

welcome student now we continue straight line and this is third lecture now we just try to reduce the general form of a straight line equation $ax + by + c = 0$ to other forms that form may be direction to slope intercept form second it may be reduction to intercept form and third it may be reaction to normal form

so first reduction to slope intercept form

so equation in general form is $ax + by + c = 0$ slope intercept form is what this is $y = mx + c$

so this can be written as $by = -ax - c$ this implies $y = \frac{-a}{b}x - \frac{c}{b}$ now you compare this equation with $y = mx + c$

so here this m is equal to $-\frac{a}{b}$ and c is equal to $-\frac{c}{b}$ it means this line $ax + by + c$ having slope $-\frac{a}{b}$ and intersect y axis at $-\frac{c}{b}$

so in this way we can reduce any general equation in $y = mx + c$ form now next reduction that is reduction to intercept form again we have equation in general form is $ax + by + c = 0$

so intercept form is what this is $\frac{x}{a} + \frac{y}{b} = 1$

so we write it as $ax + by = -c$ now in right side we have one only

so divide both side by $-c$

so when you divide both sides by $-c$ we will get $\frac{-a}{-c}x + \frac{-b}{-c}y = 1$ now we arrange this equation like this $\frac{-a}{c}x + \frac{-b}{c}y = 1$

so when you compare this equation with $\frac{x}{a} + \frac{y}{b} = 1$ you will get $a = \frac{-c}{-a}$ and $b = \frac{-c}{-b}$ it means by reducing this equation $ax + by + c = 0$ in this form we get this line intersect x axis at $-\frac{c}{a}$ and y axis at $-\frac{c}{b}$ it means x intercept $-\frac{c}{a}$ y intercept $-\frac{c}{b}$

so this is the benefit of reduction of $ax + by + c = 0$ into $\frac{x}{a} + \frac{y}{b} = 1$ now third one is very important that is how to reduce $ax + by + c = 0$ in normal form normal form means $x \cos \alpha + y \sin \alpha = p$ means we can write it as $-p = 0$ now compare these two equation

so when you compare these two equation we will get $a \cos \alpha = b \sin \alpha$ and $-c = p$ let this is equal to k

so $a \cos \alpha = k$ $b \sin \alpha = k$ and $-c = p$ or $p = -c$ squaring these two and add

so $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = k^2$ as $\sin^2 \alpha + \cos^2 \alpha = 1$ $k^2 = \frac{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}{\cos^2 \alpha + \sin^2 \alpha}$ $k = \pm \sqrt{\frac{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}{\cos^2 \alpha + \sin^2 \alpha}}$ now we have $c = -p$

so $c = -p$ $c = -p$ this implies $p = -c$ $p = -c \pm \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}$ so we have two case

so case one when $c < 0$ then $p = c + \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}$ and second when $c > 0$ then $p = c - \sqrt{a^2 \cos^2 \alpha + b^2 \sin^2 \alpha}$ but this is positive

so in this way we can reduce $ax + by + c = 0$ in normal form and this p is nothing but the distance of line from origin this p will give the distance of line from using

so finally we reduce $ax + by + c = 0$ implies $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = 0$ $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm p$

square y is equal to root under a square plus b square c by root under a square plus b a square

so this is the reduction of general equation in normal form now we have some example on various forms of equation of straight lines

so first problem is find the equation of straight line through the point minus one to making an angle of one thirty five degree with the x axis

so this line making angle one thirty five d with x axis

so given theta equal to 135 degree

so this implies m is equal to $\tan \theta$ means $\tan 135$ degree and $\tan 135$ degree is equal to minus one

so slope of this line is minus one and since this line is passing through minus one two

so line passing through minus one minus two $y - y_1 = m(x - x_1)$ means by using point slope form implies $y + 2 = -1(x + 1)$.

