0$ and second line is this is first
 line second line is $9x + 6y - 7 = 0$
 So take common three
 So $3x + 2y - 7 = 0$ in line first and second the
 coefficient of x and y are equal
 So these two lines are parallel lines now we have to find say these two lines are
 this is line one and this is line two we have to find the equation of this line which is
 equidistant and see if this is d then this is also d we have to find equation of
 this line means say this line third we have to find equation of line third
 So we have we can find it in many way but any equation is parallel to given equation of
 line then we can write it as $3x + 2y + k = 0$ say this line is parallel to
 line 1 this line is parallel to line one or you can say this is family of parallel
 lines which is parallel to line one we have to find the value of this k since this line
 $3x + 2y + k = 0$
 So we will get $y = \frac{-k - 3x}{2}$
 So say this point p the coordinate of this point p $0, \frac{-k}{2}$ it means according to
 question it is given that this line is equal distance from line in one and two
 So distance this distance and this distance must be equal and this is line $3x + 2y + 6 = 0$
 and this is line $9x + 6y - 7 = 0$
 So according to question this d_1 and d_2 are equal $d_1 = d_2$ this implies
 $\frac{3 \cdot 0 + 2 \cdot \frac{-k}{2} + 6}{\sqrt{3^2 + 2^2}} = \frac{9 \cdot 0 + 6 \cdot \frac{-k}{2} - 7}{\sqrt{9^2 + 6^2}}$
 $\frac{-k + 6}{\sqrt{13}} = \frac{-3k - 7}{\sqrt{117}}$
 So $3(-k + 6) = -3k - 7$
 $-3k + 18 = -3k - 7$
 $18 = -7$
 So these two you can cancel it
 So this implies $-k + 6 = -k - 7$
 we can write it as $k + 7 = -k - 7$
 $2k = -14$
 $k = -7$
 So $-k + 6 = 7 + 6 = 13$ implies $-k + 6 = 13$
 $-k = 7$ or $k = -7$ which is not
 possible
 So this real is not valid result
 So we can consider only one result
 So this is $-k + 6 = -3k - 7$
 $2k = -13$
 $k = -\frac{13}{2}$
 So $k = -\frac{13}{2}$
 So equation of line $3x + 2y + k = 0$ it means $3x + 2y + 11 = 0$
 $6x + 4y + 11 = 0$ it means $18x + 12y + 11 = 0$ will be the line which is
 equidistant from the given 2 line now we have another example that is find the equation
 of straight line which are perpendicular to the line $12x + 5y = 70$
 and a distance of two unit from the point $(-4, 1)$
 So given equation of line $12x + 5y = 70$
 So equation of line perpendicular to line one is $5x - 12y + k = 0$
 according to question say this is a line $5x - 12y + k = 0$
 and a point $p(-4, 1)$ is given and the distance of this point p from this line is
 two unit
 So by using distance formula $\frac{5(-4) - 12(1) + k}{\sqrt{5^2 + 12^2}} = 2$
 $\frac{-20 - 12 + k}{13} = 2$
 $k - 32 = 26$
 $k = 58$
 So $5x - 12y + 58 = 0$

equal to twenty six

So minus eight plus k is equal to plus minus twenty six

So this implies k is equal to eight plus minus twenty six

So k is equal to thirty four and minus eighteen

So equation of line equation of required line will be five x minus 12 y plus 34 equal to 0 or $5x - 12y - 18 = 0$ now another example in the triangle with vertex a two three b four minus one

So we have one triangle is given a two three b four minus one and c minus 1 2 find the equation and length of altitude from vertex a we have to find

So slope of b c is equal to slope of b c is equal to two plus one by minus 1 and minus 4 is equal to minus 3 by 5 minus three by five

So slope of a n since n is perpendicular to b c

So slope of n is five by three

So equation question is find the equation of altitude

So equation of a n equation of n is y minus three equal to five by three x minus 2

So implies 3 y minus 9 equal to 5 x minus 10

So 5 x minus 3 y and minus 1 equal to 0 now we have to find the equation length of this altitude n

So equation of b c equation of b c

So just consider one point that is b four one four minus one

So y plus one equal to and slope of b c is minus three by five

So minus 3 by 5 and x minus 4

So this is 5y plus five is equal to minus three x plus twelve is equal to three x plus five y and minus 7 equal to 0

So a n equal to n equal to mod 3 into 2 plus 5 into 3 minus 7 by square root of three

square plus five square is equal to six plus fifteen twenty one minus seven 14 mode 14 by 25 plus 9 square root of 34 is equal to 14 by square root of 34 units

So in this way we can find the length and equation of altitude of any triangular vertices are given

So ok now we shall discuss next concept in another session okay thank you you