so $x + y + 3 = 0$ is a required equation of line now find the equation of line passing through two three and making equal intercept on the coordinate axis means the situation is like this

so this is x axis this is y axis this is zero we have to find the equation of this line which makes equal intercept this is a and this is b means coordinate of this point is $(a, 0)$ and this point is $(0, b)$ and this line passing through some point p to what will be the equation of this line

so since intercept is equal

so we use intercept form let intercepts are a and b according to question

so equation of line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ this implies $x + y = a$ now this line $x + y = a$ passing through two three

so say this is line 1

so line 1 passing through two three it means this point must satisfy the equation of line

so $2 + 3 = a$ this implies $a = 5$.

so this line makes equal intercept $a = 5$ and $b = 5$

so equation of line is $\frac{x}{5} + \frac{y}{5} = 1$ this implies $x + y = 5$.

now another example that is find the equation of line passing through the point one two and zero five given line l passing through say a one two and b zero five

so equation of line a b

so equation of line a b by using two point form or you can find slope of this line

so just find slope of line

so slope of line a b $\frac{y_2 - y_1}{x_2 - x_1}$ means $\frac{5 - 2}{0 - 1} = \frac{5 - 2}{-1}$

so zero minus one means minus three now you just take either a one two or b zero five

so say this line passing through a one two

so equation of line a b a b passing through one two with slope minus three is again point slope form

so $y - y_1 = m(x - x_1)$

so $y - 2 = -3(x - 1)$

so this implies $3x + y - 5 = 0$.

so in this way we can find equation of line passing through any two points now another example determine the equation of line with $\alpha = 135$ degree

and perpendicular distance p equal to $\sqrt{2}$ from the origin here well ϕ is what α is we have to find the equation of this line and what is given this p is given and this α is given this two information is given and this is 90° p means normal or perpendicular from origin to the line l

so given α is equal to 135° and p is equal to $\sqrt{2}$

so equation of line $x \cos \alpha + y \sin \alpha = p$ means $x \cos 135^\circ + y \sin 135^\circ = \sqrt{2}$

so $x \cos 135^\circ + y \sin 135^\circ = \sqrt{2}$

so this is $-\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$

so this implies $-x + y = 2$

so $x - y + 2 = 0$ is the equation of line when these two

information is given now another example find the angle between the line when the equation of two line is given as $3x + y - 7 = 0$ and $x + 2y + 9 = 0$

so given l_1 line one is $3x + y - 7 = 0$ and l_2 $x + 2y + 9 = 0$

so slope is what

so m_1 is equal to m_2 is equal to -3 and m_2 is equal to $-\frac{1}{2}$ and just reduce this two equation in slope intercept form and we will get the value of purpose of this problem is how to reduce and get the value of m_1 and m_2 and when we get the value of m_1 and m_2 it is very easy let θ be the angle between angle between line l_1 and l_2

so $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ is equal to $\left| \frac{-3 - (-\frac{1}{2})}{1 + (-3)(-\frac{1}{2})} \right|$

so $\tan \theta = \left| \frac{-\frac{5}{2}}{1 + \frac{3}{2}} \right| = \left| \frac{-\frac{5}{2}}{\frac{5}{2}} \right| = 1$

so $\tan \theta = 1$ when you open the mod we will get $\tan \theta = 1$

so this sign plus will give the acute angle and this minus sign will give the obtuse angle the angle between

so when $\tan \theta = 1$ this implies $\theta = \frac{\pi}{4}$ and when $\tan \theta = -1$ then $\theta = \frac{3\pi}{4}$

so we just find the value of acute angle will give the value of obtuse angle also

so in this way we can find the angle between two line now another example is we have to reduce this equation $x + 3y + 4 = 0$ perpendicular for enhance find the length of the perpendicular from the origin on the straight line

so given equation $x + 3y + 4 = 0$

so a is equal to 1 b is equal to 3 and c is equal to 4

so length of perpendicular first of all reduce the equation in normal form

so $a^2 + b^2 = 1 + 9 = 10$

so $\sqrt{a^2 + b^2} = \sqrt{10}$ since c is positive since c is greater than 0 this implies $\sqrt{a^2 + b^2} = \frac{c}{\sqrt{a^2 + b^2}}$

so $\frac{x + 3y + 4}{\sqrt{10}} = \frac{4}{\sqrt{10}}$

so $x + 3y + 4 = 4$ this implies the value of a is 1 b is 3 and the value of c is 4

so this is what $x \cos \frac{\pi}{3} + y \sin \frac{\pi}{3} = 4$

so α is equal to $\frac{\pi}{3}$ and p is equal to 4

so this is what this is distance of line perpendicular distance of line from origin

so in this way we can reduce any equation in normal form now another example find the equation of straight line which passes through the point minus one three and is perpendicular to the line four x plus three y plus one equal to zero it means we have to find the equation of line say this is a line

so this is a line and we have to find the equation of this line which passing through p minus one three and perpendicular to this line four x plus three y plus one equal to zero say this line is l or say l 1 and the line which equation we have to find say this is l 2

so slope of l 1 one given equation of line given equation of line l 1 one four x plus three y plus one equal to zero

so slope of l 1 one slope of l 1 one that is m 1 is equal to minus four by three minus four by three since l 1 one perpendicular to l 2 according to question

so slope of it means say the slope of this line is m 1 and slope of this line is m 2

so these two lines are perpendicular

so m 1 cross m 2 is equal to minus one this implies m 2 is equal to minus one by m 1 means three by four now for this line l 2 we have slope is known and this line passing through p minus one two

so equation of line l 2 passing through p minus one three is y minus y 1 any y minus 3 equal to 3 by 4 x plus 1 this implies 4y minus 12 equal to 3x plus 3 this implies 3x minus 4y plus fifteen equal to zero

so in this way we can find the perpendicular line to the given line we can also find the line parallel to given line by using the concept when line is parallel to given length then slope is equal and rest is same now find the equation of line which passes through the point three one and bisect the portion of line three x plus four y equal to twelve intercepted between coordinate axis

so we have to find the equation of line which passes through the point three one and bisect the portion of line means this line is three x plus four y plus twelve three x plus four y equal to twelve this line is say this line is l 1 one is three x plus four y equal to twelve the portion of line between coordinate axis is a b

so given line three x plus four y equal to twelve

so reduce this line in intercept form

so when you reduce this line in intercept form divide both side by twelve

so when you divide both side by twelve it means this is x by four plus y by three equal to one it means a is equal to what a is equal to four zero and b is equal to zero three we have to find the equation of say this line which through the point three one passes through the point say this point is three one and bisect the portion of line it means this point say this point is say q this q q q is midpoint of heavy this q is

so according to question p q r lens say like this is line l 2 line l 2 bisect bisect l 1 one at q

so q is the midpoint

so q is the midpoint of a b

so coordinate of q is what coordinate of q is 4 plus 0 by 2 and 0 plus 3 by 2 that is q two three by two the equation of line l 2 can be easily find because we have two points of this line is known that is q two three two and p three one now p q now line l 2 that is p q passing through p three one and q two three by two slope of p q y two minus y one means three by two minus one by two minus three

so this is one by two and minus one means minus one by two

so equation of line equation of p q y minus 1 minus 1 by 2 x minus 3 implies 2 i minus 2 equal to minus x plus 3

so x plus two y minus five equal to zero next problem by using the concept of

equation of line prove that three point three zero minus two minus two and a two are collinear

so say the three points are given three points a three zero b minus two minus two and c a two

so first of all find the equation of line a b

so equation of a b equation of a b $y - 0 = \frac{0 - 2}{3 - 0}(x - 3)$ then this is slope of line a b

so $y - 0 = \frac{2}{3}(x - 3)$ implies $3y = 2x - 6$ this implies $2x - 3y - 6 = 0$ now check whether this point c i two satisfy this equation or not

so put $x = 2$ and $y = 2$ in the equation of line

so $2 \times 2 - 3 \times 2 - 6 = 4 - 6 - 6 = -8 \neq 0$ hence c a two satisfied the equation

so c a two lies on the line a b

so c a two lies on the line it means all these three points abc are collinear

so in this way we can check whether the line whether the points are collinear or note by using the concept of equation of line now find the equation of line passing through 1 2 and making an angle 30 degree with the y axis

so what is given this is x this is the y axis and this is zero

so this line makes angle 30 degree with the y axis it means situation is like this

so this angle is what this angle is 30 degree line all max 30 degree with y axis

so if this angle is 30 degree it means this angle is also 30 degree

so this angle is what this angle is 60 degree

so this angle is sixty degree

so this line l makes sixty degree with the x axis

so this implies l max 60 degree with x axis

so slope equal to $\tan 60^\circ$ means $\sqrt{3}$

so equation of line

so equation of l which passing through p one two l passing through p 1 2 is $y - 2 = \sqrt{3}(x - 1)$

so this implies $\sqrt{3}x - y - \sqrt{3} + 2 = 0$

so $\sqrt{3}x - y + 2 - \sqrt{3} = 0$ here two minus $\sqrt{3}$ the value of c

so this is the equation of line for the given formation now another problem the perpendicular from the origin to the line $y = mx + c$ meet it at the point $(-\frac{c}{m^2+1}, \frac{c}{m^2+1})$ find the value of m and c

so again this is situation like this this is say l $y = mx + c$ and meet at perpendicular

so this is origin this is y axis x axis

so perpendicular from this origin this meet at $(-\frac{c}{m^2+1}, \frac{c}{m^2+1})$

so this information is given and we have to find the value of this m and c

so slope of o p is equal to $\frac{y_2 - y_1}{x_2 - x_1}$ means say zero minus two by zero plus one is equal to minus two equal to my slope of this o p is minus two

so equation of slope of is minus 2 since this o p is perpendicular to l since o p is perpendicular to l that is $y = mx + c$

so slope of l is equal to minus one by say this is m one m one

so minus one by m one is equal to one by two and say this slope of l is m two

so equation of line l $y - 2 = -2(x - 1)$ because this slope of line l is one by two and this line passing through p minus one two

so this implies $2y - 4 = -2x + 2$ this implies $2y = -2x + 6$

equal to x plus five this implies y is equal to one by two x plus five by two
now compare this with y equal to $m x$ plus c

so you this implies m is equal to one by two and c is equal to five by two
so in this way we can find the value of m and c of the line l now another
problem that is the point p $1\ 2$ and r $0\ -1$ are the two opposite vertices of
rhombus $p\ q\ r\ s$ find the equation of the diagonal $q\ s$

so vertices of rhombus is given that is only two vertices given lets say
so rhombus is a parallelogram who all the sides are equal now we have given $p\ q$
 $r\ s$

so p $1\ 2$ and r $0\ -1$ and we have to find the equation of diagonal $q\ s$
equation of diagonal $q\ s$ equal to what we know that the diagonal of rhombus
bisect each other and perpendicular to each other means diagonal of diagonals of
rhombus are perpendicular bisector of each other it means this angle is 90
degree and this point say o is the midpoint of both $p\ r$ and $q\ s$ and this o
obviously lies on both diagonals in the figure o is the midpoint of $p\ r$

so coordinate of o is one plus zero by two and two minus one by two that is one
by two one by two one information about this diagonal $q\ s$ is now known that one
point one by two one by two lies on this $q\ s$ now slope of this $p\ r$ slope of $p\ r$
say this is m is equal to y two minus y one

so say two plus one by one minus zero

so slope of $p\ r$ is three since this $p\ r$ is perpendicular to $q\ s$

so slope of $q\ s$ is equal to minus one by three this diagonal $q\ s$ having slope
minus one by three and passing through point one by two one by two

so this point is one by two one by two

so equation of diagonal $q\ s$ y minus one by two minus one by three x minus one
by two implies implies

so this is two i minus one by two equal to minus one by three two x minus one
by two two two cancel

so this is what this is six y minus $6y$ minus $6y$ plus 3 minus $6y$ plus 3 is equal
to $2x$ minus one this implies two x plus six y and minus four equal to zero and
when you simplify it you will get x plus $3y$ minus 2 equal to 0 will be the
equation of diagonal of diagonal $q\ s$ ok we discuss in another section ok thank
you you